Coexistence of Ferromagnetism and Superconductivity

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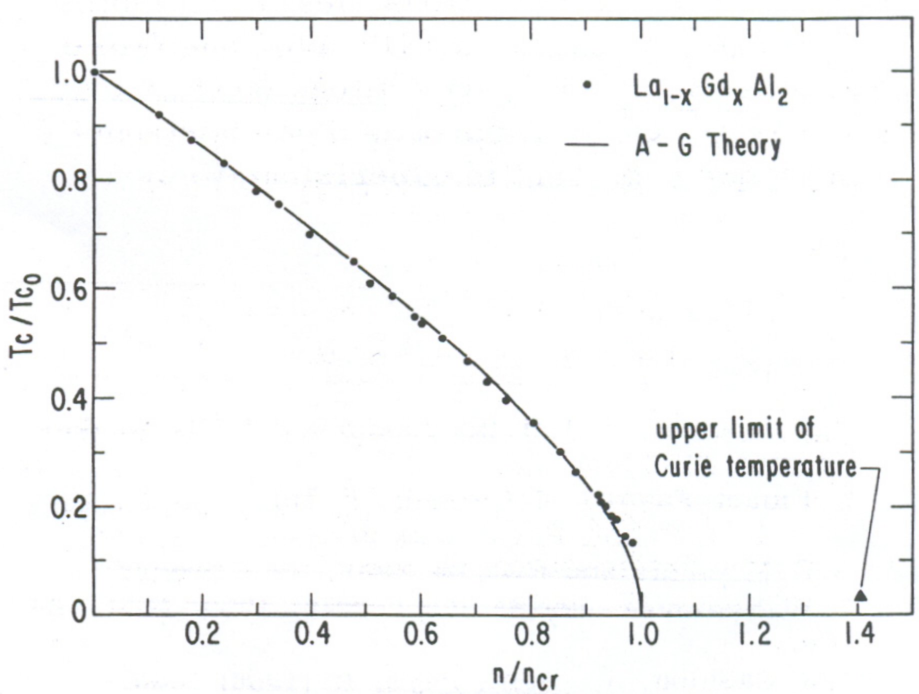
- Recall on magnetism and superconductivity coexistence
- Origin and the main peculiarities of the proximity effect in superconductor-ferromagnet systems.
- Josephson π-junction, the role of the magnetic scattering.
- Domain wall superconductivity. Spin-valve effet.
- Inversion of the proximity effect in atomic F/S/F structures.
- φ-junctions.
- Possible applications

Magnetism and Superconductivity Coexistence



$$\frac{dT_c}{dx} \approx -\Theta_m x$$

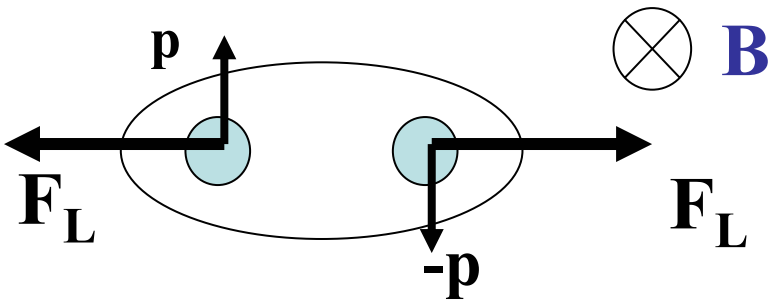
(Abrikosov and Gorkov, 1960)

The critical temperature variation versus the concentration n of the Gd atoms in $La_{1-x}Gd_xAl_2$ alloys (Maple, 1968). T_{c0}=3.24 K and n_{cr}=0.590 atomic percent Gd.

The earlier experiments (Matthias *et al.*, 1958) demonstrated that the presence of the magnetic atoms is very harmful for superconductivity.

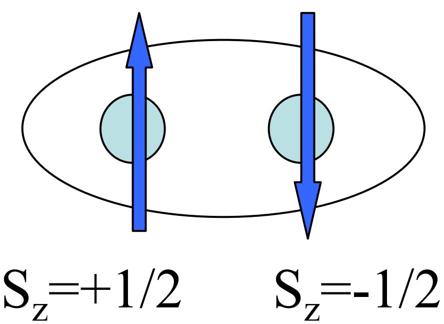
Antagonism of magnetism (ferromagnetism) and superconductivity

- Orbital effect (Lorentz force)



Electromagnetic mechanism (breakdown of Cooper pairs by magnetic field induced by magnetic moment)

- Paramagnetic effect (singlet pair)



$\mu_B H \sim \Delta \sim T_c$

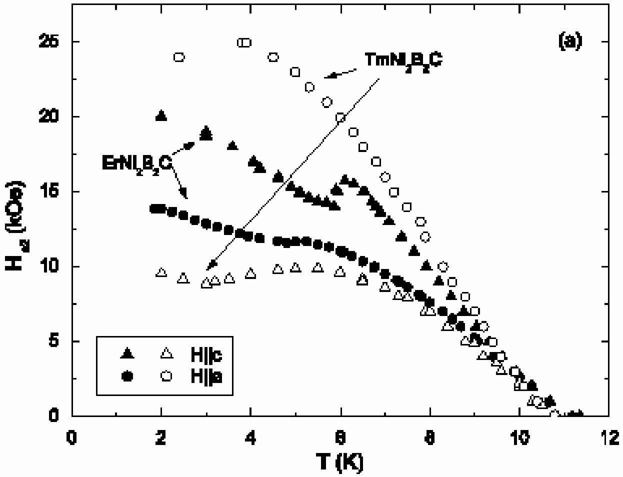
$$I\left(\vec{S}\cdot\vec{s}\right)\approx T_c$$

Exchange interaction

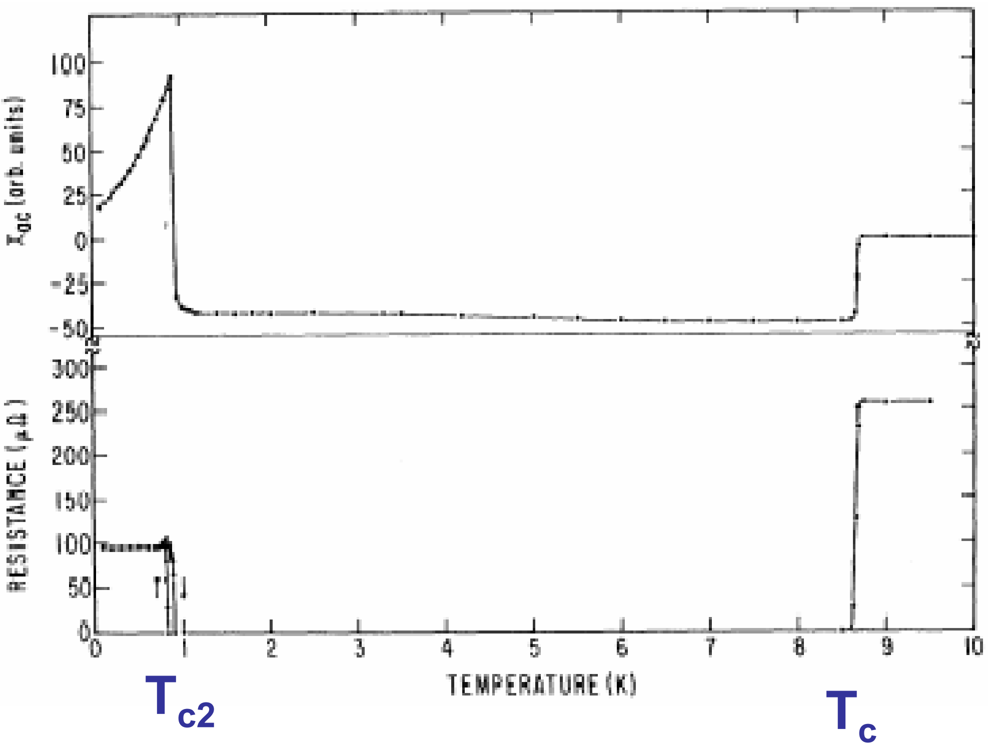
No antagonism between antiferromagnetism and superconductivity

	T_c (K)	T_N (K)
$NdRh_4B_4$	5.3	1.31
$SmRh_4B_4$	2.7	0.87
$TmRh_4B_4$	9.8	0.4
$GdMo_6S_8$	1.4	0.84
$TbMo_6S_8$	2.05	1.05
$DyMo_6S_8$	2.05	0.4
$ErMo_6S_8$	2.2	0.2
$GdMo_6Se_8$	5.6	0.75
$ErMo_6Se_8$	6.0	1.1
$DyNi_2B_2C$	6.2	11
$ErNi_2B_2C$	10.5	6.8
$TmNi_2B_2C$	11	1.5
$HoNi_2B_2C$	8	5

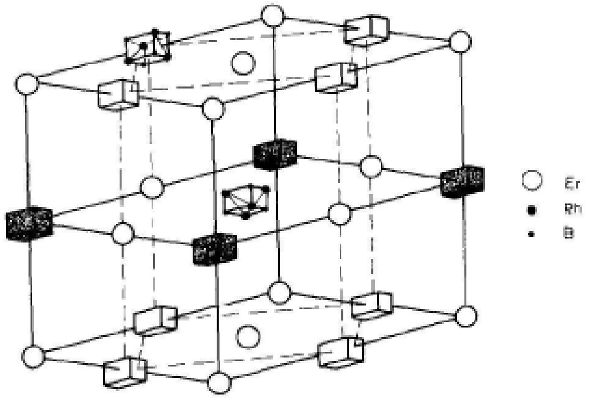
Usually $T_c > T_N$



FERROMAGNETIC CONVENTIONAL (SINGLET) SUPERCONDUCTORS

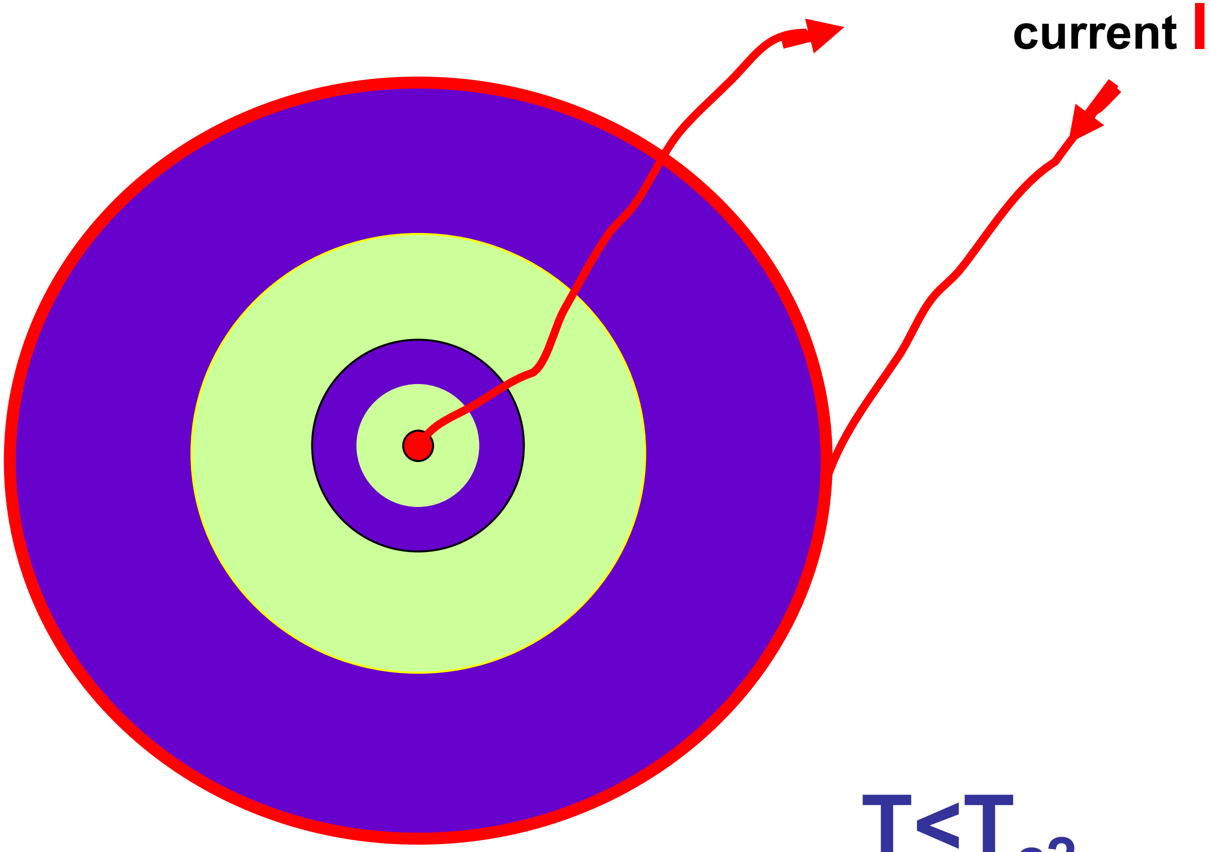


A. C. susceptibility and resistance versus temperature in $ErRh_4B_4$ (Fertig *et al.*,1977).



RE-ENTRANT SUPERCONDUCTIVITY in $ErRh_4B_4$, $HoMo_6S_8$

Auto-waves in reentrant superconductors?



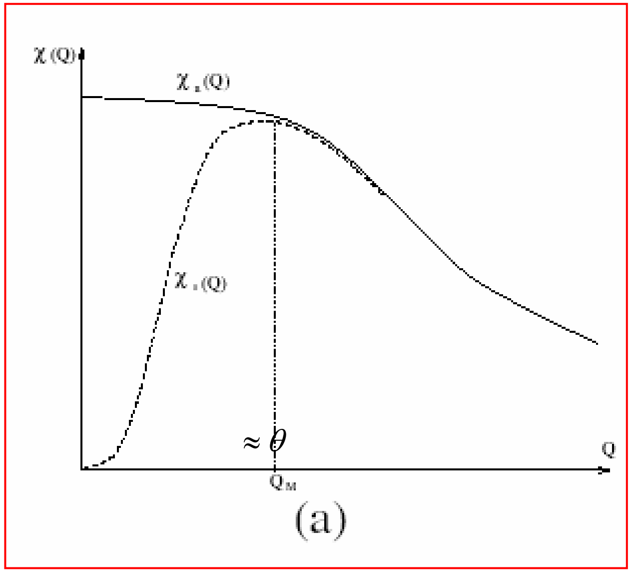
$T<T_{c2}$

Coexistence phase

$$E_m = -\sum_Q \frac{\chi(Q)}{2} |h_Q|^2$$

At T=0 and $Q\xi_0 >> 1$ following (**Anderson and Suhl, 1959**)

$$\frac{\chi_s(Q) - \chi(Q)}{\chi(0)} = \frac{\pi}{2Q\xi_0}$$

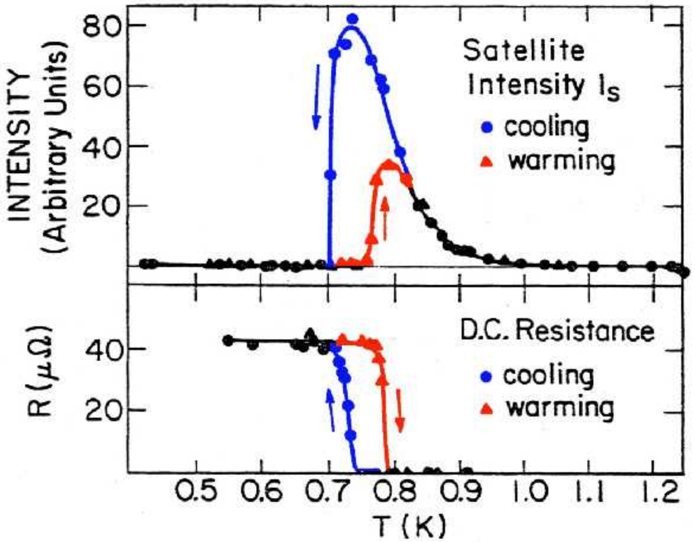


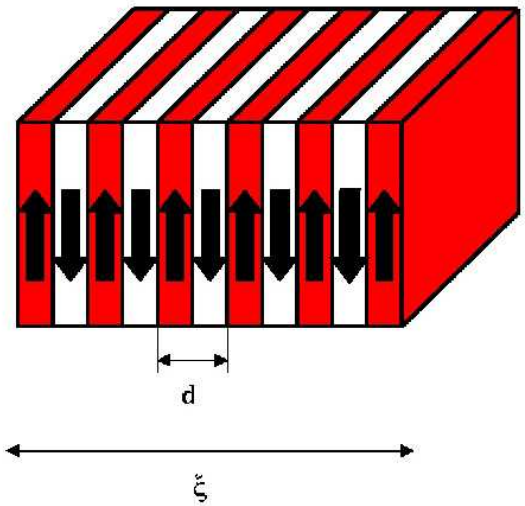
Energy per atom/electron :
magnetic, superconducting

$\approx \theta$

$\approx T_c^2 \; / E_F$

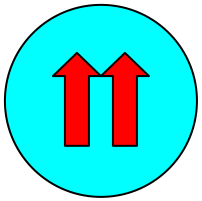
$d \propto \sqrt{a\xi}$





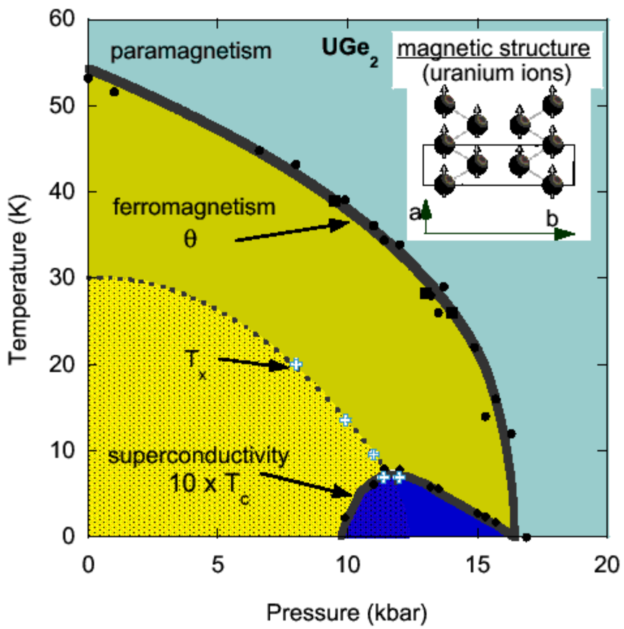
Intensity of the neutron Bragg scattering and resistance as a function of temperature in an $ErRh_4B_4$ (**Sinha *et al.*,1982**). The satellite position corresponds to the wavelength of the modulated magnetic structure around 92 Å.

FERROMAGNETIC UNCONVENTIONAL (TRIPLET) SUPERCONDUCTORS

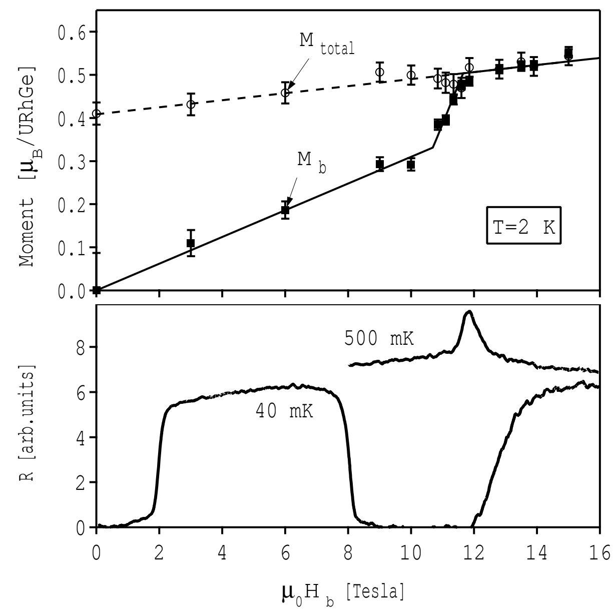


UGe_2 (Saxena *et al.*, 2000) and URhGe (Aoki *et al.*, 2001)

Triplet pairing

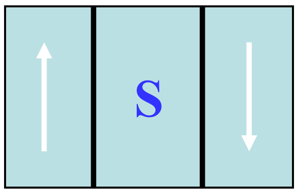


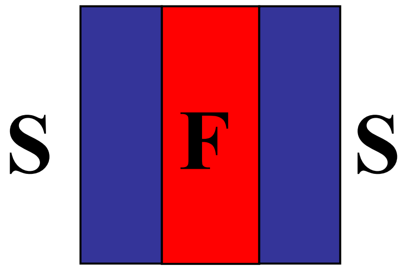
UGe_2

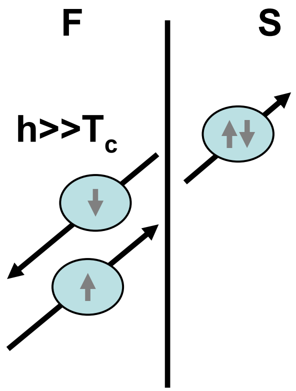


URhGe (a) The total magnetic moment M total and the component M_b measured for H// to the b axis .
In (b), variation of the resistance at 40 mK and 500 mK with the field re-entrance of SC between 8-12 T
(Levy et al 2005).

The coexistence of singlet superconductivity and ferromagnetism is basically impossible in the same compound but may be easily achieved in artificially fabricated superconductor/ferromagnet heterostructures.







Due to the proximity effect, the Cooper pairs penetrate into the F layer and we have the unique possibility to study the properties of superconducting electrons under the influence of the huge exchange field.

