Magnetization plateaux: the intriguing case of $SrCu_2(BO_3)_2$

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Collaborators

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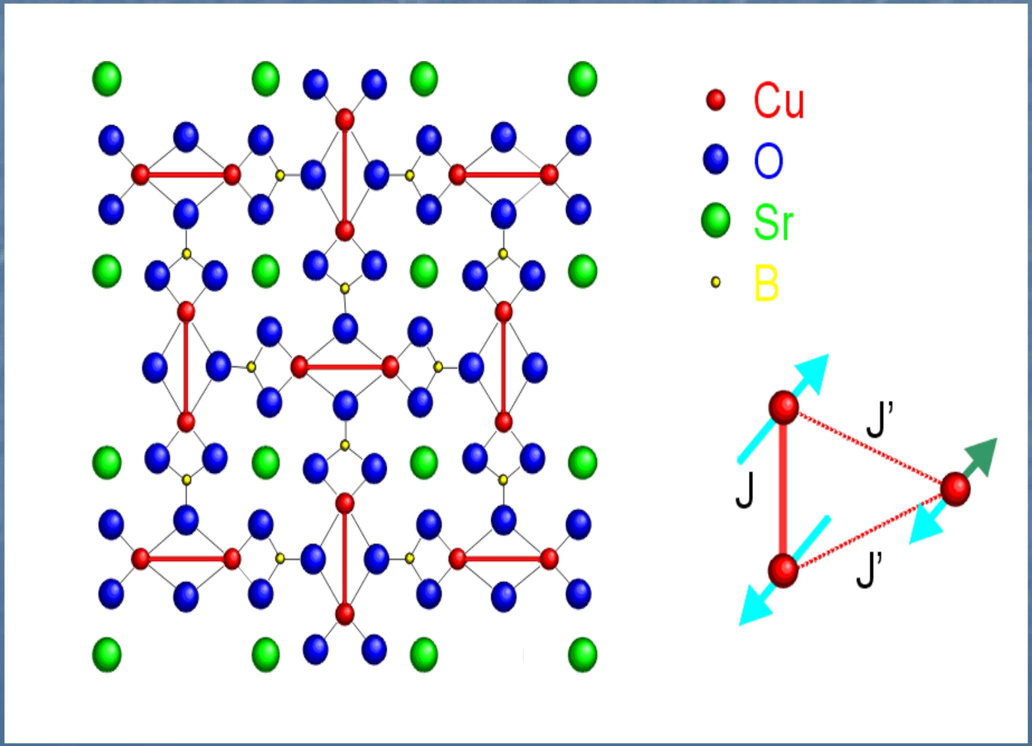
Scope

- $SrCu_2(BO_3)_2$: early experiments
- Mechanism of plateau formation
- Evidence of new plateaux: brief review
- Theory of plateaux: early results
- High order perturbation theory
- Conclusions and perspectives

$SrCu_2(BO_3)_2$

Kageyama et al, PRL '99

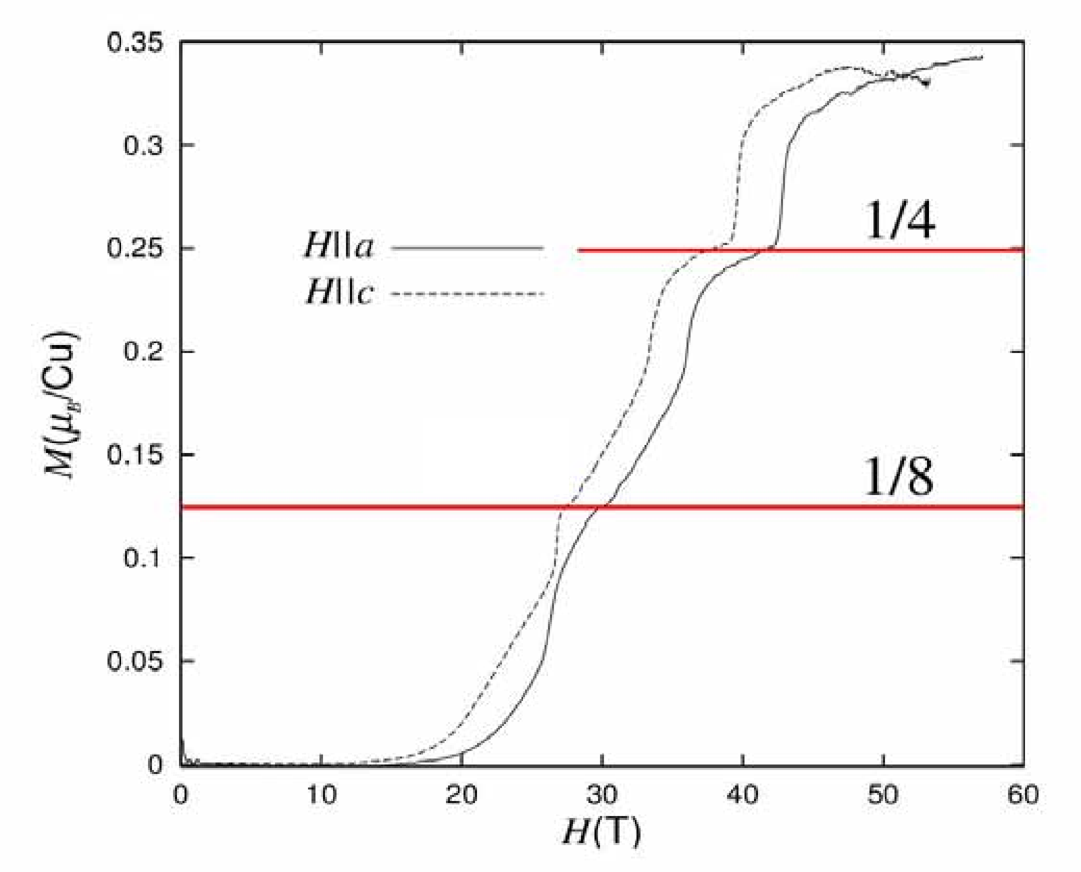


Cu^{2+} -> Spin 1/2

$J \simeq 85$ K

$J'/J \simeq 0.63$

Magnetization of $SrCu_2(BO_3)_2$

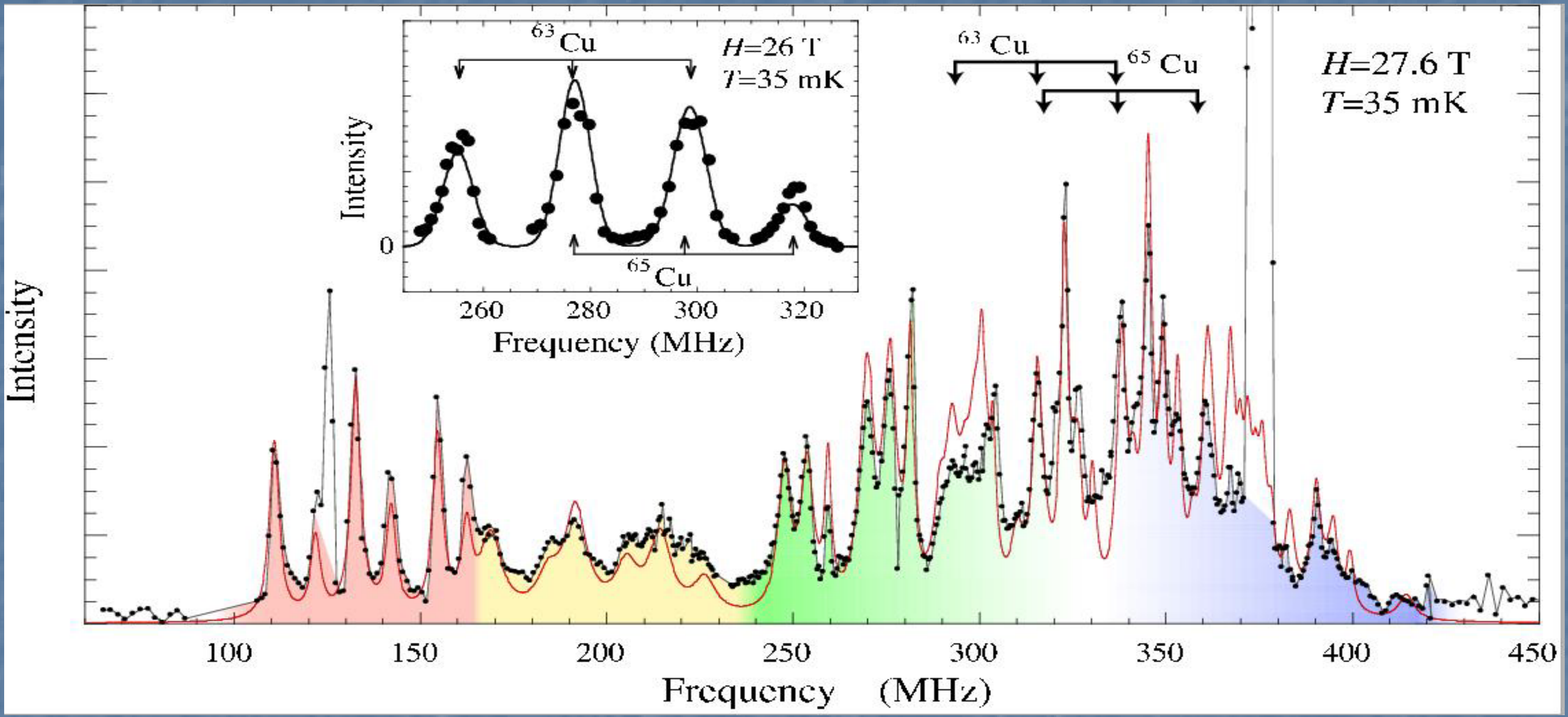


Kageyama et al
PRL '99

Plateaux

- M=0
- M=1/8
- M=1/4
- M=1/3

NMR at 1/8-plateau



Many different sites → **Broken translation symmetry**

K. Kodama, M. Takigawa, M. Horvatic, C. Berthier, H. Kageyama,
Y. Ueda, S. Miyahara, F. Becca, FM, Science '02

Heisenberg model

$$\mathcal{H} = \sum_{i,j} J_{ij} \vec{S}_i . \vec{S}_j + H \sum_i S_i^z$$

SU(2)