Varying in the controllable manner the thicknesses of the ferromagnetic and superconducting layers it is possible to change the relative strength of two competing ordering. Interesting effects at the nanoscopic scale.

The Josephson junctions with ferromagnetic layers reveal many unusual properties quite interesting for applications, in particular the so-called π-Josephson junction (with the π-phase difference in the ground state).

Superconducting order parameter behavior in ferromagnet

Standard Ginzburg-Landau functional:

$$F = a|\Psi|^2 + \frac{1}{4m}|\nabla\Psi|^2 + \frac{b}{2}|\Psi|^4$$

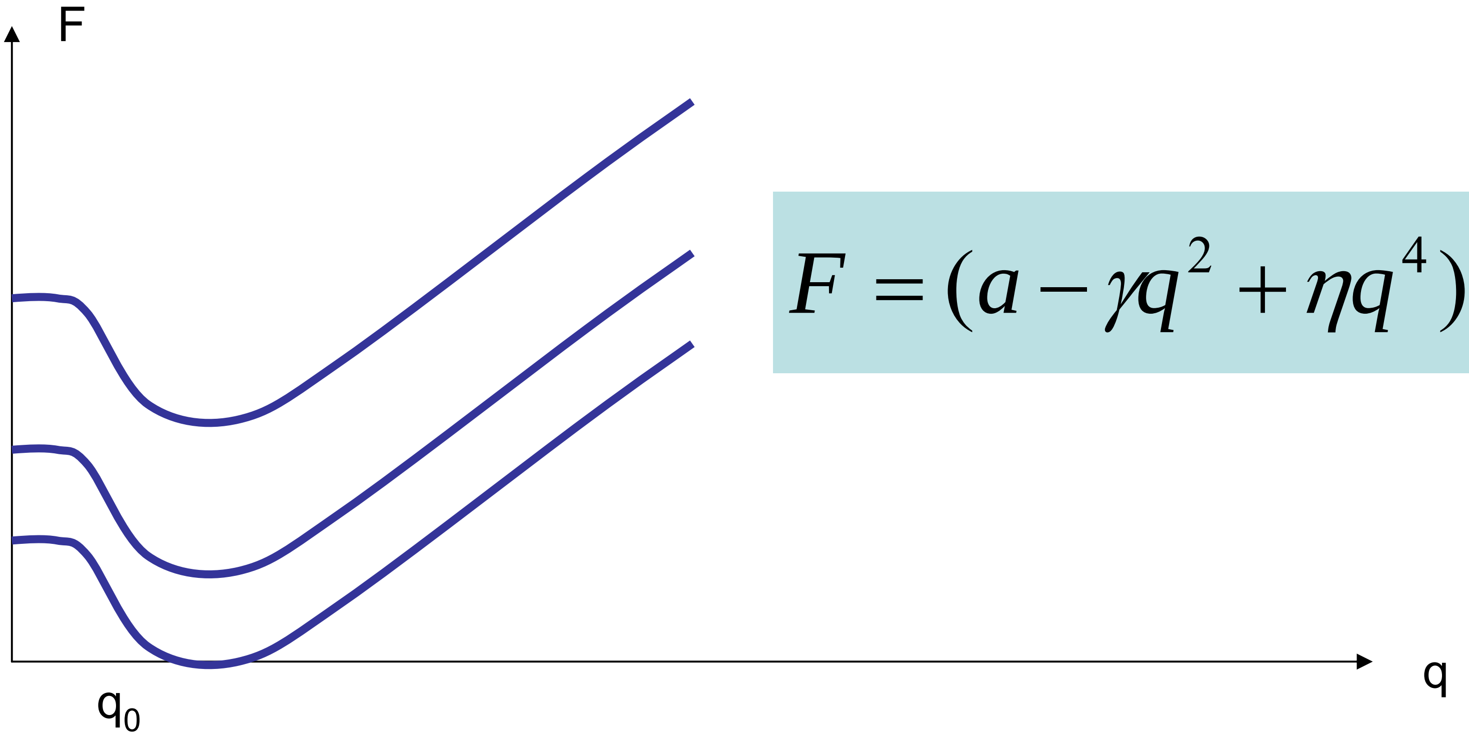
The minimum energy corresponds to Ψ=const

The coefficients of GL functional are functions of internal exchange field h !

Modified Ginzburg-Landau functional ! :

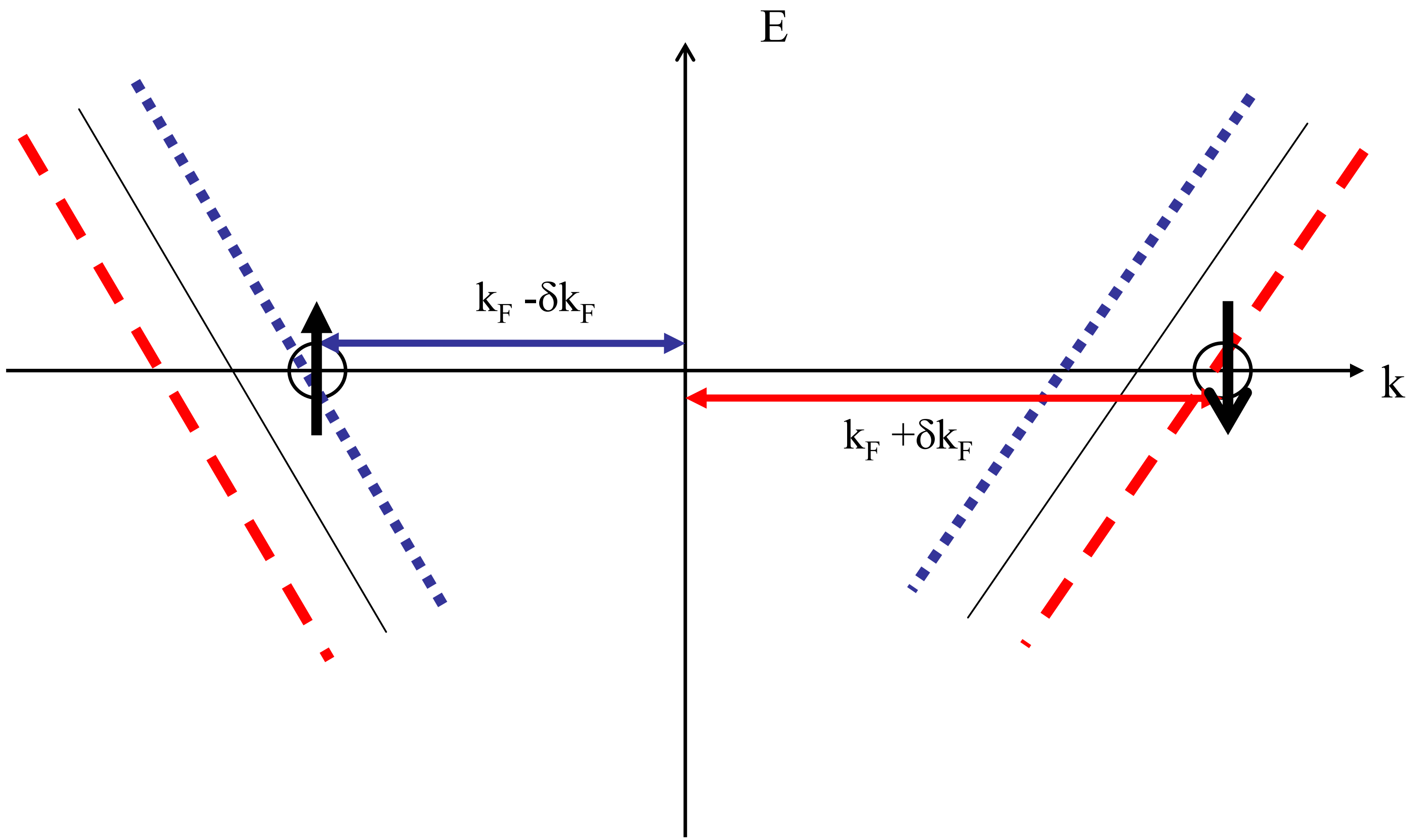
$$F = a|\Psi|^2 - \gamma|\nabla\Psi|^2 + \eta|\nabla^2\Psi|^2 + \ldots$$

The **non-uniform** state Ψ~exp(iqr) will correspond to minimum energy and higher transition temperature



$$F = (a - \gamma q^2 + \eta q^4)\left|\Psi_q\right|^2$$

$\Psi \sim \exp(iqr)$ - **Fulde-Ferrell-Larkin-Ovchinnikov state (1964).**
Only in pure superconductors and in the very narrow region.



The total momentum of the Cooper pair is
$-(k_F - \delta k_F) + (k_F - \delta k_F) = 2\,\delta k_F$

Proximity effect in a ferromagnet ?

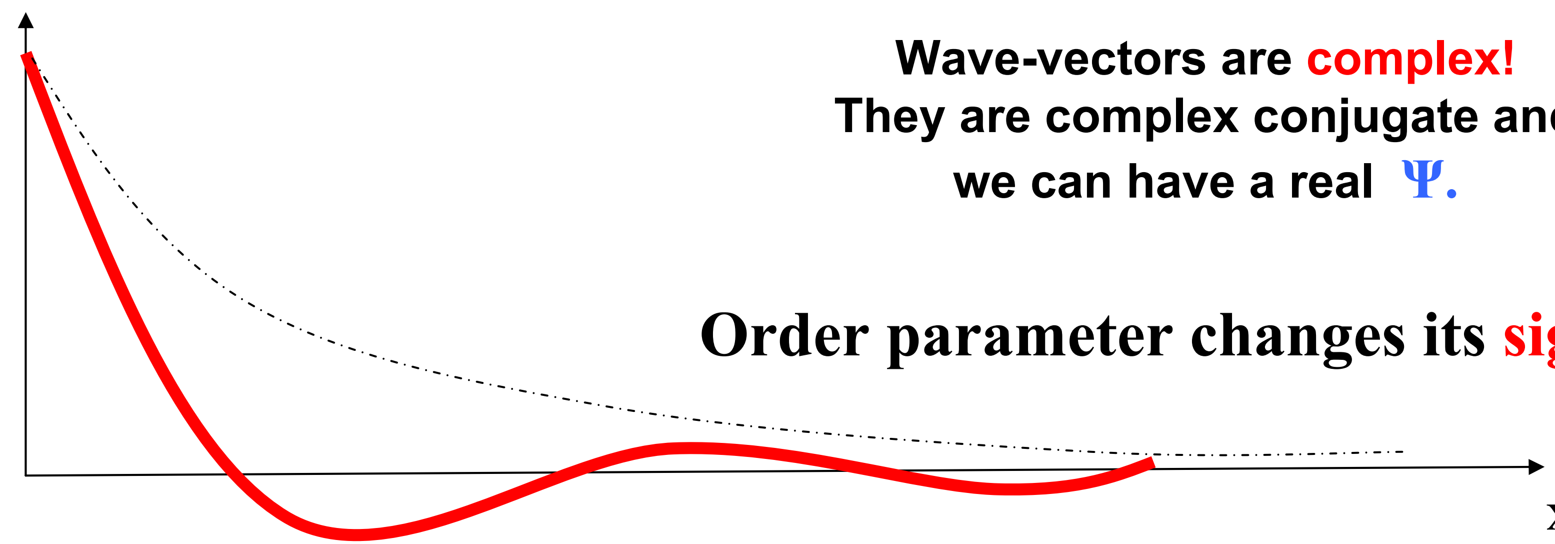
In the usual case (normal metal):

$$a\Psi - \frac{1}{4m}\nabla^2\Psi = 0, \text{ and solution for } T > T_c \text{ is } \Psi \propto e^{-qx}, \text{ where } q = \sqrt{4ma}$$

In **ferromagnet** (in presence of exchange field) the equation for superconducting order parameter is different

$$a\Psi + \gamma\nabla^2\Psi - \eta\nabla^4\Psi = 0$$

Its solution corresponds to the order parameter which decays with oscillations! $\Psi \sim \exp[-(q_1 \pm iq_2)x]$

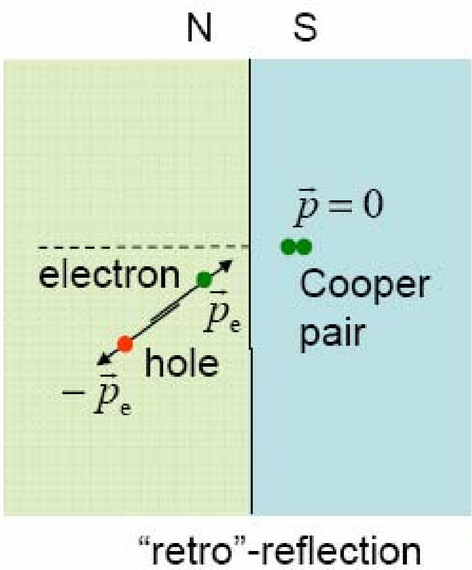


Wave-vectors are complex!
They are complex conjugate and
we can have a real Ψ.

Order parameter changes its sign!

Proximity effect as Andreev reflection

p_F

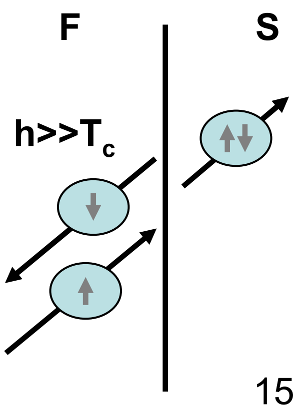


Classical Andreev reflection

Quantum Andreev reflection

(A.F. Andreev, September, 2008)

$$p_{F\uparrow} \neq p_{F\downarrow}$$



Theory of S-F systems in dirty limit

Analysis on the basis of the Usadel equations

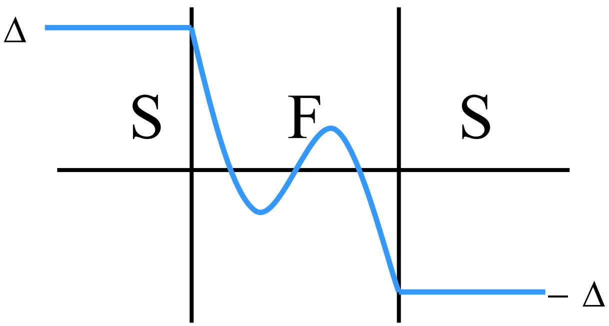
$$-\frac{D_f}{2}\vec{\nabla}^2 F_f(x,\omega,h)+(\omega+ih)F_f(x,\omega,h)=0$$

$$G_f^{\,2}(x,\omega,h)+F_f(x,\omega,h)F_f^{\,*}(x,\omega,-h)=1$$

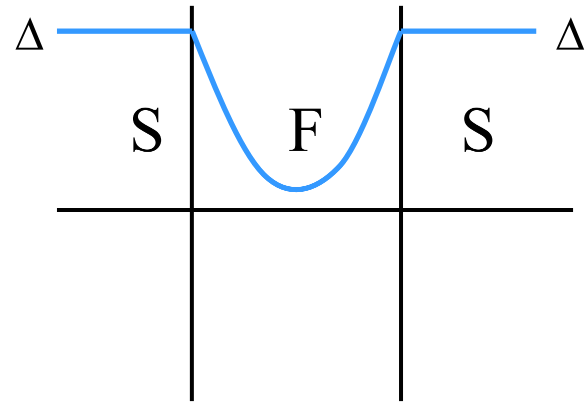
.

leads to the prediction of the oscillatory - like dependence of the critical current on the exchange field h and/or thickness of ferromagnetic layer.

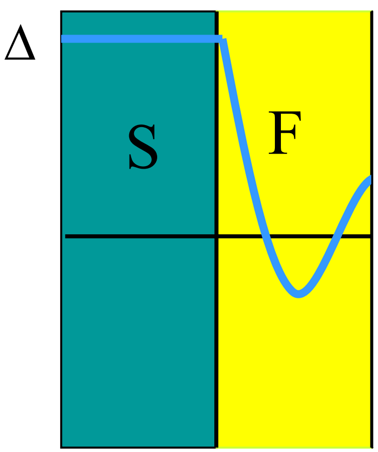
Remarkable effects come from the possible shift of sign of the wave function in the ferromagnet, allowing the possibility of a « π-coupling » between the two superconductors (π-phase difference instead of the usual zero-phase difference)



« π phase »



« 0 phase »



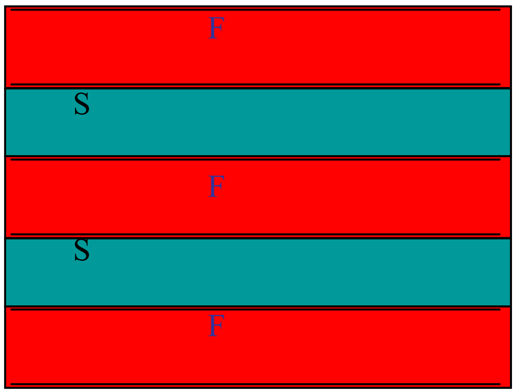
S/F bilayer

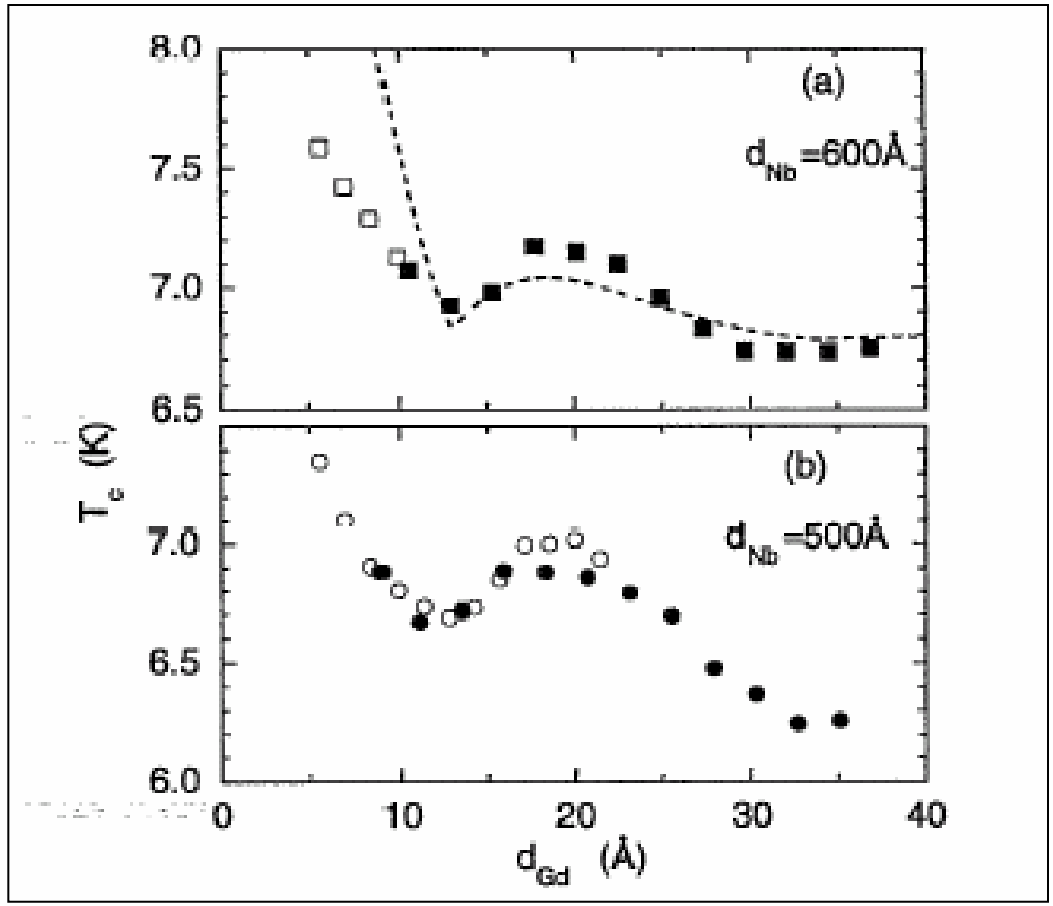
$$\xi_f = \sqrt{D_f / h}$$

h-exchange field,

D_f-diffusion constant

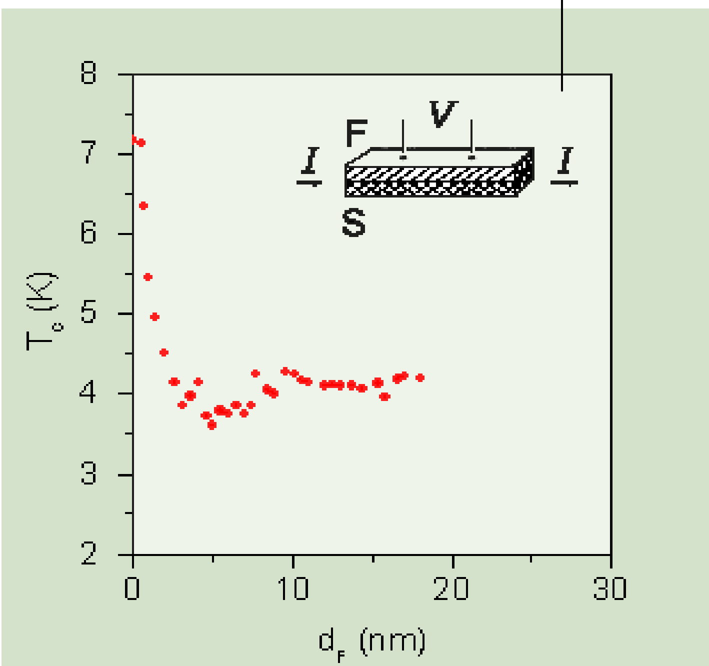
The oscillations of the critical temperature as a function of the thickness of the ferromagnetic layer in S/F multilayers has been predicted in 1990 and later observed on experiment by Jiang et al. PRL, **1995**, in Nb/Gd multilayers