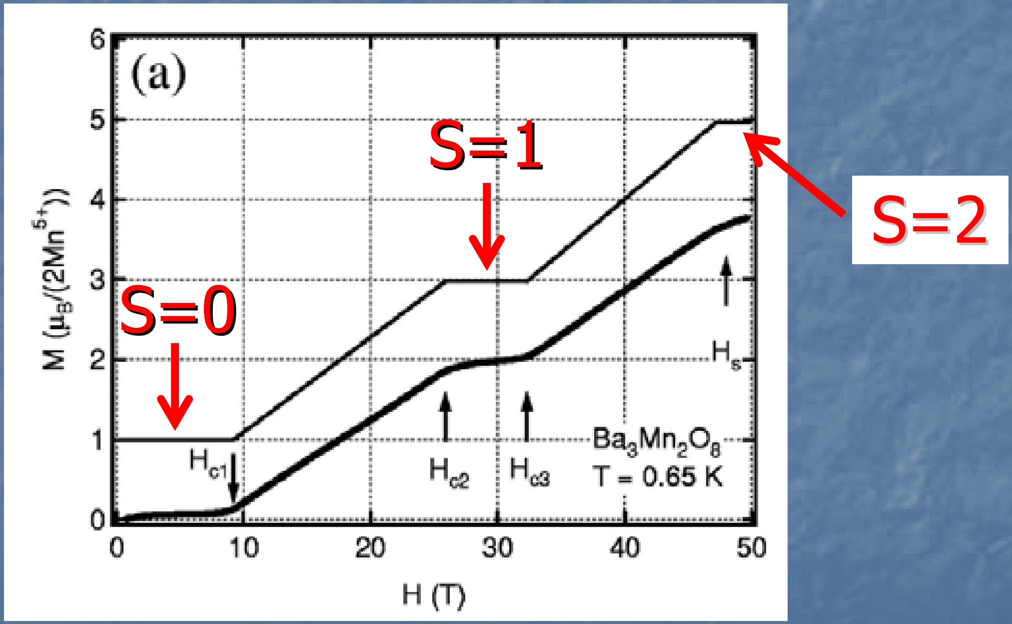
U(1) spin rotation around z

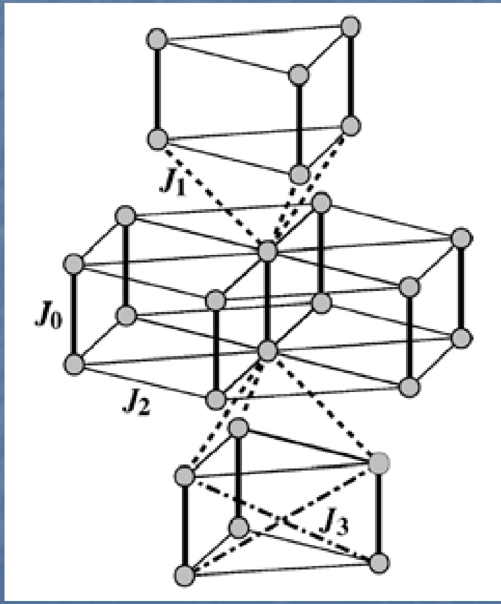
+ spatial symmetries
(translations and point group)

‘Natural’ plateaux

No broken translational symmetry

Trimerized chain, Hida, JPSP ’94; Odd-leg ladders, Cabra et al, PRL ’97





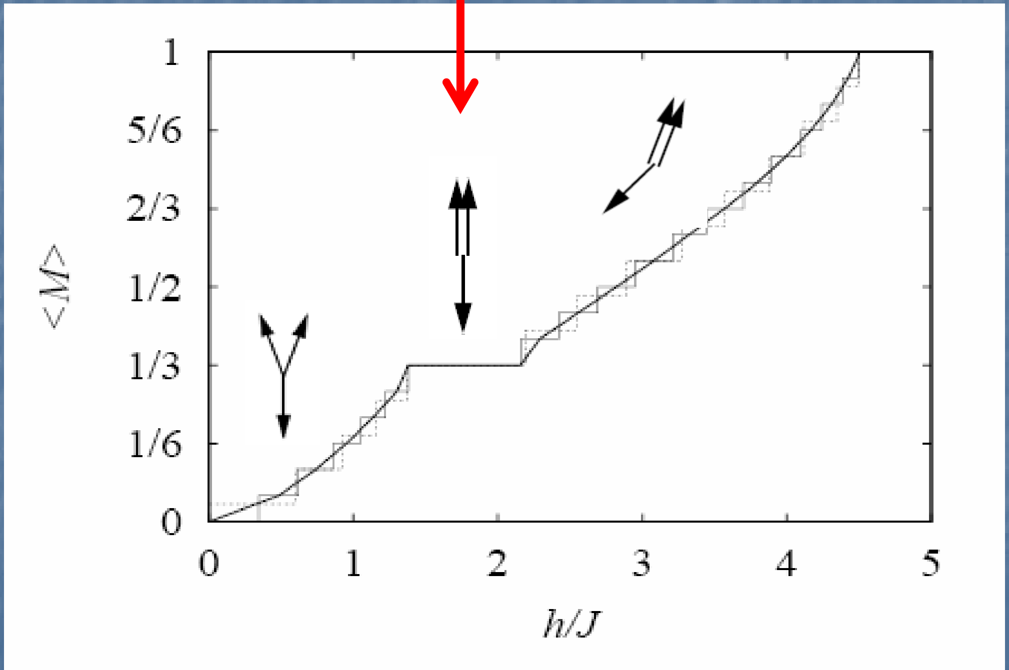
M. Uchida et al, PRB 2002

Spin-1 dimers

'Classical' plateaux

Classical GS stabilized in a finite field range by quantum fluctuations (order by disorder)

Chubukov, 1990



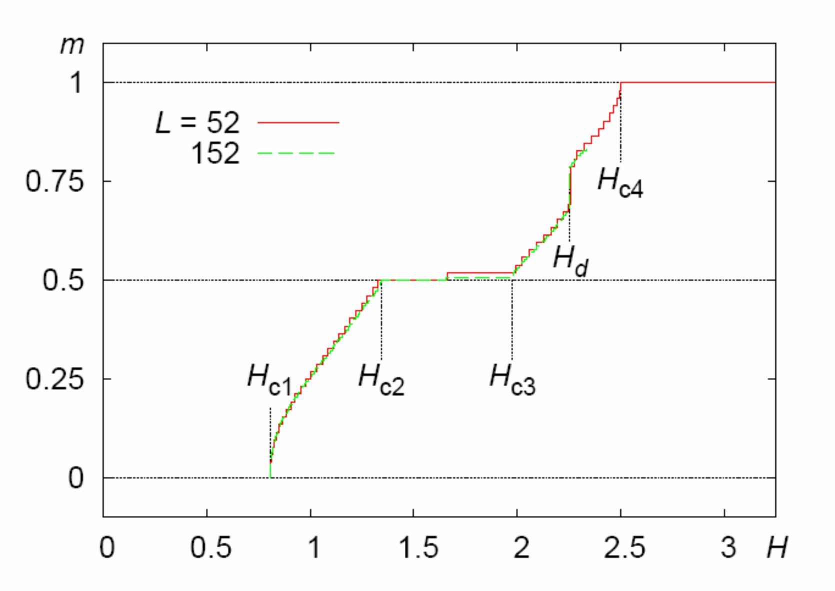
Triangular lattice

S=1/2

Review: Honecker et al, 2004

‘Quantum’ plateaux

GS without classical analog

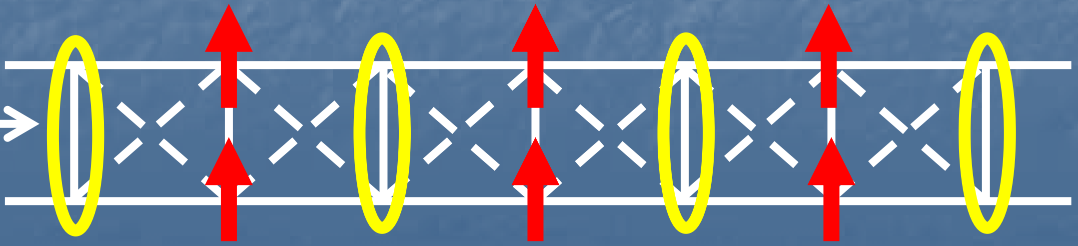


FM, EPJB ’98

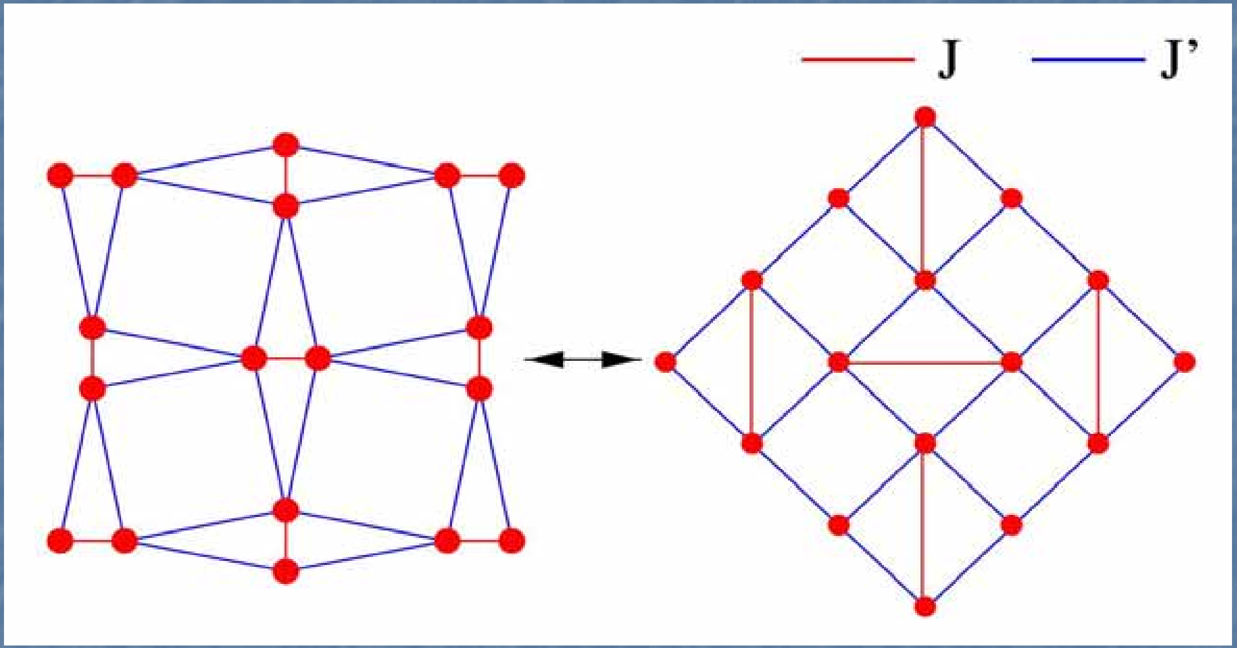
K. Totsuka, PRB ’98

← J.- B. Fouet et al, PRB 2006