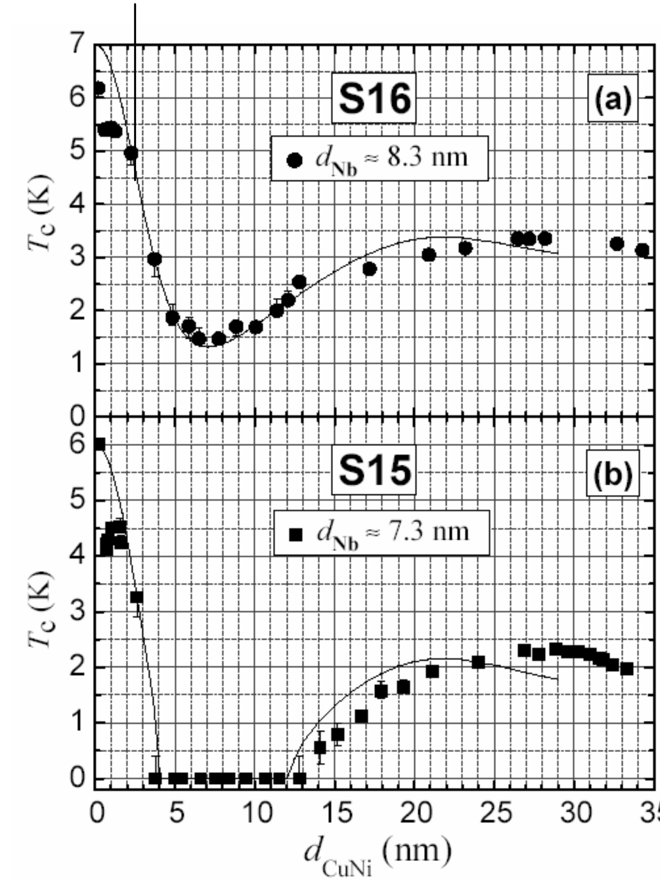


SF-bilayer T_c-oscillations

Ryazanov et al. JETP Lett. 77, 39 (2003) $Nb\text{-}Cu_{0.43}Ni_{0.57}$



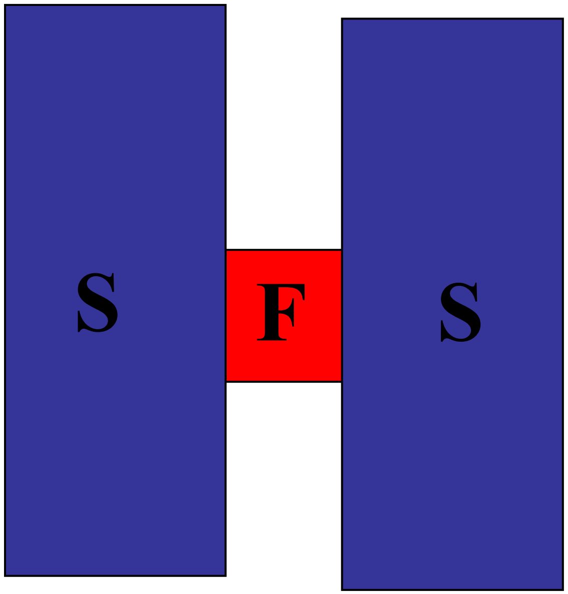
V. Zdravkov, A. Sidorenko et al PRL (2007) $Nb\text{-}Cu_{0.41}Ni_{0.59}$



$d_{Fmin} = (1/4)\,\lambda_{ex}$ largest T_c-suppression

S-F-S Josephson junction in the clean limit

(Buzdin, Bulaevskii and Panjukov, JETP Lett. 81)

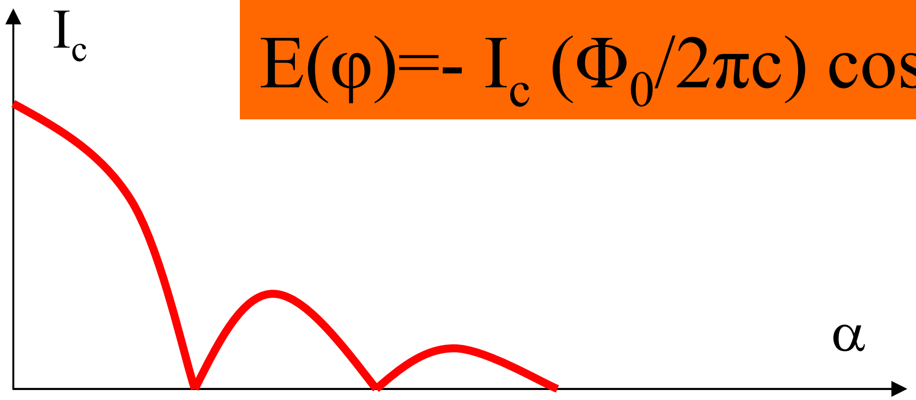


Damping oscillating dependence of the critical current I_c as the function of the parameter $\alpha = hd_F / v_F$ has been predicted.

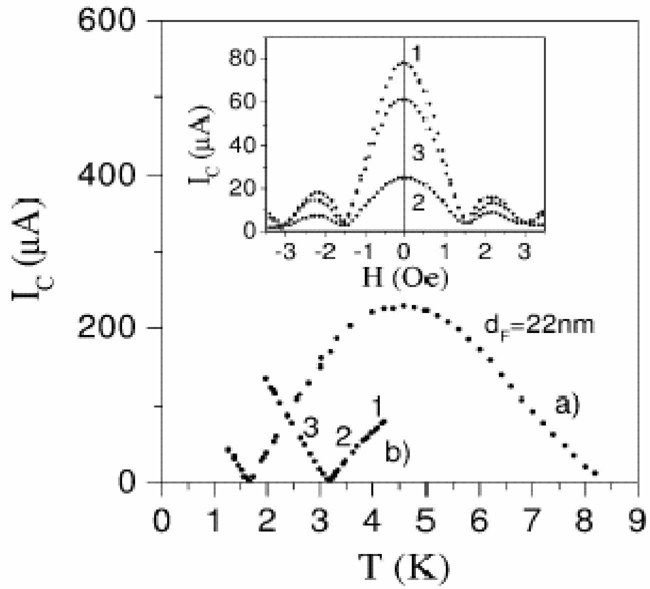
h- exchange field in the ferromagnet,
d_F - its thickness

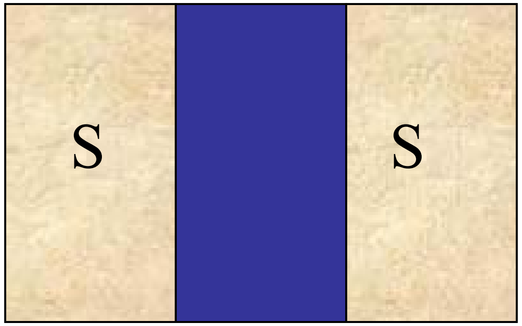
$E(\varphi) = - I_c (\Phi_0/2\pi c) \cos\varphi$

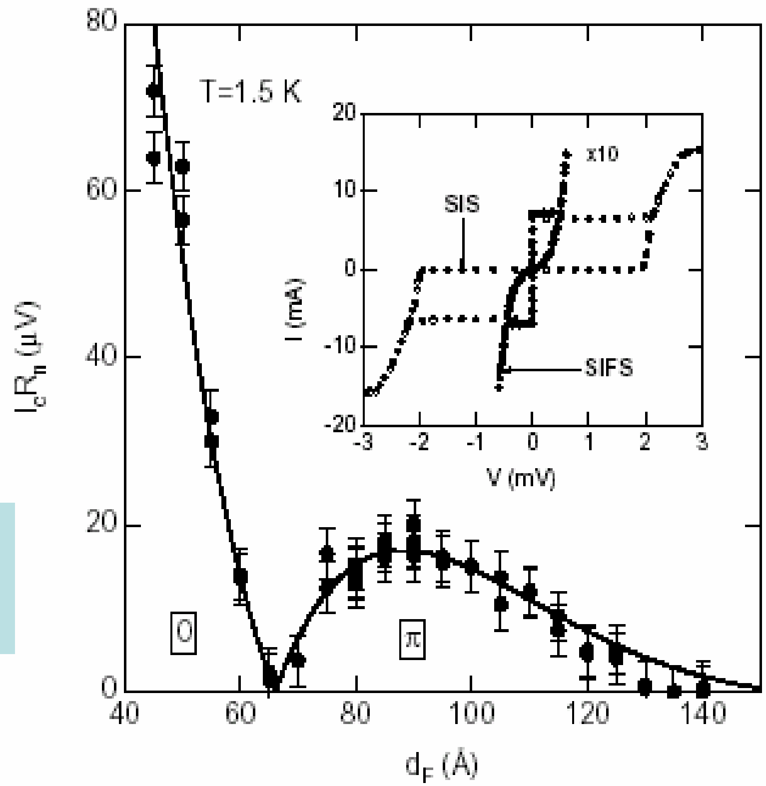
$J(\varphi) = I_c \sin\varphi$



The oscillations of the critical current as a function of temperature (for different thickness of the ferromagnet) in S/F/S trilayers have been observed on experiment by Ryazanov et al. 2000, PRL







and as a function of a ferromagnetic layer thickness by Kontos et al. 2002, PRL

Phase-sensitive experiments

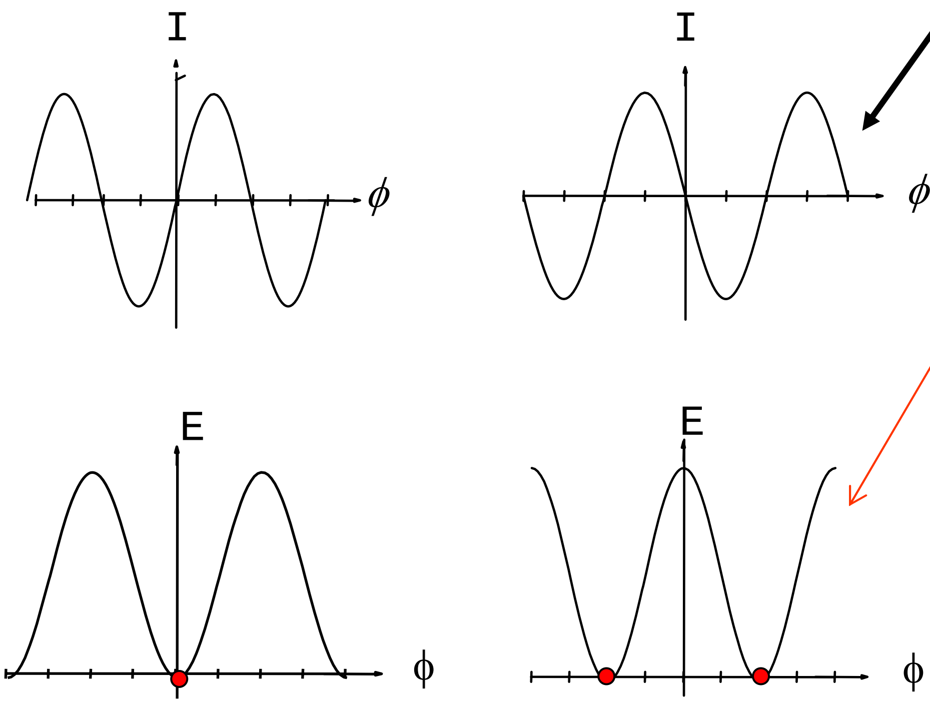
π-junction in one-contact interferometer

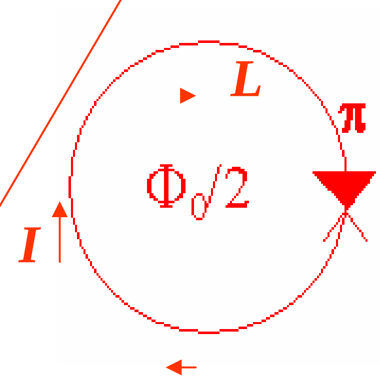
0-junction
minimum energy at 0

π-junction
minimum energy at π

$I = I_c \sin(\pi + \phi) = -I_c \sin\phi$

$E = E_J[1 - \cos(\pi + \phi)] = E_J[1 + \cos\phi]$





$2\pi L I_c > \Phi_0/2$

$\phi = \pi = (2\pi/\Phi_0)\int A dl = 2\pi\,\Phi/\Phi_0$

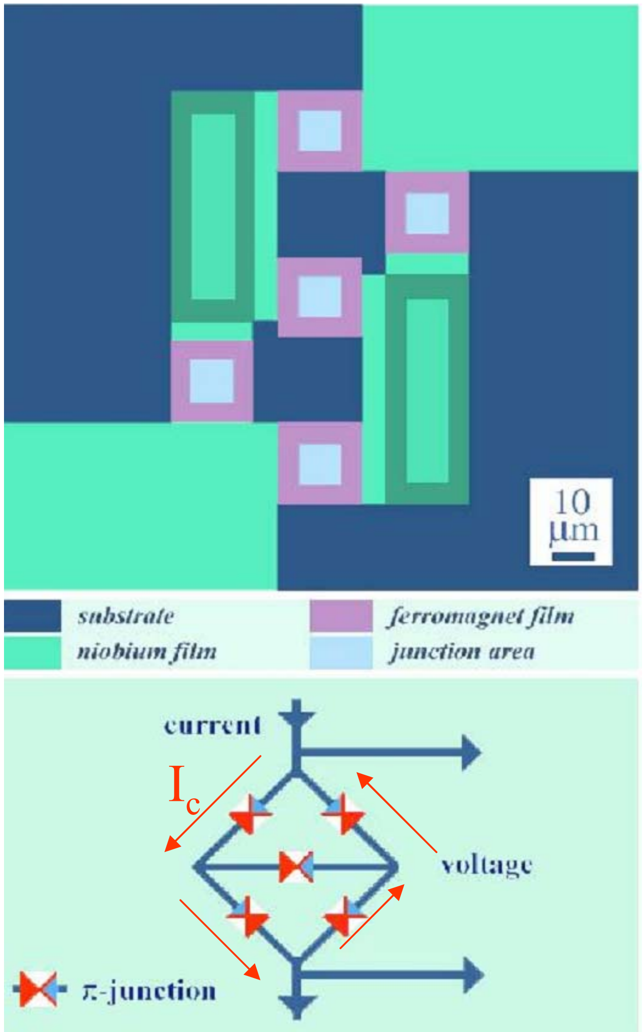
Spontaneous circulating current in a closed superconducting loop when $\beta_L > 1$ with NO applied flux

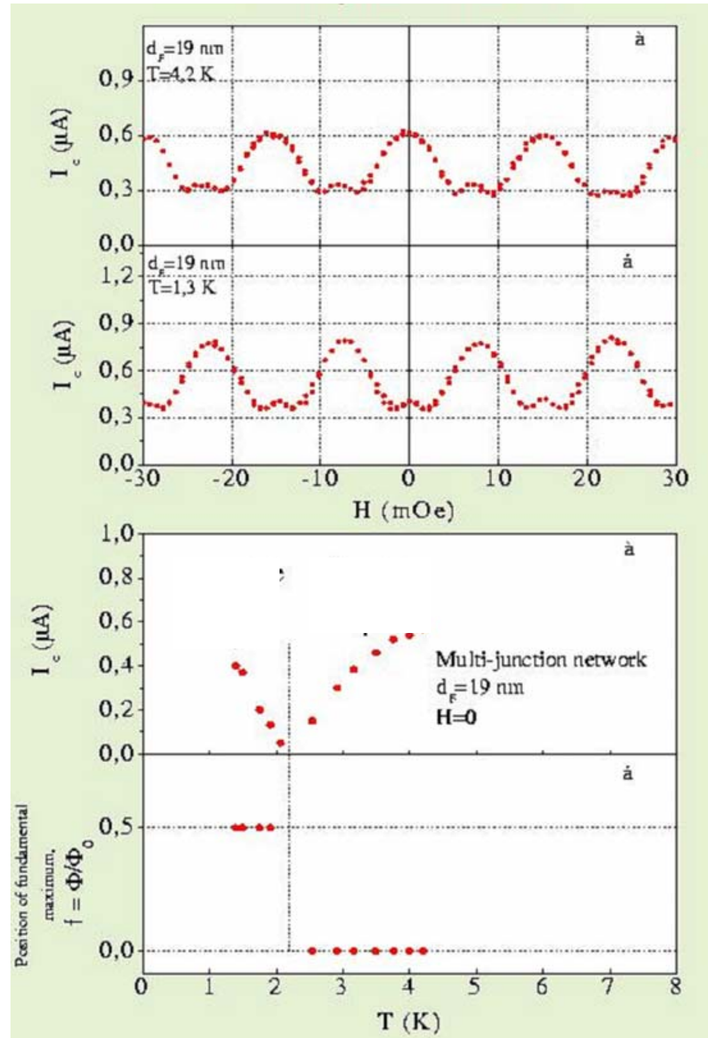
$\beta_L = \Phi_0/(4\pi L I_c)$

$\Phi = \Phi_0/2$

Bulaevsky, Kuzii, Sobyanin, JETP Lett. 1977

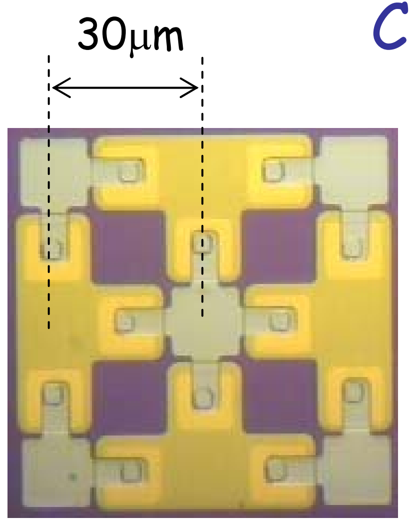
Current-phase experiment.
Two-cell interferometer



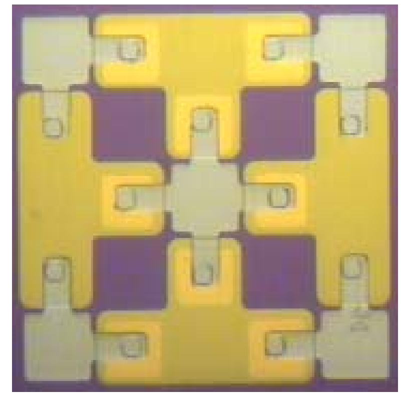


Cluster Designs (Ryazanov et al.)