Singlet →



Shastry-Sutherland model



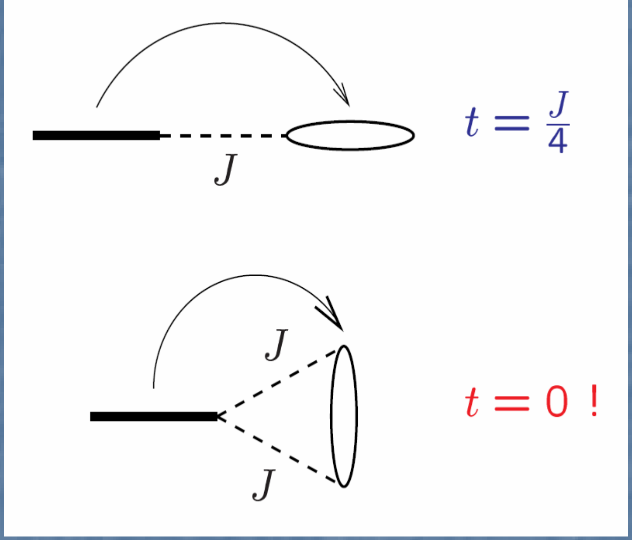
Ground-state Product of singlets on J-bonds (Shastry, Sutherland, ’81)

Triplets Almost immobile and repulsive (Miyahara et al, ’99)

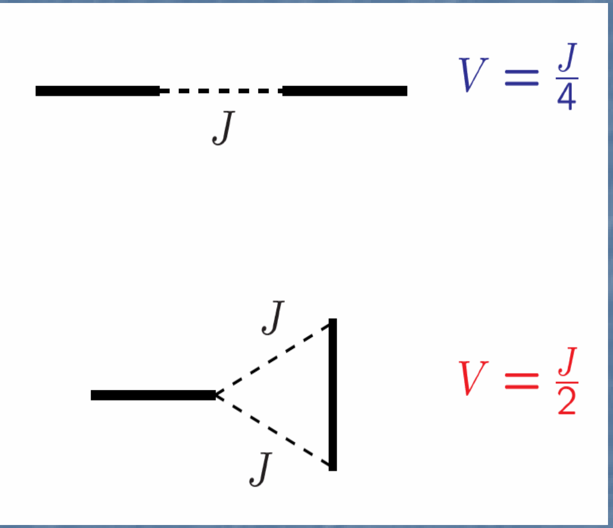
→ **Plateaux** (Miyahara et al, ’00)