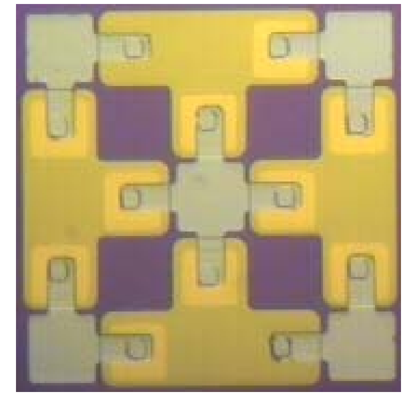
2 x 2



unfrustrated

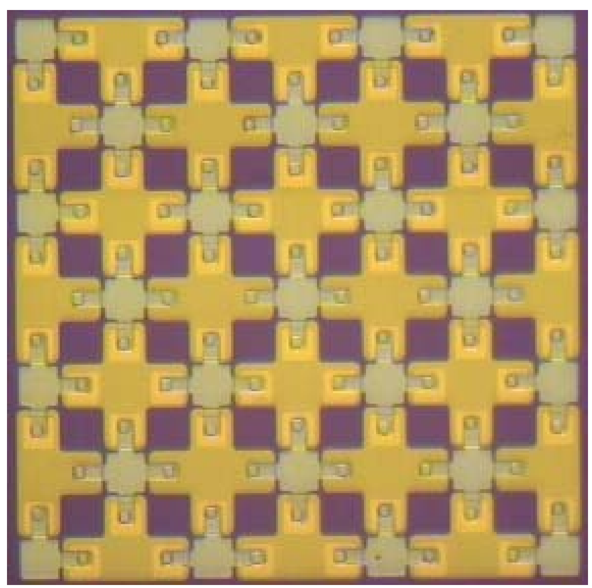


fully-frustrated

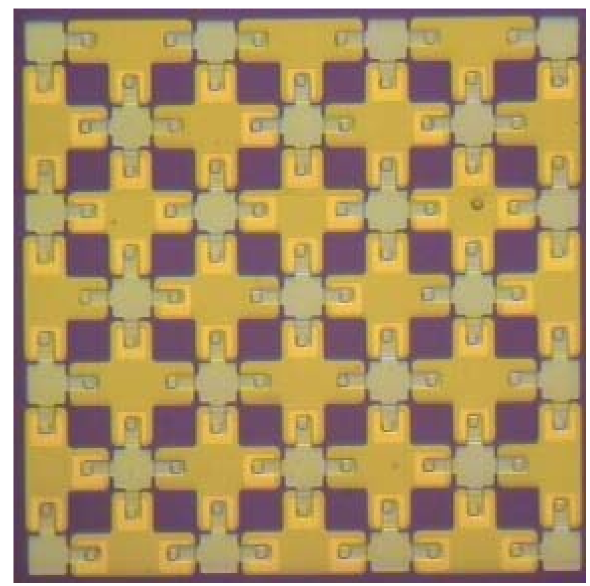


checkerboard-frustrated

6 x 6



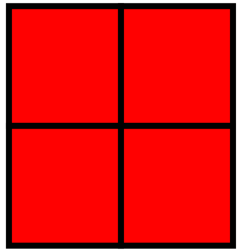
fully-frustrated

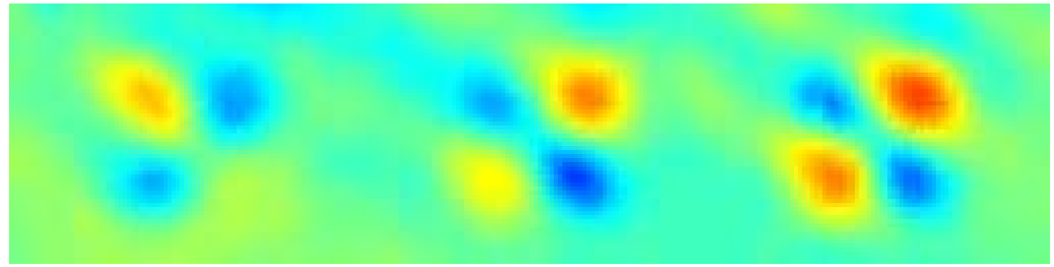


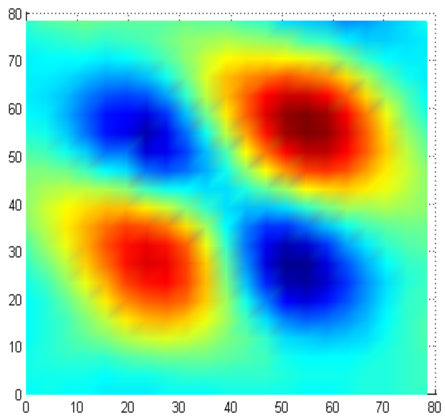
checkerboard-frustrated

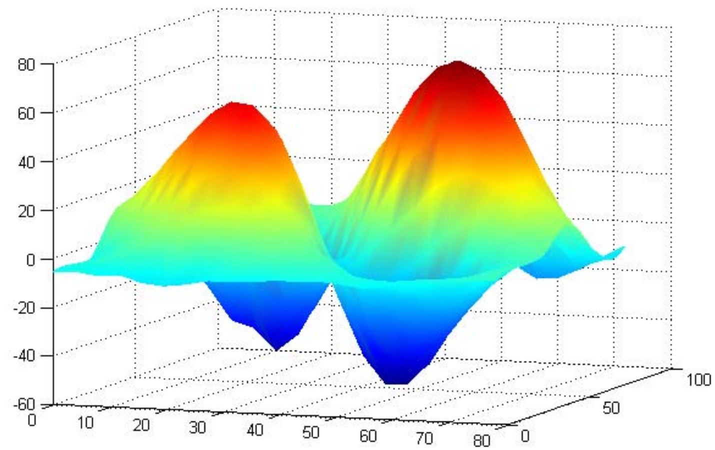
2 x 2 arrays: spontaneous vortices

Fully frustrated

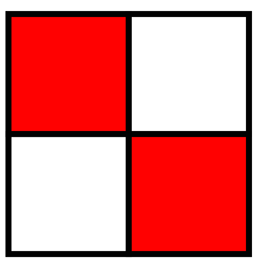


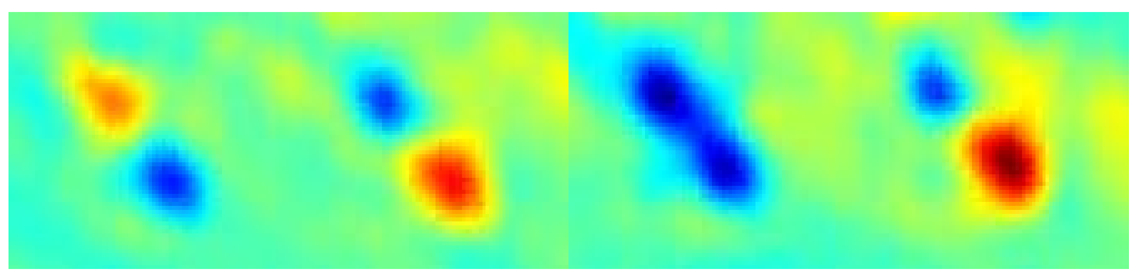






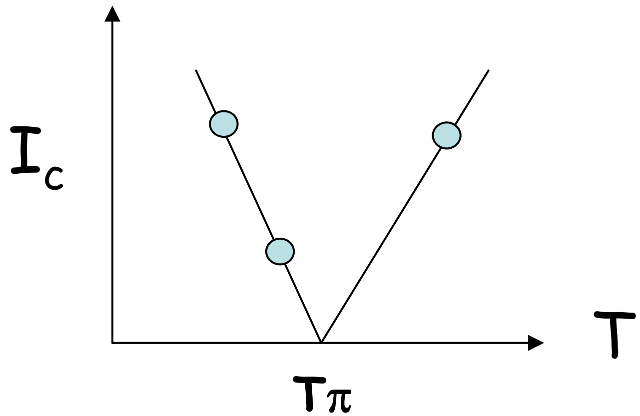
Checkerboard frustrated





Scanning SQUID Microscope images

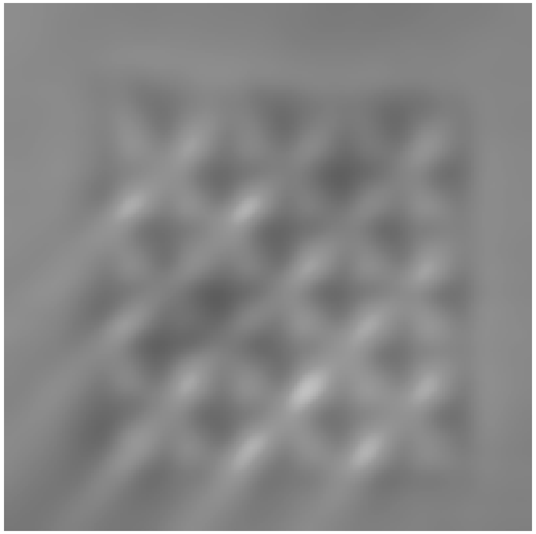
(Ryazanov et al.)



T = 1.7K

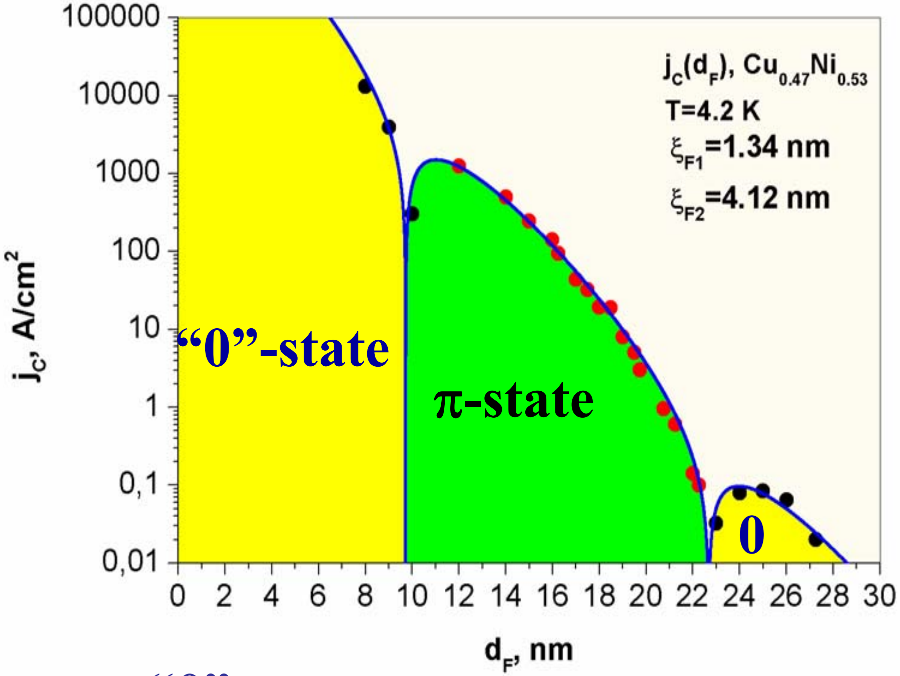
T = 2.75K

T = 4.2K



Critical current density vs. F-layer thickness (V.A.Oboznov et al., PRL, 2006)

$$I_c = I_{c0} \exp(-d_F/\xi_{F1}) \left|\cos(d_F/\xi_{F2}) + \sin(d_F/\xi_{F2})\right|$$



$d_F \gg \xi_{F1}$

Spin-flip scattering decreases the decaying length and increases the oscillation period.

$\xi_{F2} > \xi_{F1}$

$Nb\text{-}Cu_{0.47}Ni_{0.53}\text{-}Nb$

"0"-state

$I = I_c \sin\varphi$

π-state

$I = I_c \sin(\varphi + \pi) = -I_c \sin(\varphi)$

Critical current vs. temperature

Nb-$Cu_{0.47}Ni_{0.53}$-Nb
d_F=9-24 nm
h=E_{ex} ~ 850 K (T_{Curie}= 70 K)

"Temperature dependent" spin-flip scattering

$$\frac{1}{\xi_{F1}} = \frac{1}{\xi_F}\sqrt{\sqrt{1+\left(\frac{1}{h\tau_s}\right)^2}+\left(\frac{1}{h\tau_s}\right)},$$

$$\frac{1}{\xi_{F2}} = \frac{1}{\xi_F}\sqrt{\sqrt{1+\left(\frac{1}{h\tau_s}\right)^2}-\left(\frac{1}{h\tau_s}\right)}$$

$$\xi_{F2} > \xi_{F1}$$

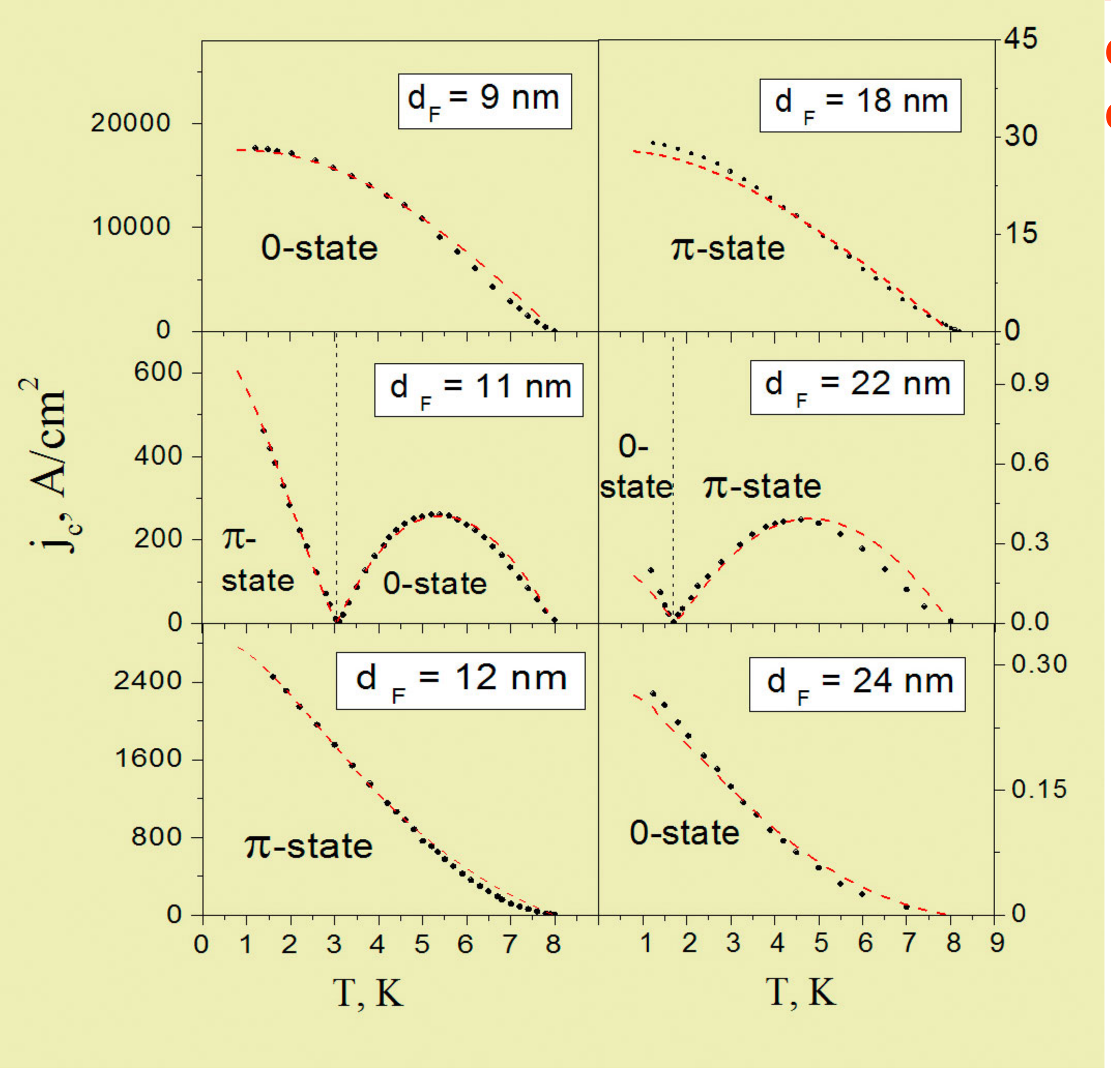
$$\left(\omega + iE_{ex} + \frac{\hbar\cos\Theta}{\tau_s}\right)\sin\Theta - \frac{\hbar D}{2}\frac{\partial^2\Theta}{\partial x^2} = 0$$

G = cos Θ(T); F = sin Θ(T)

Effective spin-flip rate
Γ(T)= cos Θ(T)/τ_S;

Critical current vs. temperature (0-π- and π-0- transitions)

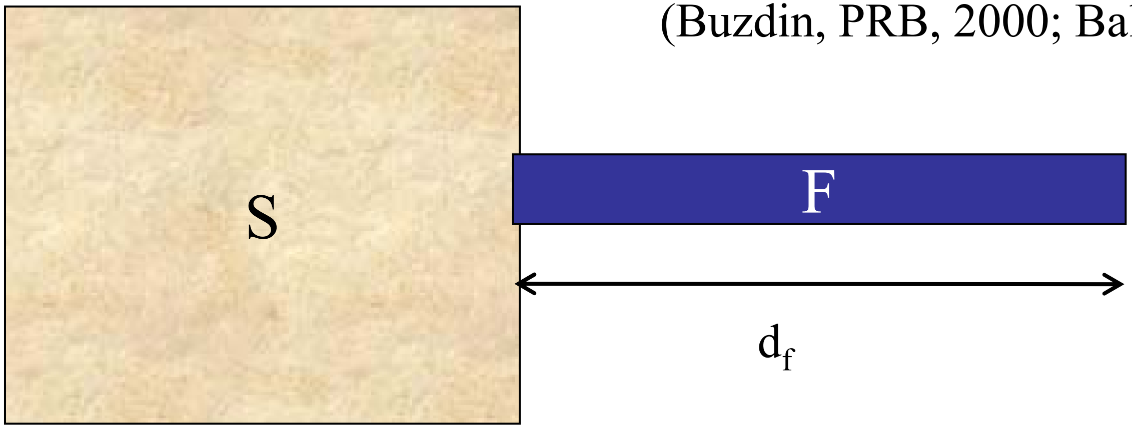
Nb-$Cu_{0.47}Ni_{0.53}$-Nb
d_{F1}=10-11 nm
d_{F2}=22 nm



(V.A.Oboznov et al., PRL, 2006)

Density of states in the ferromagnet in contact with superconductor

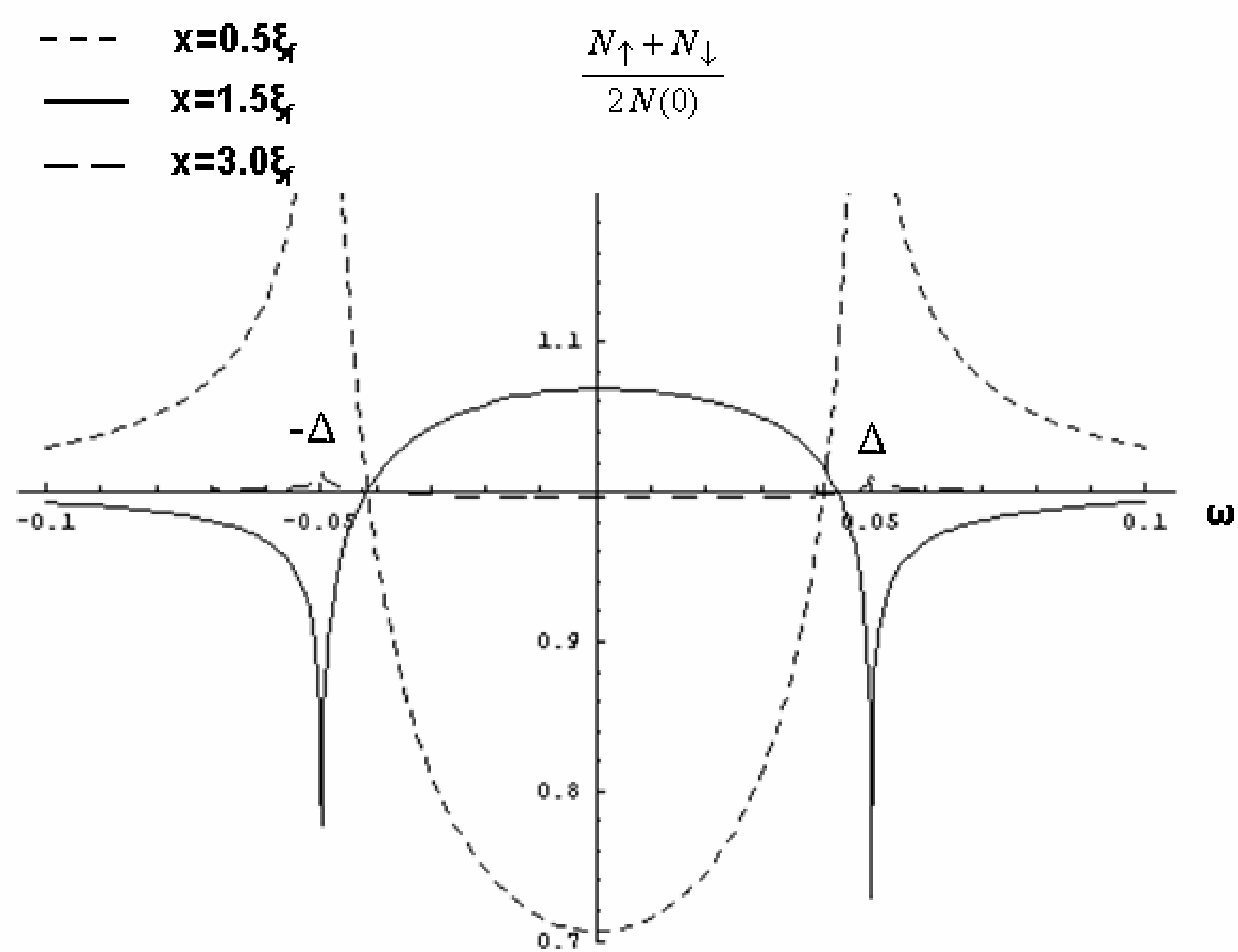
(Buzdin, PRB, 2000; Baladie and Buzdin, PRB, 2001)



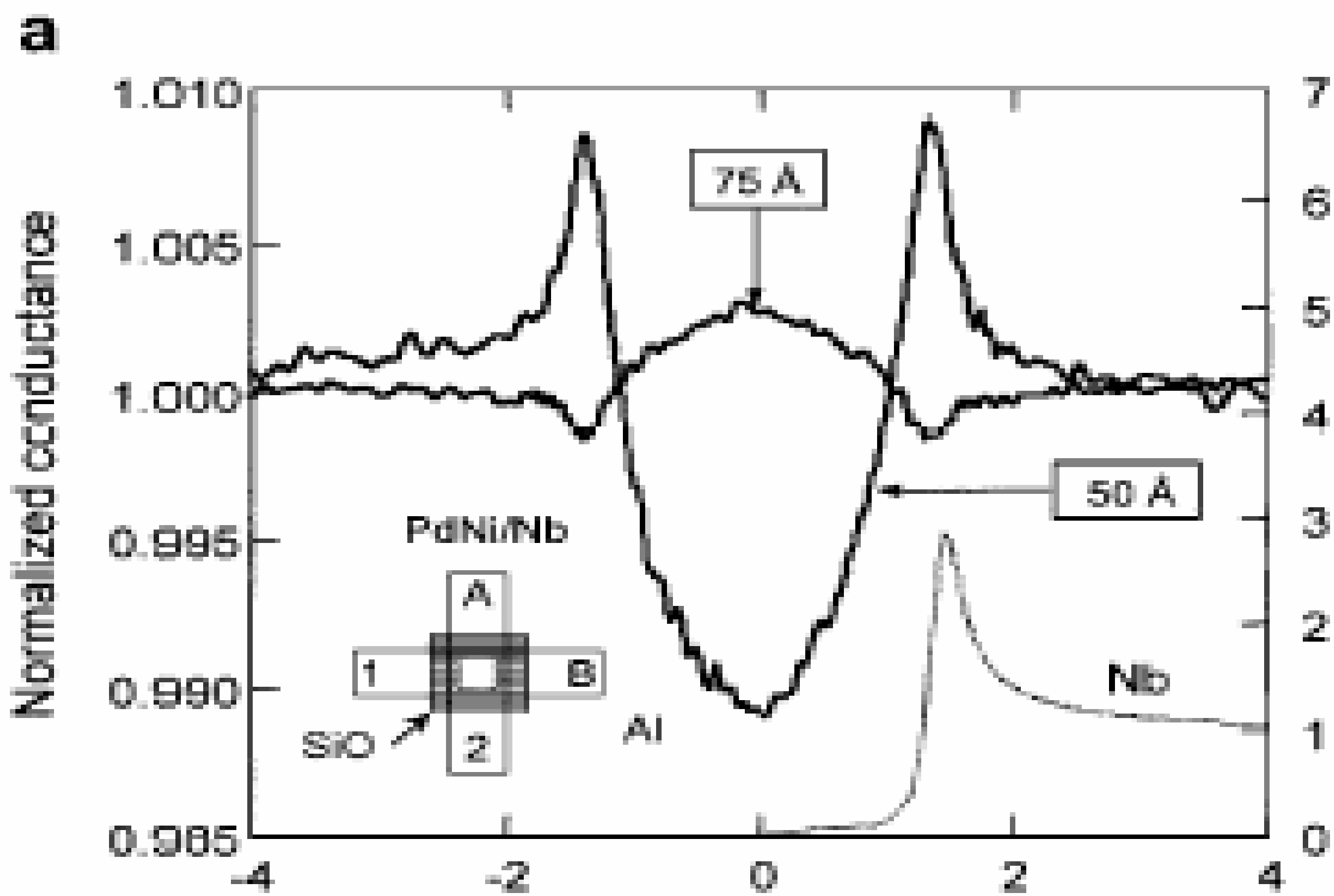
In the case of a weak proximity effect (weak influence of the ferromagnet on the superconducting order parameter) we can derive the superconducting density of states induced in the ferromagnet by the proximity effect.

In the clean limit and in the dirty limit, far away from T_c , close to T_c we see that the oscillatory behavior of the density of states close to the S/F interface is really robust to the variation of parameters characterizing the system.

DOS structures at different distances from S layer



Density of states measured by Kontos et al (PRL 2001) on Nb/PdNi bilayers



Triplet correlations

Bergeret, Volkov Efetov -as a review see Bergeret et al., Rev. Mod. Phys. (2005).

$$D\partial_X^2 \hat{f} - 2|\omega|\hat{f} + i\,\mathrm{sgn}(\omega)\left(\hat{f}\hat{V}^* - \hat{V}\hat{f}\right) = 0$$

$$\hat{V} = J\begin{pmatrix} \cos\alpha & \pm i\sin\alpha \\ \mp i\sin\alpha & -\cos\alpha \end{pmatrix}$$

Structure of the functions f:

$$f = i\hat{\tau}_2\left(f_3(x)\hat{\sigma}_3 + f_0(x)\right) + i\hat{\tau}_1\hat{\sigma}_1 f_1(x)$$

σ, τ -Pauli matrices (spin, Nambu)

$f_3 \propto \langle \psi_\uparrow \psi_\downarrow \rangle - \langle \psi_\downarrow \psi_\uparrow \rangle$ -Singlet condensate

$f_0 \propto \langle \psi_\uparrow \psi_\downarrow \rangle + \langle \psi_\downarrow \psi_\uparrow \rangle$ -Triplet condensate (with projection 0 on z-axis)

$f_1 \propto \langle \psi_\uparrow \psi_\uparrow \rangle \propto \langle \psi_\downarrow \psi_\downarrow \rangle$ -Triplet condensate (with projection +1,-1)

8

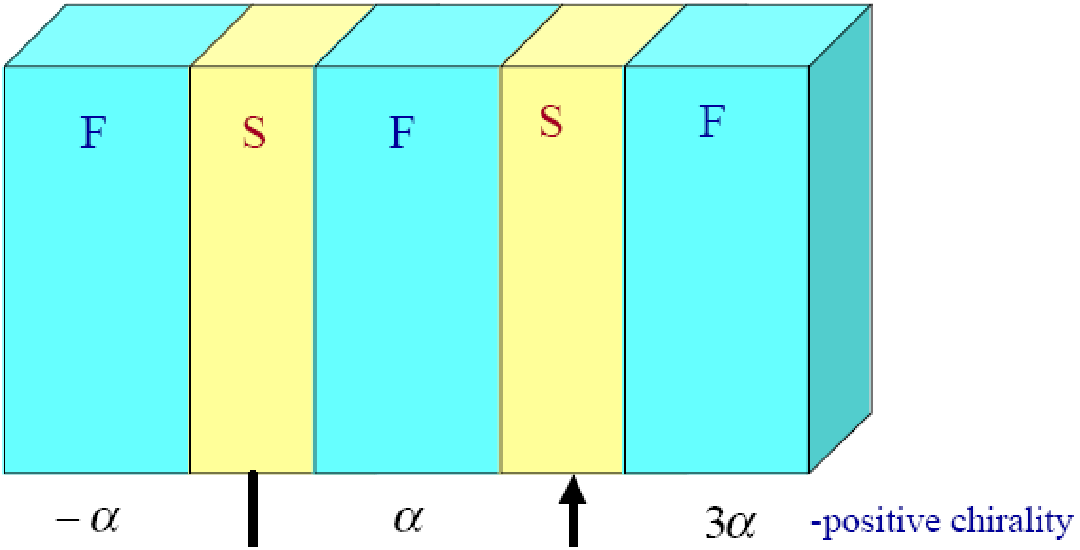


TABLE I. Characteristic length scales of S/F proximity effect.

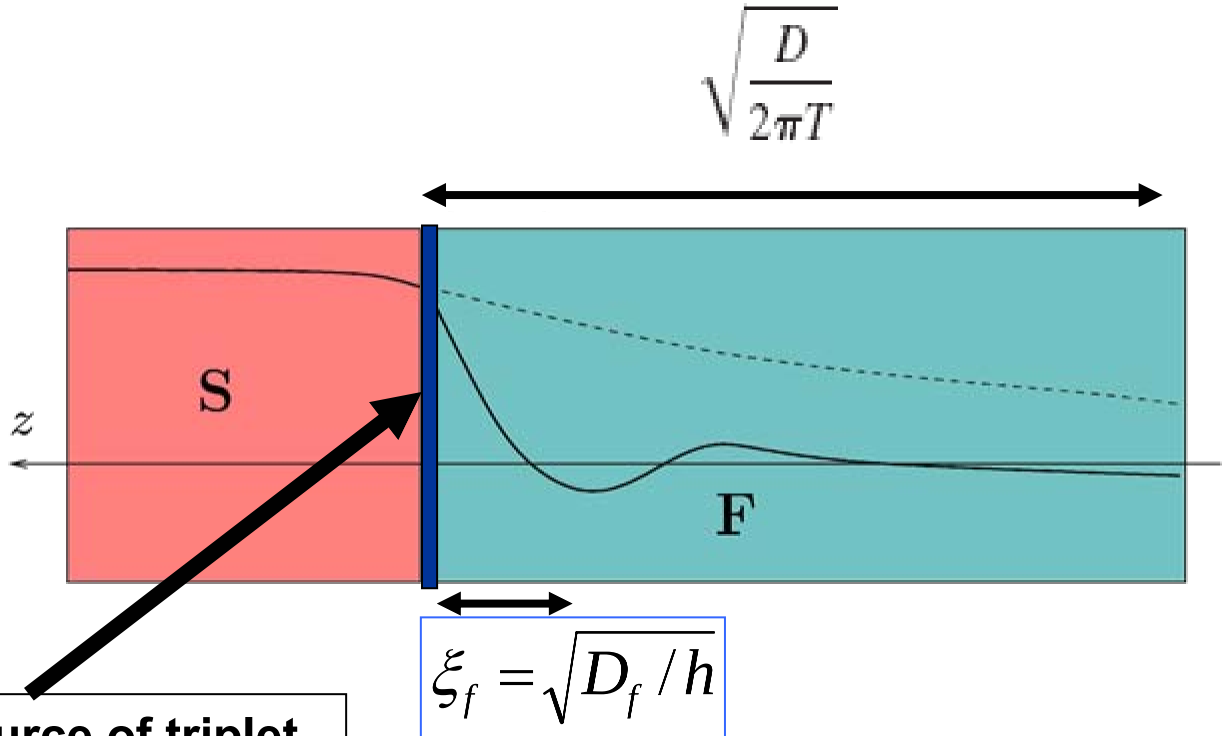
Thermal diffusion length L_T	$\sqrt{\frac{D}{2\pi T}}$
Superconducting coherence length ξ_s	$\frac{v_{Fs}}{2\pi T_c}$ in pure limit $\sqrt{\frac{D_s}{2\pi T_c}}$ in dirty limit
Superconducting correlation decay length ξ_{1f} in a ferromagnet	$\frac{v_{Ff}}{2\pi T}$ in pure limit $\xi_f = \sqrt{\frac{D_f}{h}}$ in dirty limit
Superconducting correlation oscillating length ξ_{2f} in a ferromagnet	$\frac{v_{Ff}}{2h}$ in pure limit $\xi_f = \sqrt{\frac{D_f}{h}}$ in dirty limit

Triplet proximity effect may substantially increase the decaying length in the dirty limit.

The same, but larger amplitude

$$\sqrt{\frac{D}{2\pi T}}$$

No oscillations



Some source of triplet correlations ?

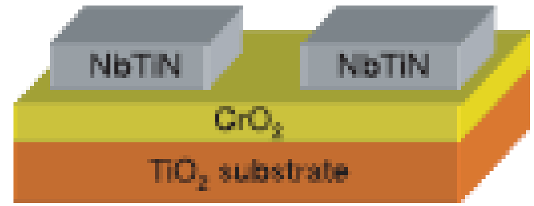
Why difficult to observe ?

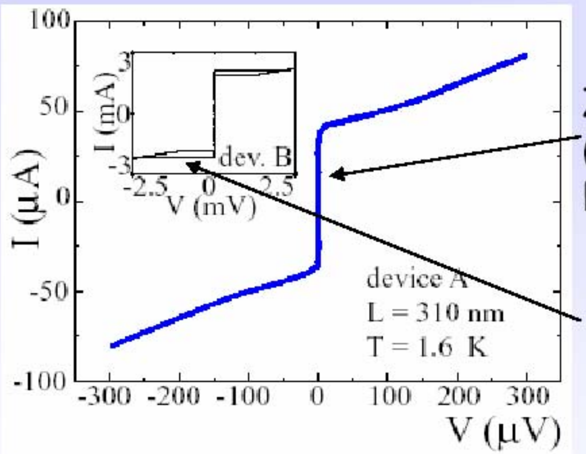
Magnetic scattering and spin-orbit scattering are harmful for long ranged triplet component.

Magnetic disorder, spin-waves…

Supercurrent measured in NbTiN/CrO2/NbTiN junctions

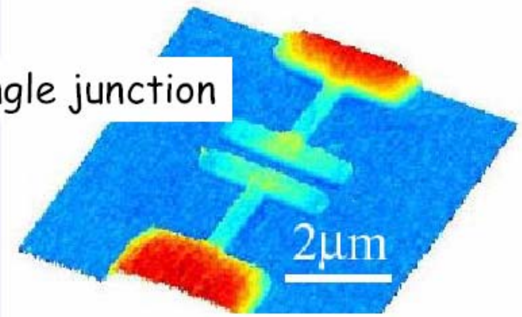
Klapwijk's group in Delft



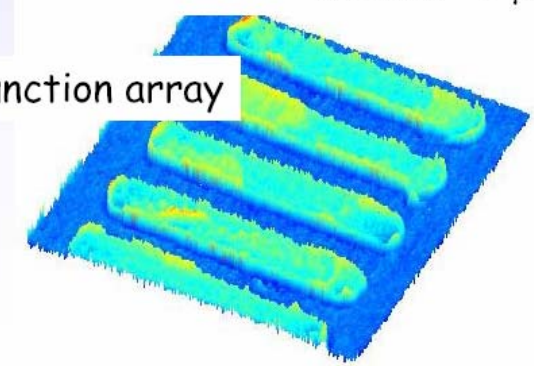


Zero resistance (supercurrent) branch

High I_c devices show hysteresis

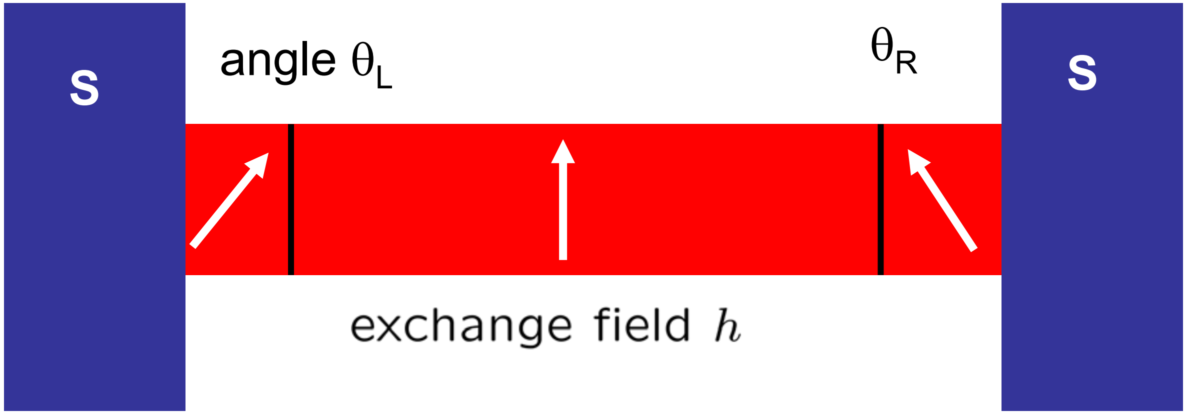


Typical junction length:
300nm - 1 µm



Long junctions with « large » I_c

CrO_2 is half-metallic !



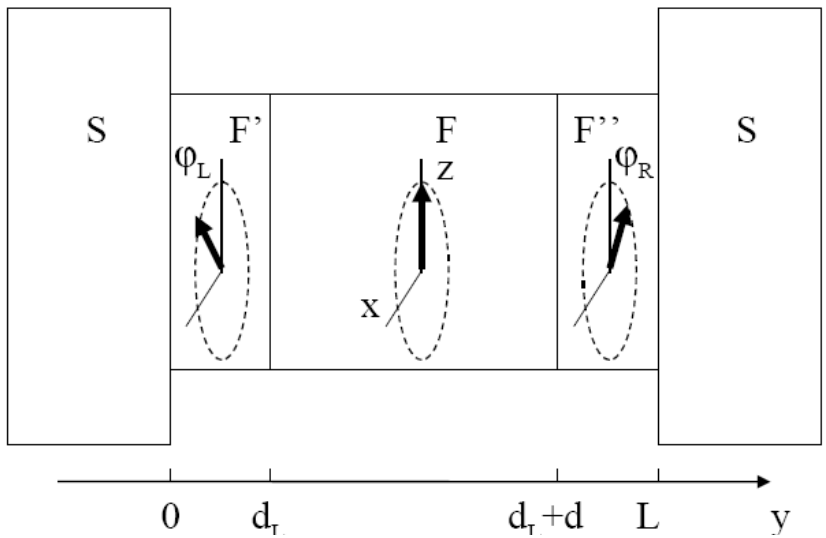


FIG. 1: Geometry of S/F'/F/F''/S junction. The arrows indicate non-colinear orientations of magnetizations in each layer with thickness d_L, d, d_R, respectively ($L = d_L + d + d_R$).

$\xi_f \ll L \ll \xi_0$

$$eR_F I_c = -\frac{2\Delta(T)^2 h_0^2}{\pi^3 T_c^3} \sin\theta_R \sin\theta_L$$

(+ small term)

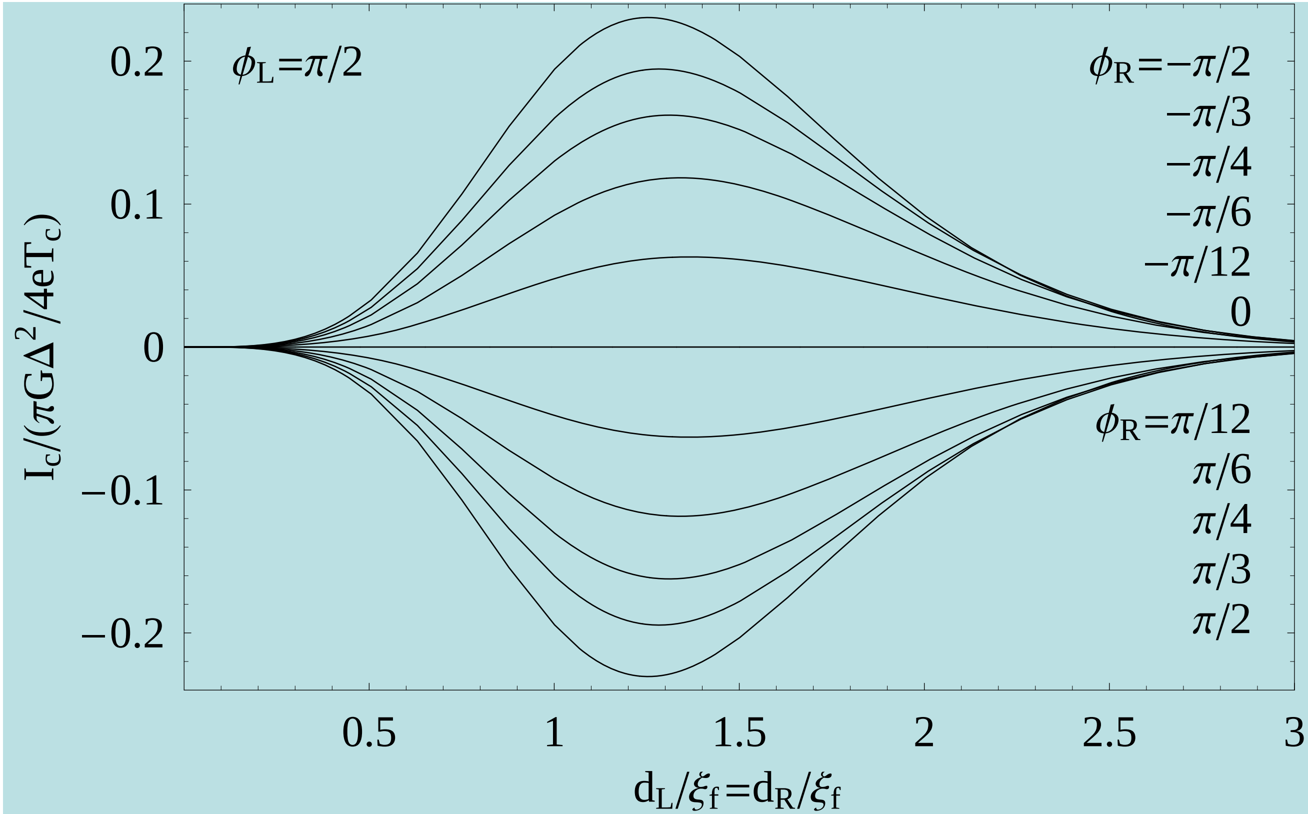


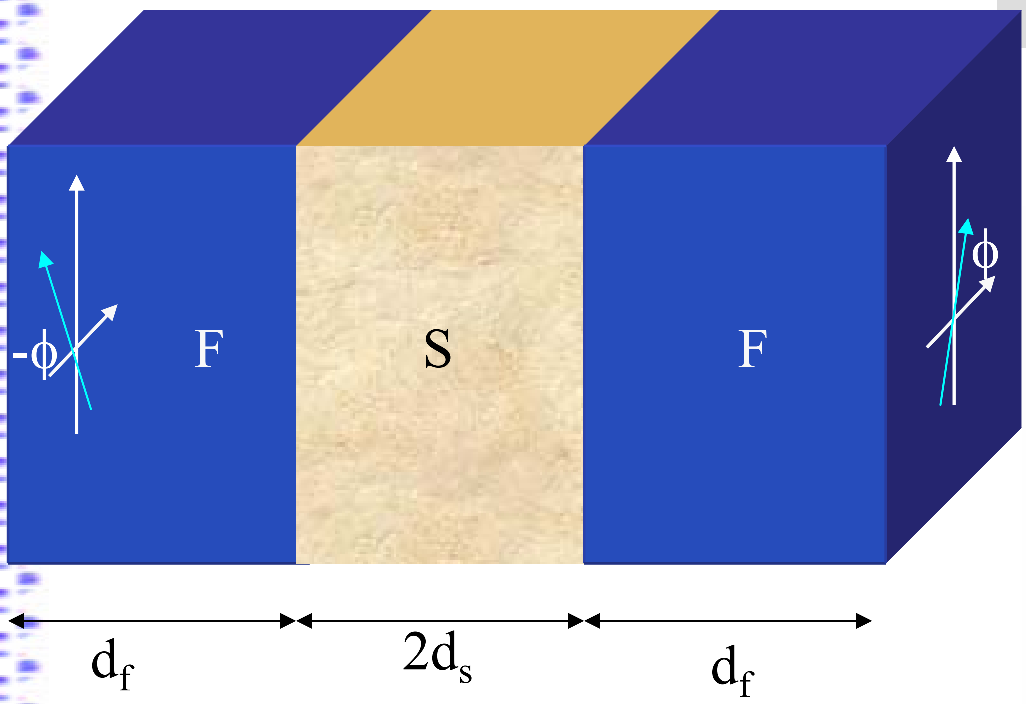
FIG. 2: Critical current induced by long range triplet proximity effect in S/F'/F/F"/S junction, in units of $(\pi G\Delta(T)^2/4eT_c)$, for varying length of F' and F" layers, at $d_L = d_R \sim \xi_f \ll d \ll \xi_0$, and for different orientations of the magnetization in the layers.

Rather sharp maximum of the critical current at $d_L=d_R=\xi_f$

More details - M. Houzet and A. Buzdin « Long range triplet Josephson effect through a ferromagnetic trilayer », PRB, 2007.