Basic mechanism

Triplet Hopping



Triplet Repulsion



Kinetic energy

Frustration → Repulsion → **Metal-insulator transition**

Magnetization plateau

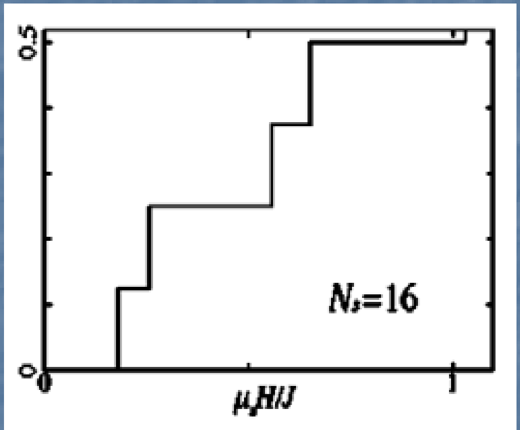
Recent experiments

- Takigawa et al, PRL 2008, NMR
  → Translation symmetry broken above 1/8 plateau
- Levy et al, EPL 2008, torque
  → New phase above 1/8
- Sebastian et al, unpublished, torque
  → Plateaux at 1/p (p=2,…,9) and 2/9
- Takigawa et al, unpublished, NMR
  → Plateaux at 1/8+ϵ, 1/6, 1/4

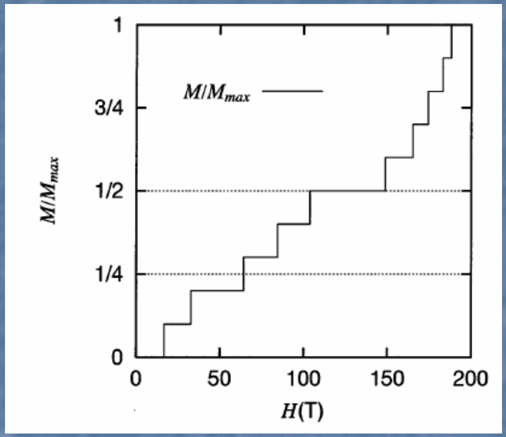
→ Theorists: Calculate magnetization plateaux!