F/S/F trilayers, spin-valve effect

If d_s is of the order of magnitude of ξ_s, the critical temperature is controlled by the proximity effect.



Firstly the FI/S/FI trilayers has been studied experimentally in 1968 by Deutscher et Meunier.
In this special case, we see that the critical temperature of the superconducting layers is reduced when the ferromagnets are polarized in the same direction

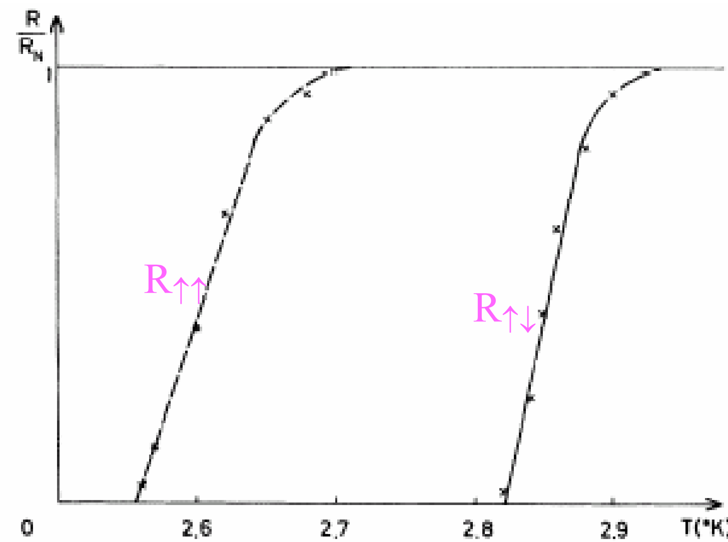


FIG. 1. Resistive measurements of the critical temperatures (R_N = resistance in the normal state) in zero field after the following: dashed line, application of 10 000 G ($T_{c\uparrow\uparrow}$) (all fields are applied parallel to the plane of the films); solid line, application of −10 000 G and subsequently +300 G to return the magnetization of the FeNi layer ($H_1 < 300$ G $< H_2$) ($T_{c\uparrow\downarrow}$).

In the dirty limit, we used the quasiclassical Usadel equations to find the new critical temperature T^*_c.
We solved it self-consistently supposing that the order parameter can be taken as :

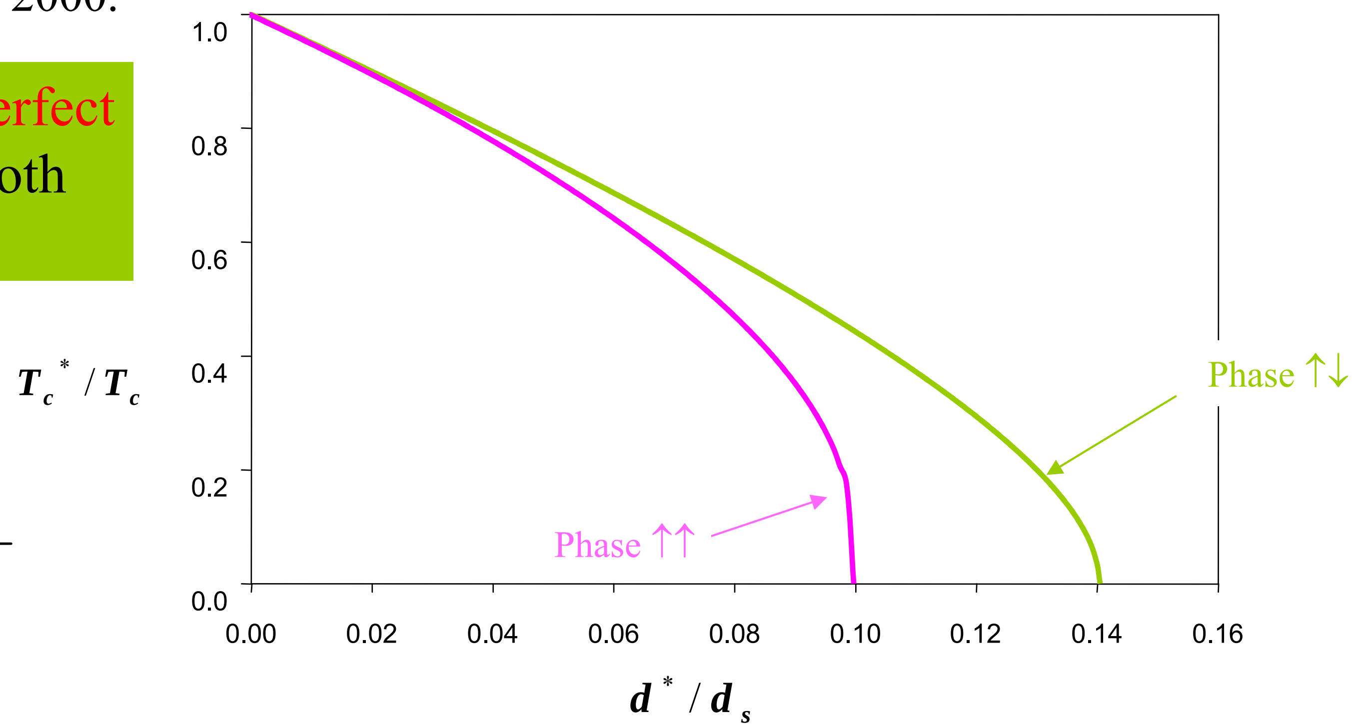
$$\Delta = \Delta_0 \left(1 - \frac{x^2}{L^2}\right)$$

with $L >> d_S$

Buzdin, Vedyaev, Ryazhanova, Europhys Lett. 2000,
Tagirov, Phys. Rev. Let. 2000.

In the case of a perfect transparency of both interfaces

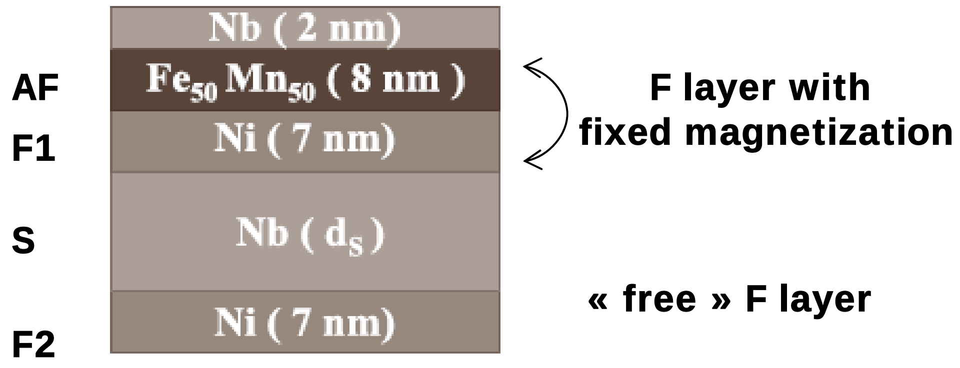
$$d^* = \gamma \sqrt{\frac{h}{D_n}} \frac{D_s}{4\pi T_c}$$

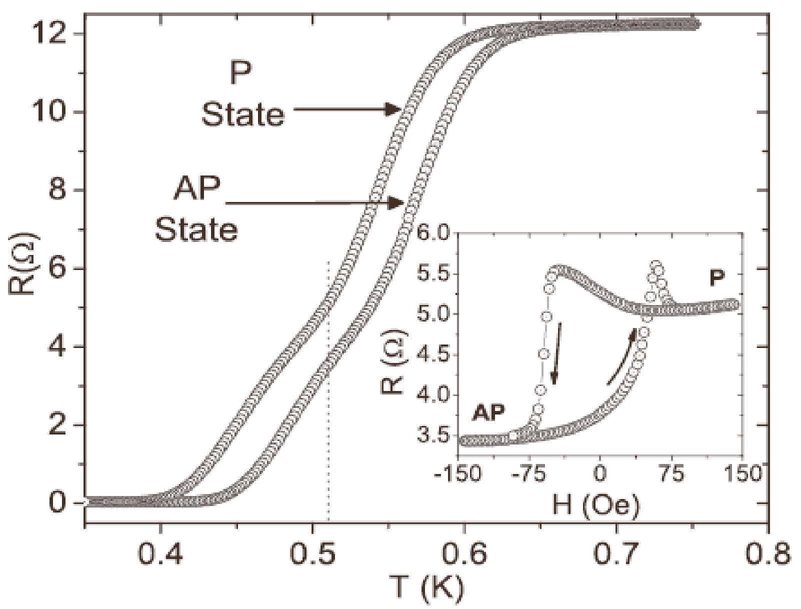


$$\ln\left(\frac{T_{c\uparrow\uparrow}^*}{T_c}\right) = \Psi\left(\frac{1}{2}\right) - \mathrm{Re}\,\Psi\left(\frac{1}{2} + \frac{d^* T_c}{d_s T_{c\uparrow\uparrow}^*}(1+i)\right)$$

$$\ln\left(\frac{T_{c\uparrow\downarrow}^*}{T_c}\right) = \Psi\left(\frac{1}{2}\right) - \Psi\left(\frac{1}{2} + \frac{d^* T_c}{d_s T_{c\uparrow\downarrow}^*}\right)$$

Recent experimental verifications

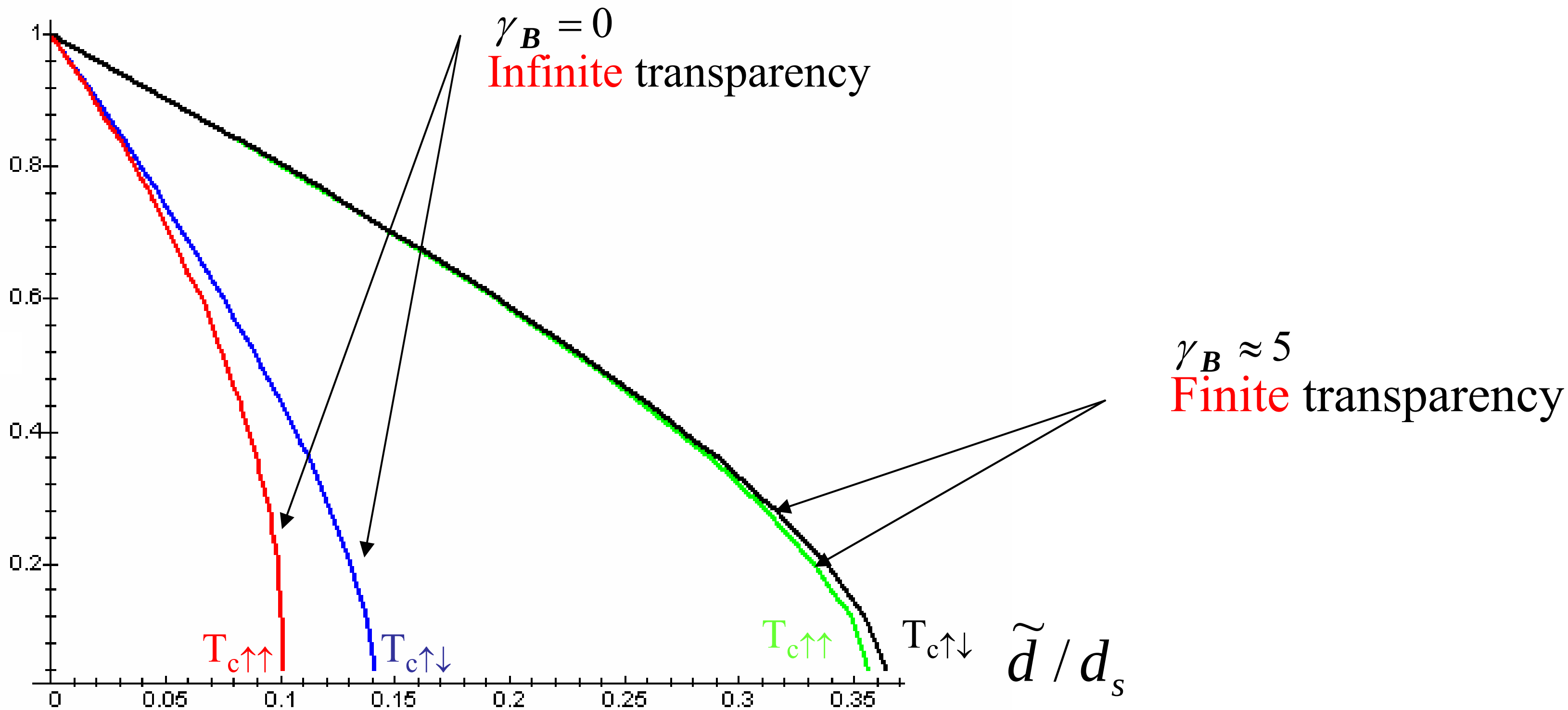




CuNi/Nb/CuNi
Gu, You, Jiang, Pearson, Bazaliy, Bader, 2002

Ni/Nb/Ni
Moraru, Pratt Jr, Birge, 2006

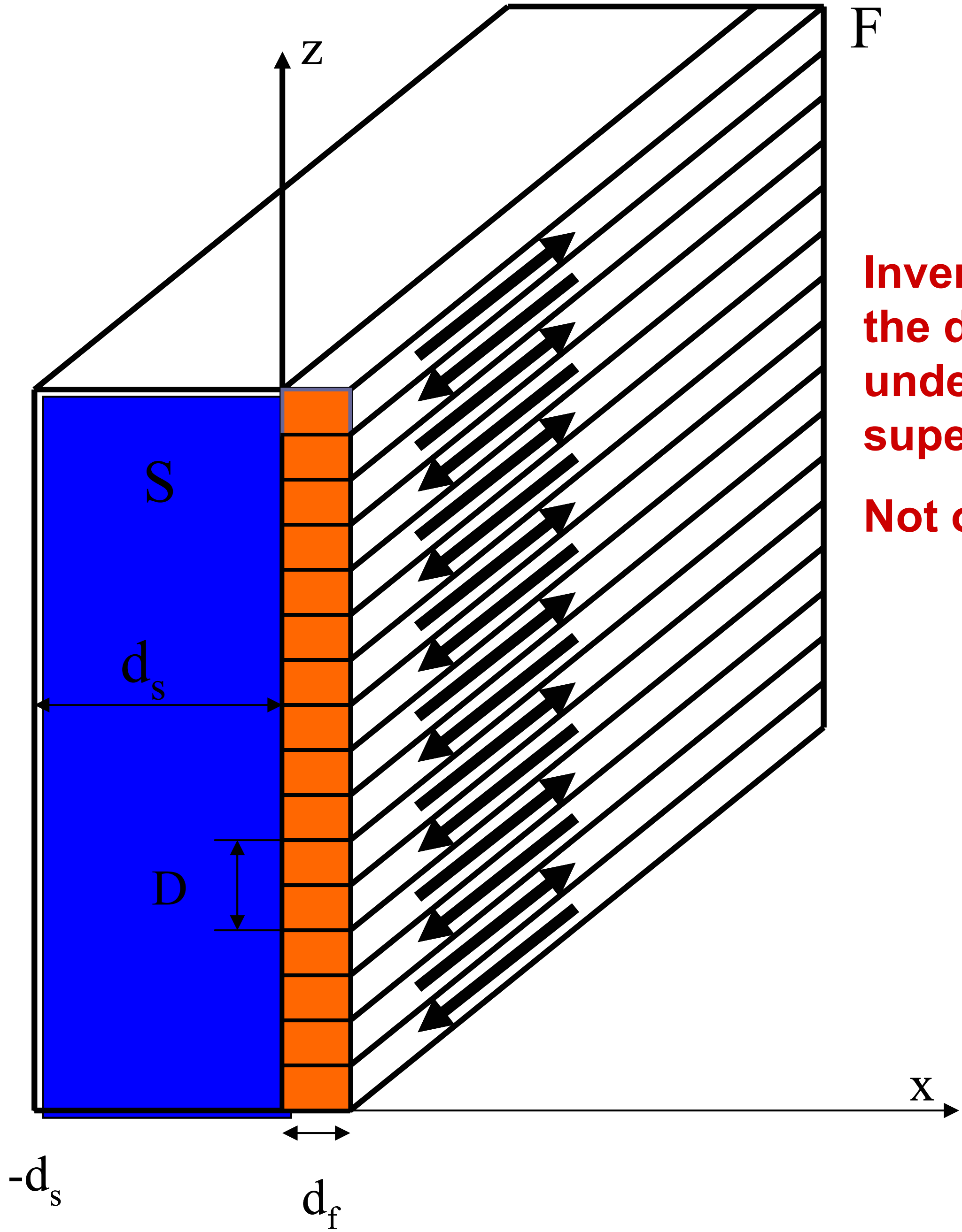
Evolution of the difference between the critical temperatures as a function of interfaces' transparency



$$\ln\left(\frac{T_{c\uparrow\uparrow}^{*}}{T_c}\right) = \Psi\left(\frac{1}{2}\right) - \mathrm{Re}\,\Psi\left(\frac{1}{2} + \frac{\tilde{d}T_c}{d_s T_{c\uparrow\uparrow}^{*}}(1+i)\right)$$

$$\ln\left(\frac{T_{c\uparrow\downarrow}^{*}}{T_c}\right) = \Psi\left(\frac{1}{2}\right) - \Psi\left(\frac{1}{2} + \left(\frac{\tilde{d}T_c}{d_s T_{c\uparrow\downarrow}^{*}}\right)\right)$$

$$\tilde{\boldsymbol{d}} = \frac{\boldsymbol{D_s}}{4\pi \boldsymbol{T_c}} \frac{\gamma\sqrt{\frac{\boldsymbol{h}}{\boldsymbol{D_n}}}}{1+(1+\boldsymbol{i})\gamma_{\boldsymbol{B}}\gamma\sqrt{\frac{\boldsymbol{h}}{\boldsymbol{D_n}}}}$$



Inverse effect: appearence of the dense domain structure under the influence of superconductivity.

Not observed yet.

$D \ll \xi$

Ni$_{0.80}$Fe$_{0.20}$/Nb (20nm)

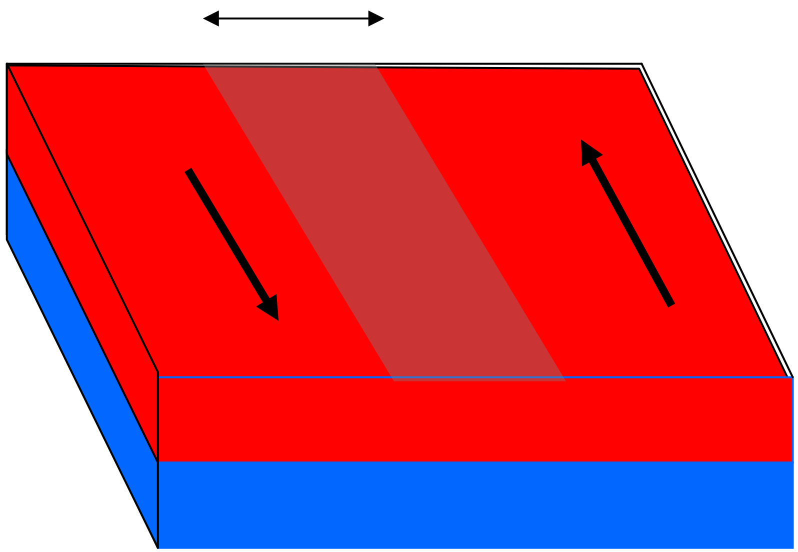
Thin films : Néel domains

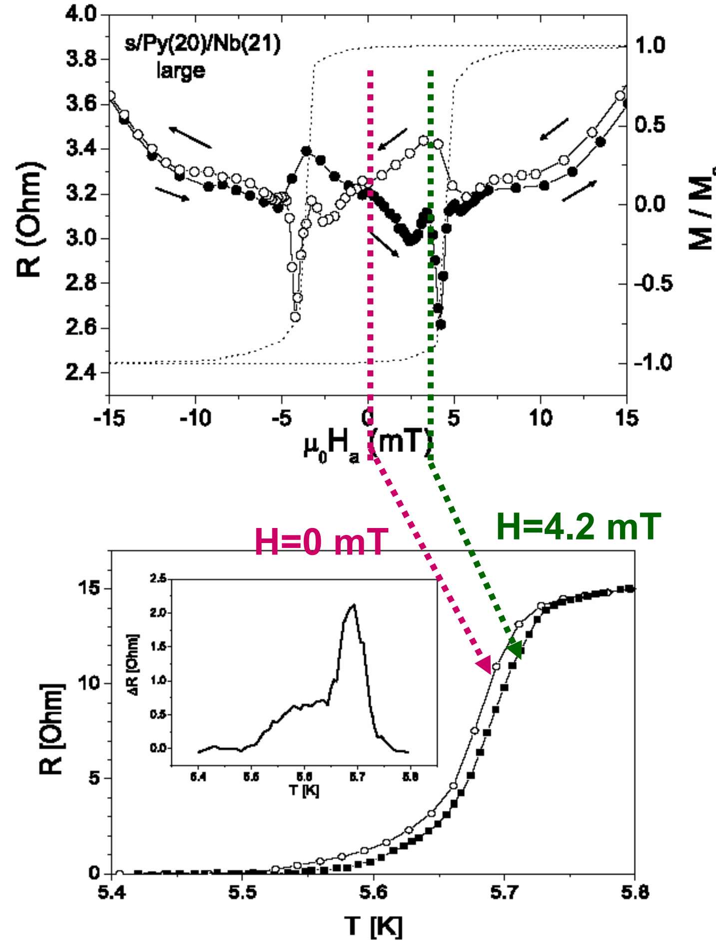
Rusanov et al., PRL, 2004

Similar physics in F/S bilayers

In practice, magnetic domains appear quite easily in ferromagnets

w: width of the domain wall



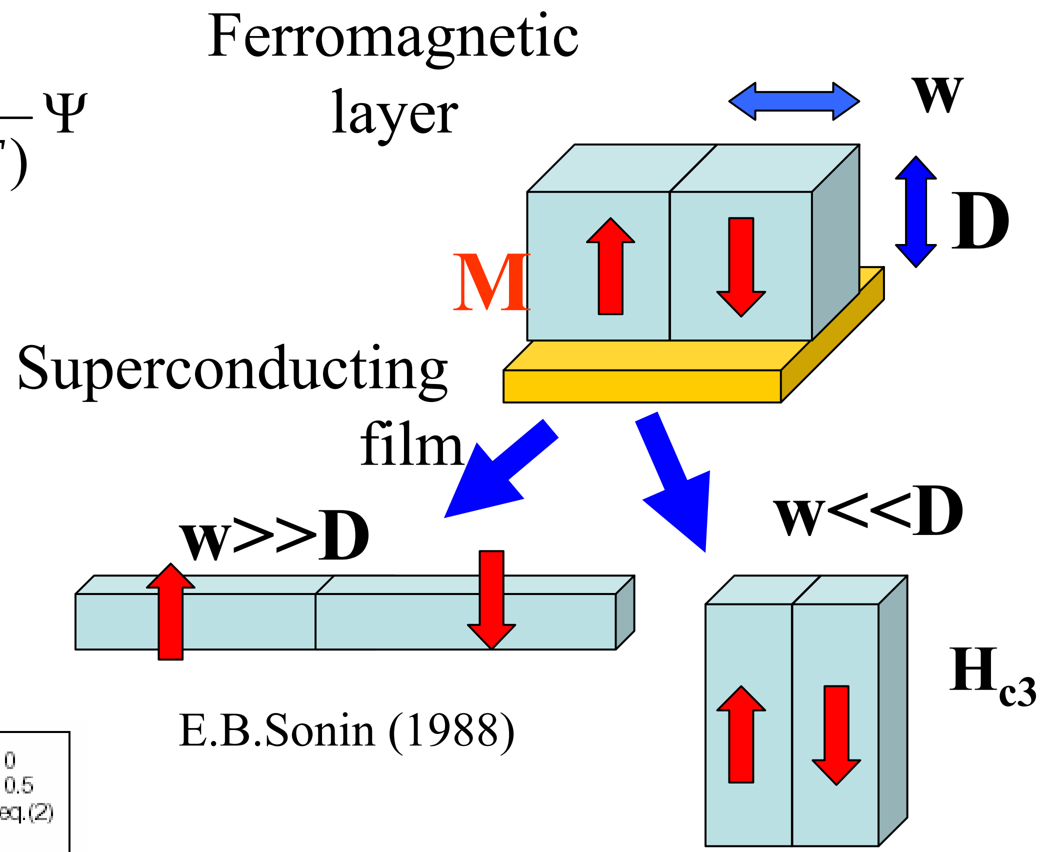


Localized (domain wall) superconducting phase.

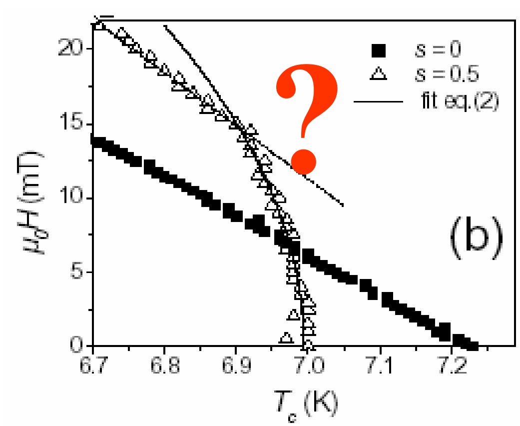
Theory - Houzet and Buzdin, Phys. Rev. B (2006).

Domain wall superconductivity in purely electromagnetic model

$$-\left(\nabla + \frac{2\pi i}{\Phi_0}\vec{A}(\vec{r})\right)^2 \Psi = \frac{1}{\xi^2(T)}\Psi$$



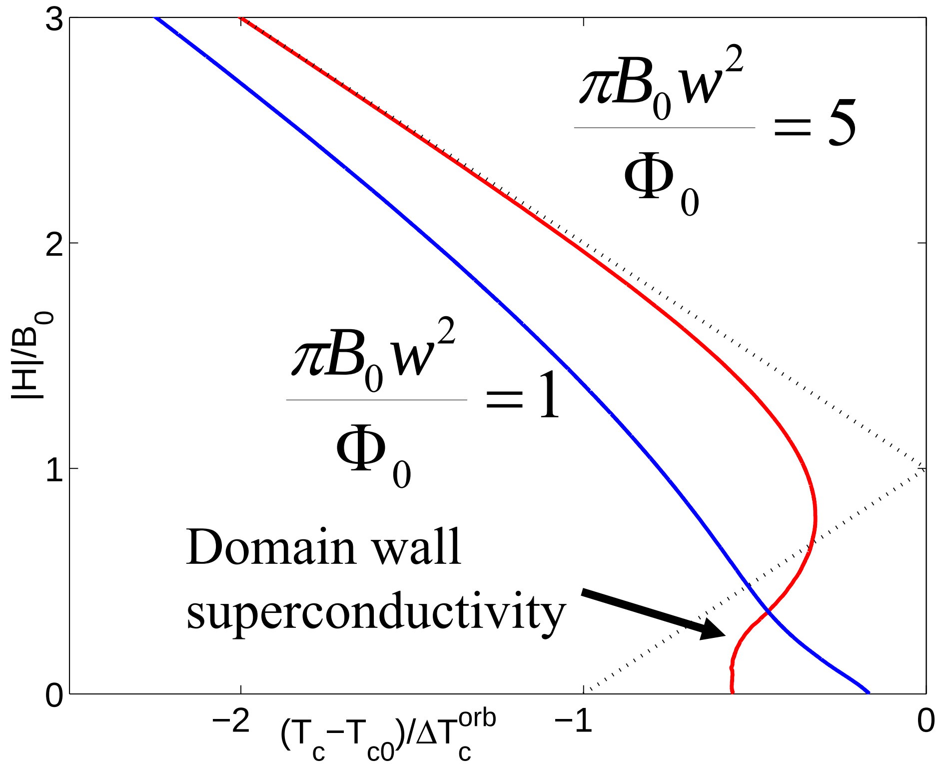
E.B.Sonin (1988)

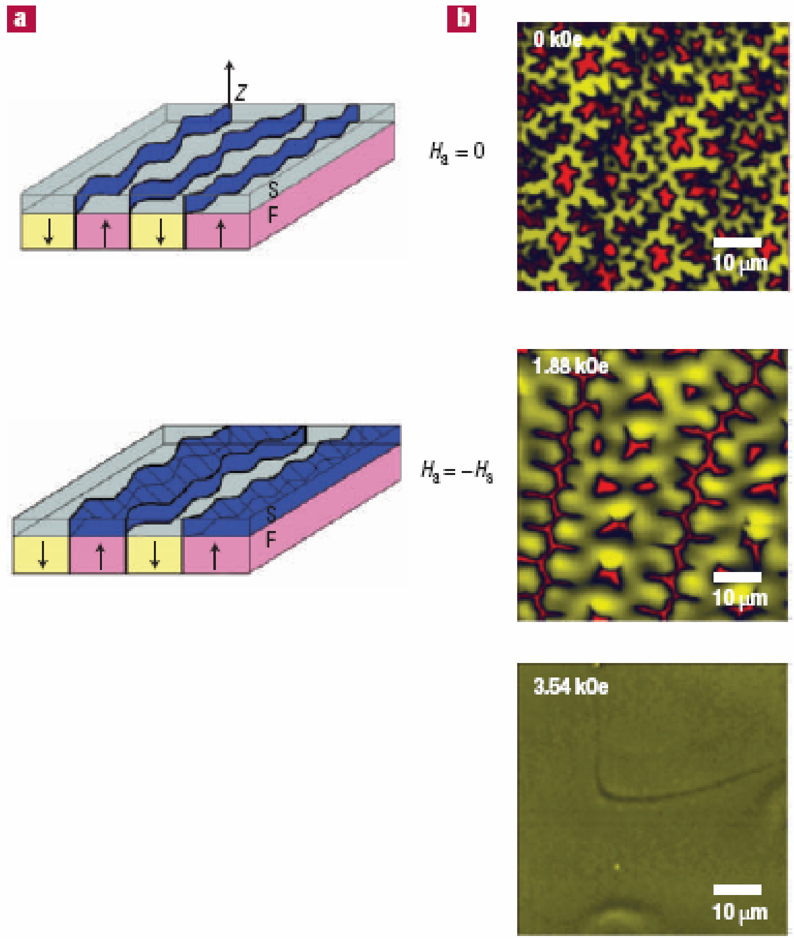


Pb-Co/Pt

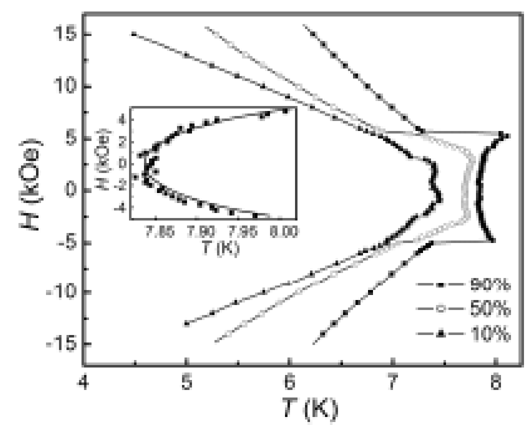
Superconducting nucleus in a periodic domain structure in an external field

$H \neq 0$





$Nb/BaFe_{12}O_{19}$

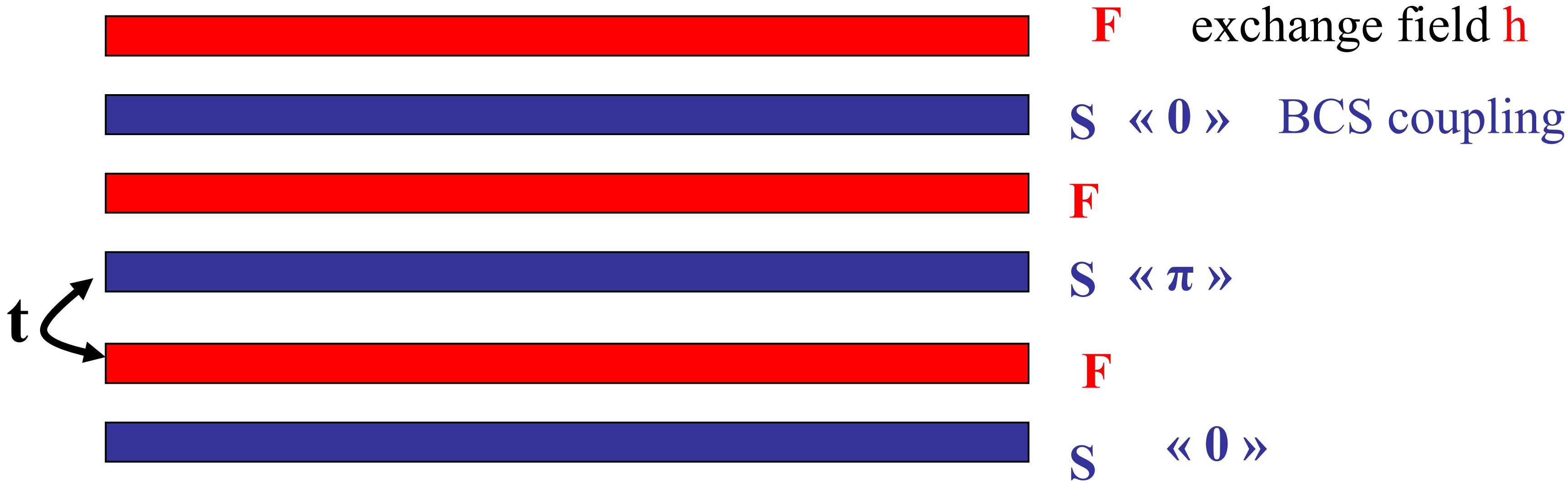


Z. YANG et al, Nature Materials, 2004

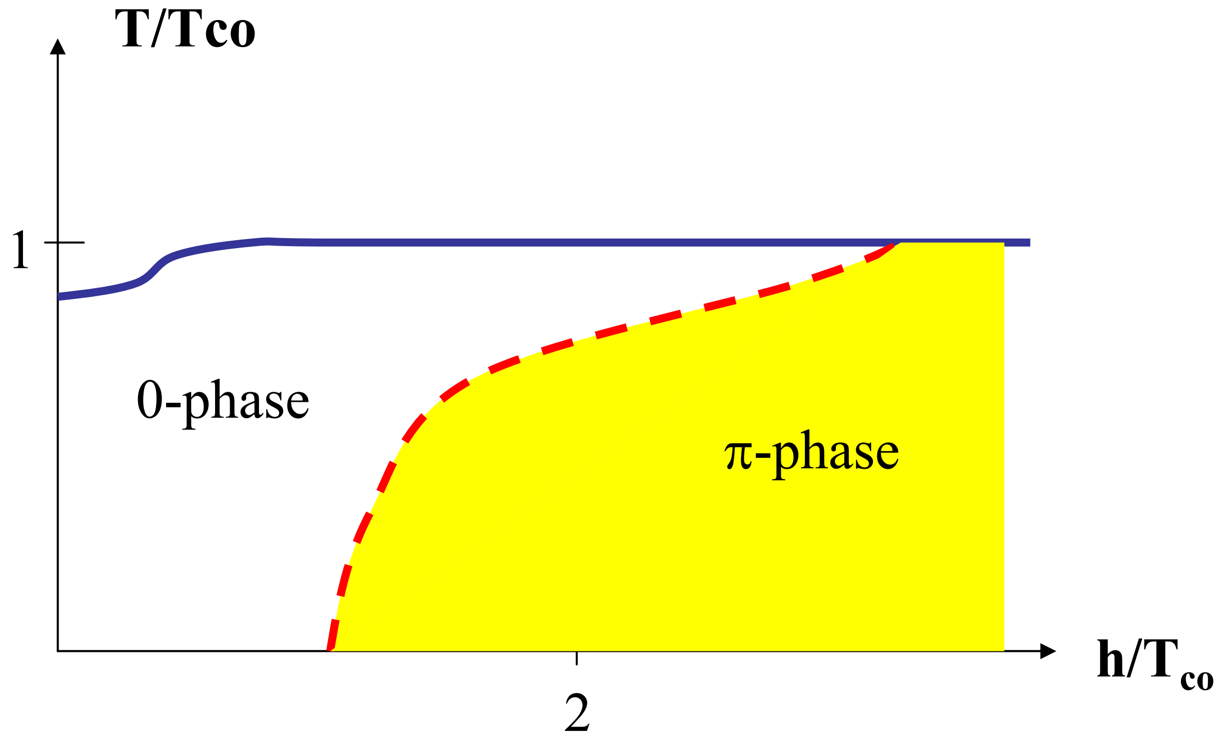
Atomic layered S-F systems

(Andreev et al, PRB 1991, Houzet et al, PRB 2001, Europhys. Lett. 2002)

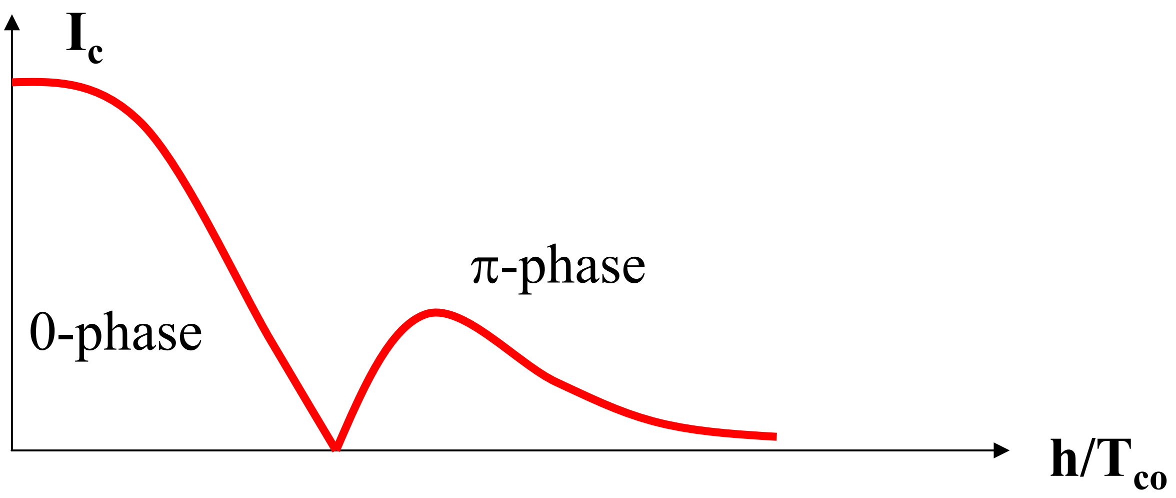
Magnetic layered superconductors like $RuSr_2GdCu_2O_8$



Also even for the quite small exchange field ($h>T_c$) the **π-phase** must appear.



The limit **t** <<T_{co}



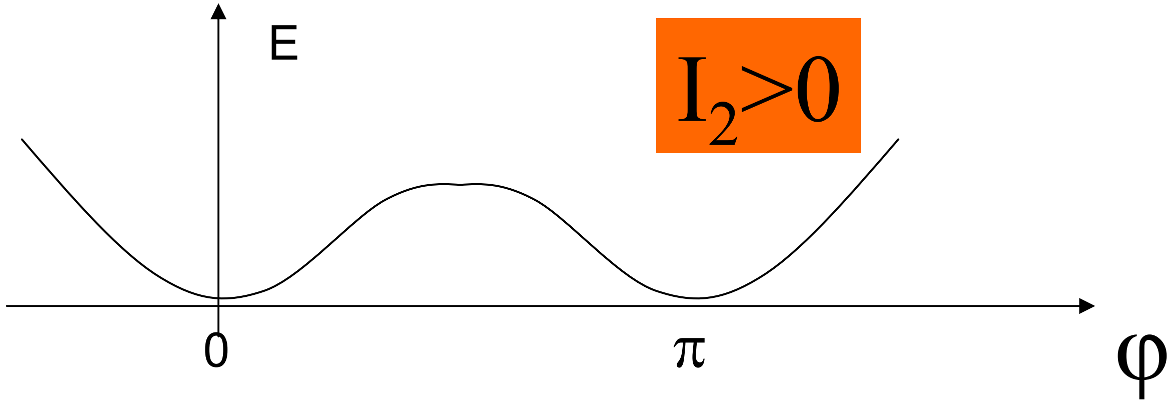
How the transition from 0- to π – state occurs?

$J(\varphi)=I_c \sin\varphi$; $I_c>0$ in the **0- state and**

$I_c<0$ in the **π – state**

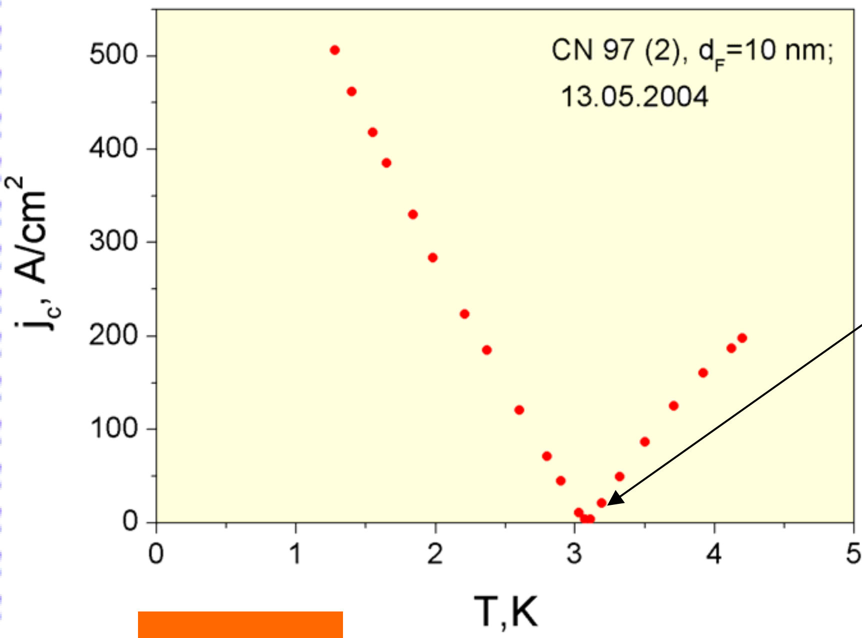
$J(\varphi)=I_1\sin\varphi + I_2\sin 2\varphi$

Energy $E(\varphi)=(\Phi_0/2\pi c)[-I_1\cos\varphi - (I_2/2)\cos 2\varphi]$



$I_2>0$

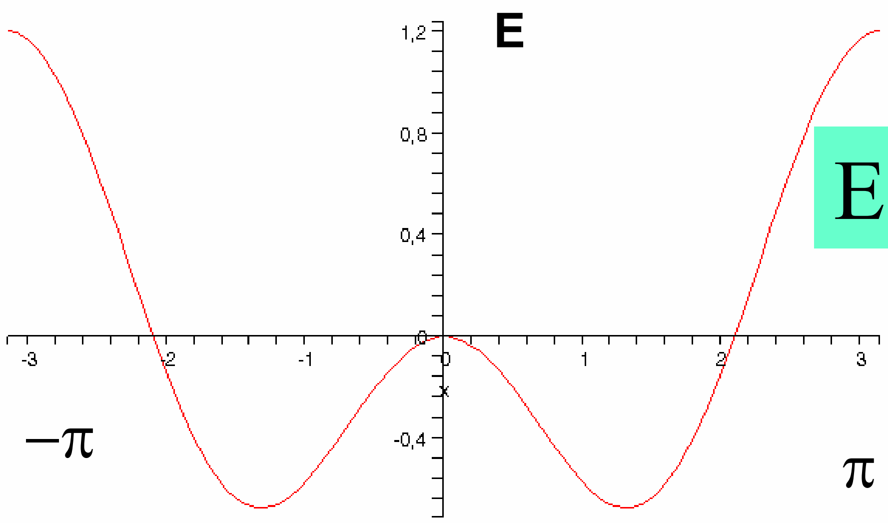
$I_c=|I_2|$



$J(\varphi)=I_2\sin 2\varphi$

The realization of the equilibrium phase difference $0<\varphi_0<\pi$

$I_2<0$



$E(\varphi=\varphi_0)=E(\varphi=-\varphi_0)$

Different mechanism for the φ_0 - Josephson junction realization.

Recently the broken inversion symmetry (BIS) superconductors (like $CePt_3Si$) have attracted a lot of interest.

Very special situation is possible when the weak link in Josephson junction is a non-centrosymmetric magnetic metal with broken inversion symmetry !

Suitable candidates : MnSi, FeGe.

Josephson junctions with time reversal symmetry: $j(-\varphi) = -j(\varphi)$;

i.e. higher harmonics can be observed $\sim j_n \sin(n\varphi)$ –the case the π junctions.

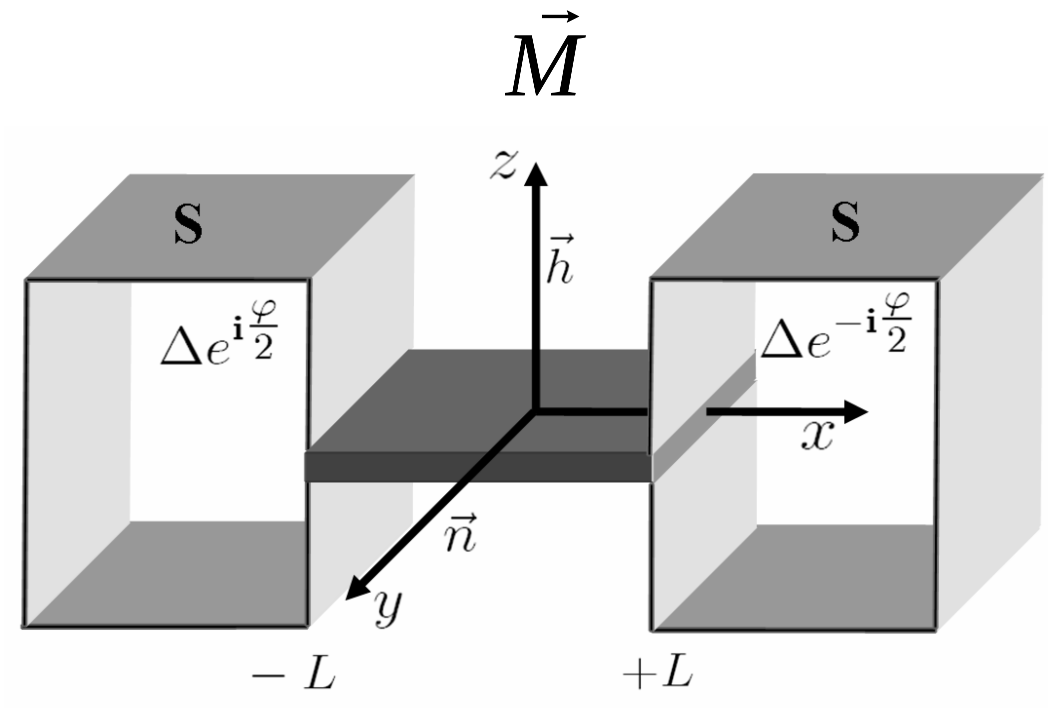
Without this restriction a more general dependence is possible

$$j(\varphi) = j_0 \sin(\varphi + \varphi_0).$$

Rashba-type spin-orbit coupling $\alpha\left(\vec{\sigma} \times \vec{p}\right) \cdot \vec{n}$

$\vec{n}$ is the unit vector along the asymmetric potential gradient.

Geometry of the junction with BIS magnetic metal



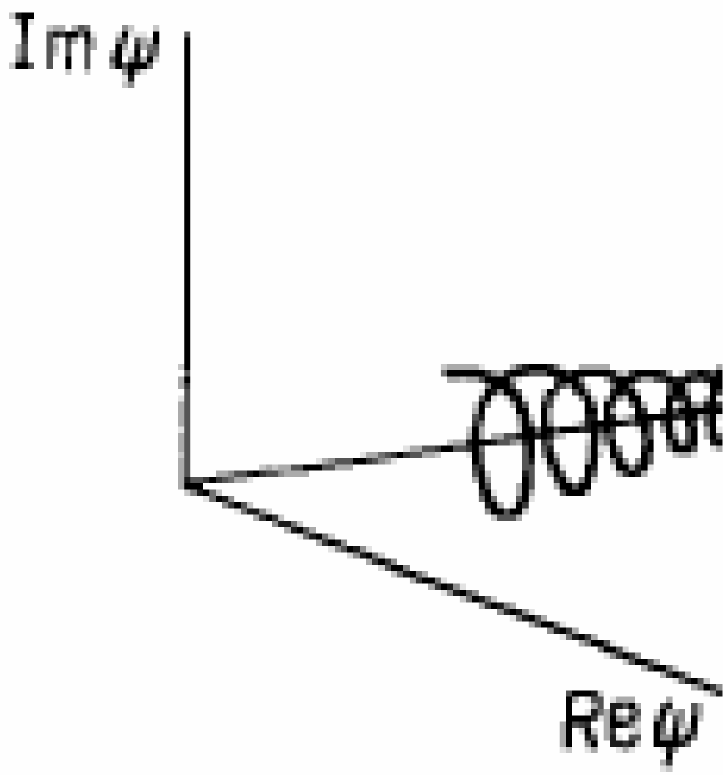
$$F = a|\Psi|^2 + \gamma|\vec{D}\Psi|^2 + \frac{b}{2}|\Psi|^4 - \varepsilon\vec{n}\left[\vec{h} \times \left(\Psi\left(\vec{D}\Psi\right)^* + \Psi^*\left(\vec{D}\Psi\right)\right)\right],$$

$$D_i = -i\partial_i - 2eA_i$$

$$a\Psi - \gamma\frac{\partial^2\Psi}{\partial x^2} + 2i\varepsilon h\frac{\partial\ \Psi}{\partial x} = 0,$$

$$\Psi \propto \exp(i\tilde{\varepsilon}x)\exp(-x\sqrt{\frac{a-a_c}{\gamma}}), \quad where\ \tilde{\varepsilon} = \frac{\varepsilon h}{\gamma}$$

$$\Psi \propto \exp(i\tilde{\varepsilon}x)\exp\left(-x\sqrt{\frac{a-a_c}{\gamma}}\right), \quad where\ \tilde{\varepsilon} = \frac{\varepsilon h}{\gamma}$$



In contrast with a π junction it is not possible to choose a real Ψ function !

φ_0 Josephson junction

$$j(\varphi) = j_c \sin(\varphi + \varphi_o)$$

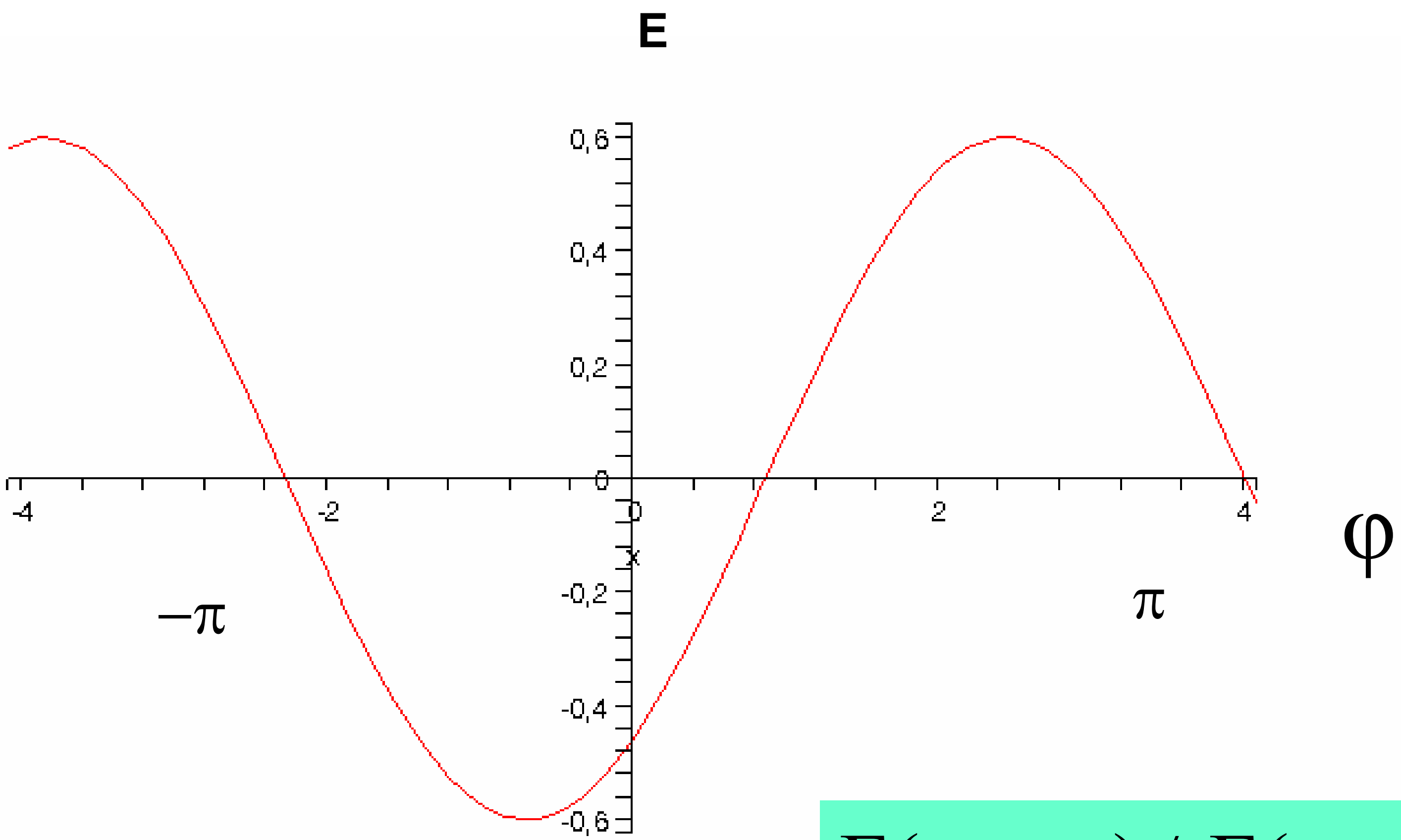
where $$\varphi_o = \frac{2\varepsilon hL}{\gamma}$$

The phase shift φ_0 is proportional to the length and the strength of the BIS magnetic interaction.

The φ_0 Junction is a natural phase shifter.

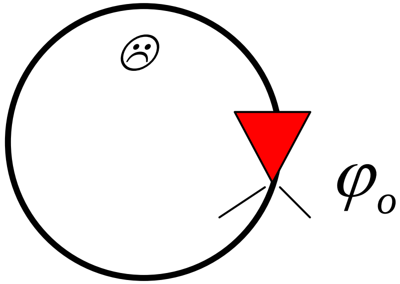
Energy $E_J(\varphi) \sim -j_c \cos(\varphi + \varphi_0)$

$$E_J(\varphi) \sim -j_c \cos(\varphi + \varphi_0)$$



$$E(\varphi = \varphi_0) \neq E(\varphi = -\varphi_0)$$

Spontaneous flux (current) in the superconducting ring with φ_0 - junction.



$$E(\varphi) = \frac{j_c}{2e}\left(-\cos(\varphi + \varphi_0) + \frac{k\varphi^2}{2} \right)$$

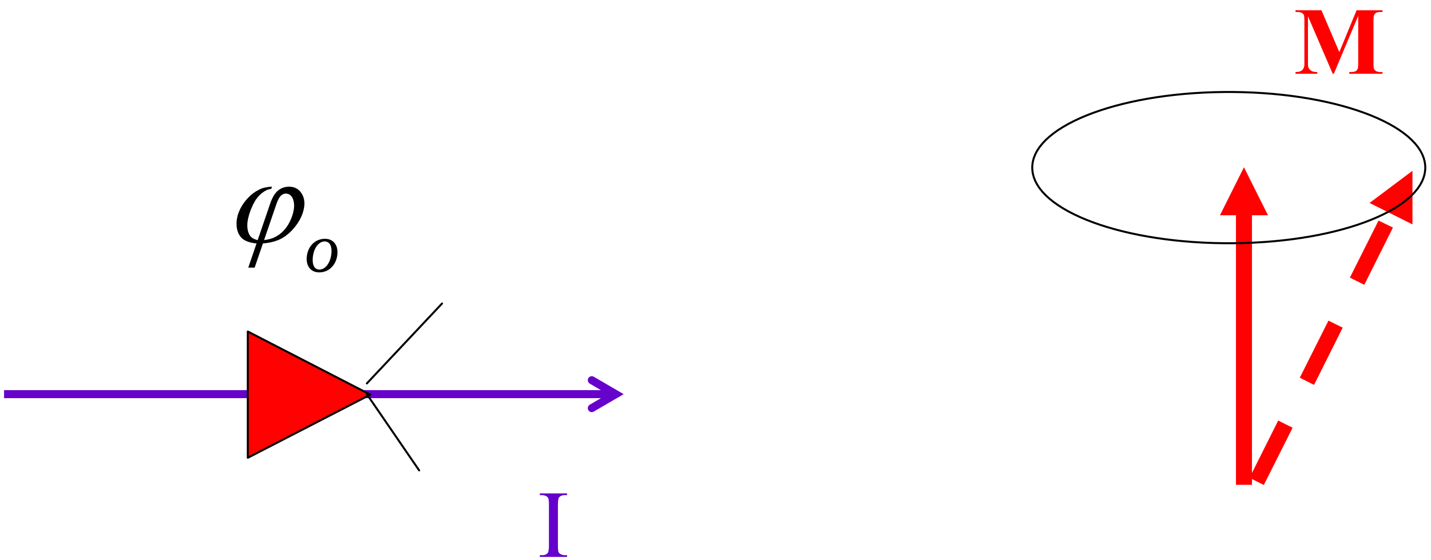
$$k = \frac{c\Phi_0}{2\pi L j_c}$$

In the k<<1 limit the junction generates the flux $\Phi = \Phi_0(\varphi_0/2\pi)$

$$\varphi_o = \frac{2\varepsilon h L}{\gamma}$$

Very important : The φ_0 junction provides a mechanism of a direct coupling between supercurrent (superconducting phase) and magnetic moment (z component).

Applying to the φ_0 - junction the current (phase difference) we can generate the magnetic moment rotation.



Inversely the magnetic moment precession would create the a. c. current.

More – see F. Konschelle and A. Buzdin –Cond. Mat. 2008 (submitted to PRL).

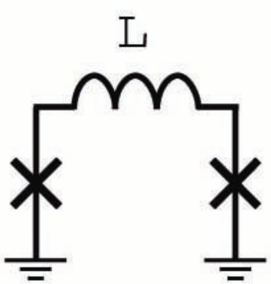
Complementary Josephson logic

RSFQ-logic using π-shifters

A.V.Ustinov, V.K.Kaplunenko. Journ. Appl. Phys. **94**, 5405 (2003)

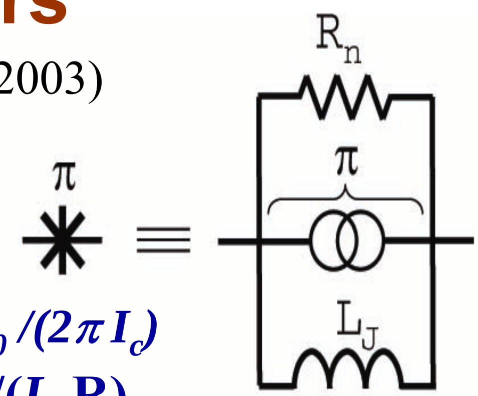
RSFQ- logic: Rapid Single Quantum logic

Conventional RSFQ-cell



$LI_c > \Phi_0$

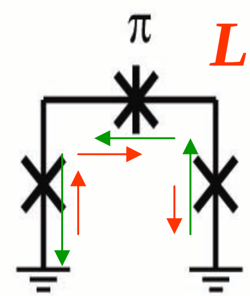
Fluxon memorizing cell



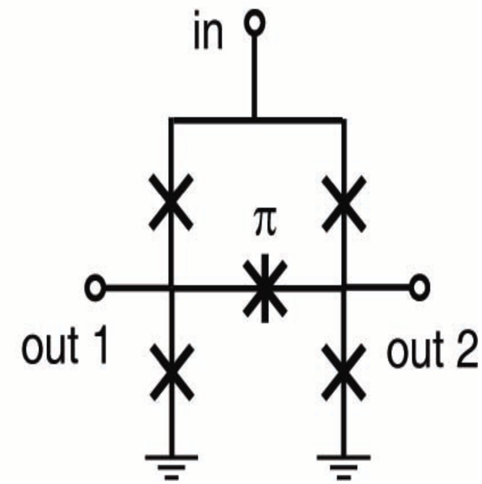
$L_J = \Phi_0/(2\pi I_c)$

$\tau \sim 1/(I_c R)$

RSFQ -*π*– cell



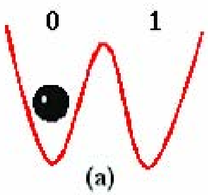
L → 0



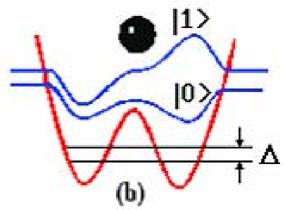
π-RSFQ –***Toggle Flip-Flop***

To operate at 20 GHz clock rate
$I_c R$~50 μV has to be
We have $I_c R$ > 0.1 μV for the present

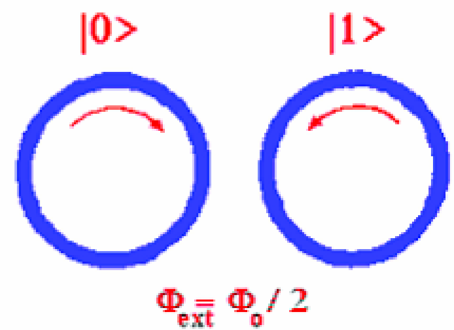
Superconducting phase qubit



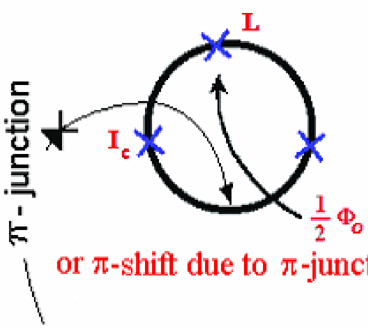
Digital bit



Quantum bit



or π-shift due to π-junction



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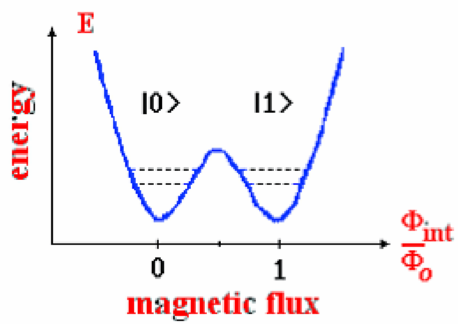

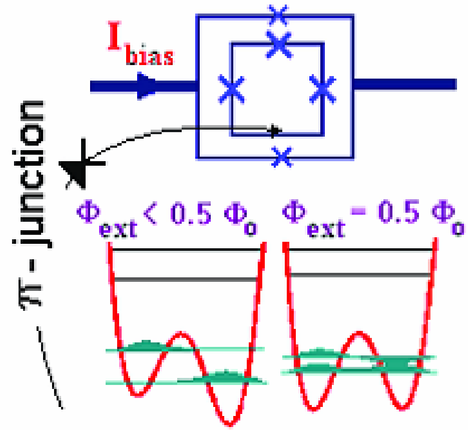

qubit operation

Conclusions

- **Superconductor-ferromagnet heterostructures permit to study superconductivity under huge exchange field ($h \gg T_c$).**

- **The π-junction realization in S/F/S structures is quite a general phenomenon.**

- **Domain wall superconductivity. Spin - valve effects.**

- **The BIS magnets provide a mechanism of the realization of the novel φ_0 - junctions with the very special properties.**

- **In these φ_0 - junctions a direct (linear) coupling between superconductivity and magnetism is realized. Seems to be Ideal for superconducting spintronics.**

Refs.: **Magnetic superconductors-** M. Kulic and A. Buzdin in **Superconductivity,** Springer, 2008 (eds. Benneman and Ketterson).

S/F proximity effect - A. Buzdin, Rev. Mod. Phys. (2005).

φ_0 - junctions -A. Buzdin, PRL (2008).

It seems that the perspectives for the applications are very interesting.

But…

In the theory there is no difference between practice and theory, but in practice there is.

(from the Conference on Symbolic Logic)