Exact diagonalizations

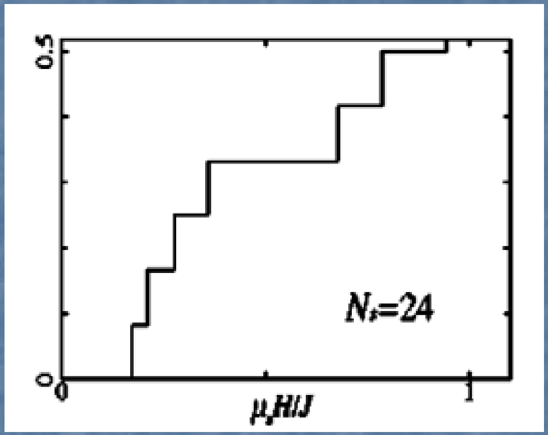
Miyahara and Ueda, 1999, 2000



16 sites



20 sites

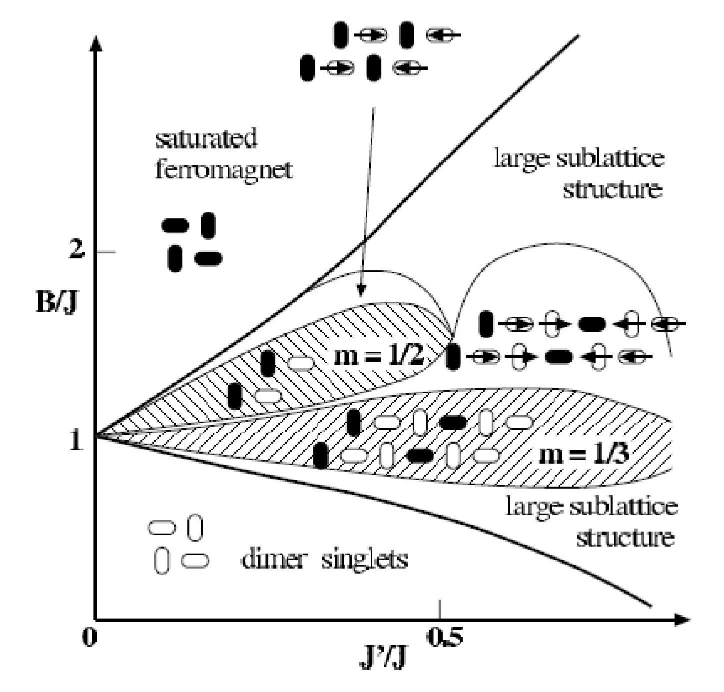


24 sites

Huge finite-size effects

Perturbation theory in J'/J

Momoi and Totsuka, 2000



3rd order perturbation theory

↓

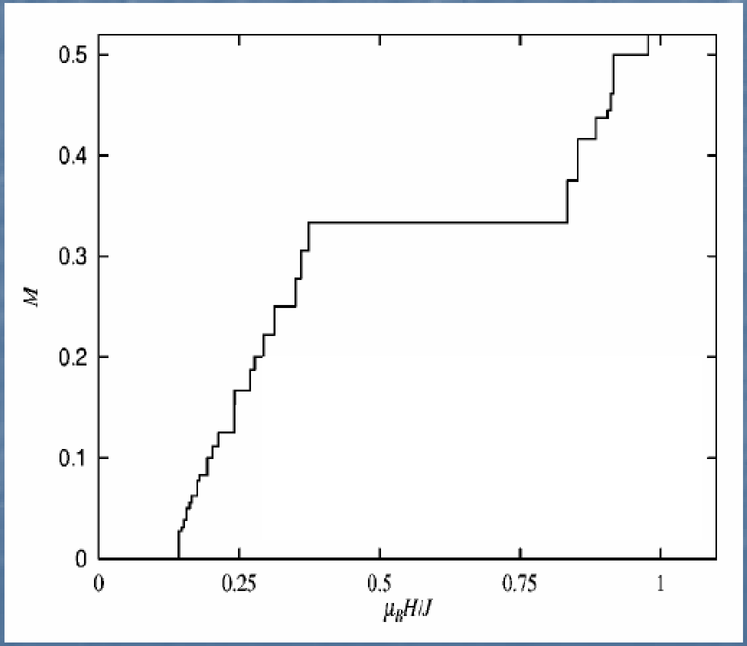
Repulsion to 3rd neighbour

↓

Plateaux at 1/2 and 1/3

Phenomenological approach

Triplet-triplet repulsion $\propto e^{(-r/r_0)}$ beyond 3rd neighbour



Miyahara and Ueda, 2000

No kinetic energy

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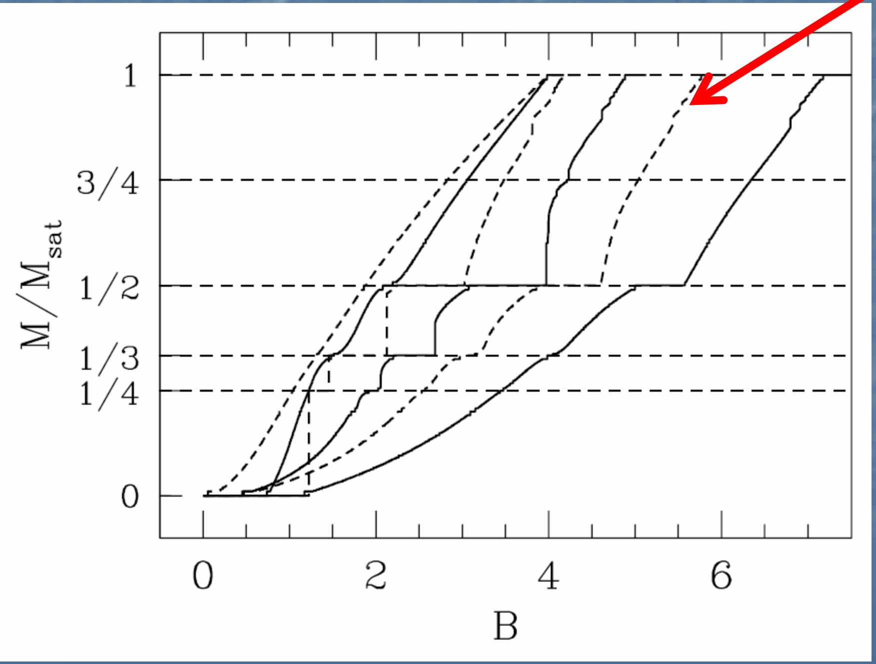
Many plateaux

Not predictive

Chern-Simons theory

G. Misguich, T. Jolicoeur, S. Girvin, PRL 2001

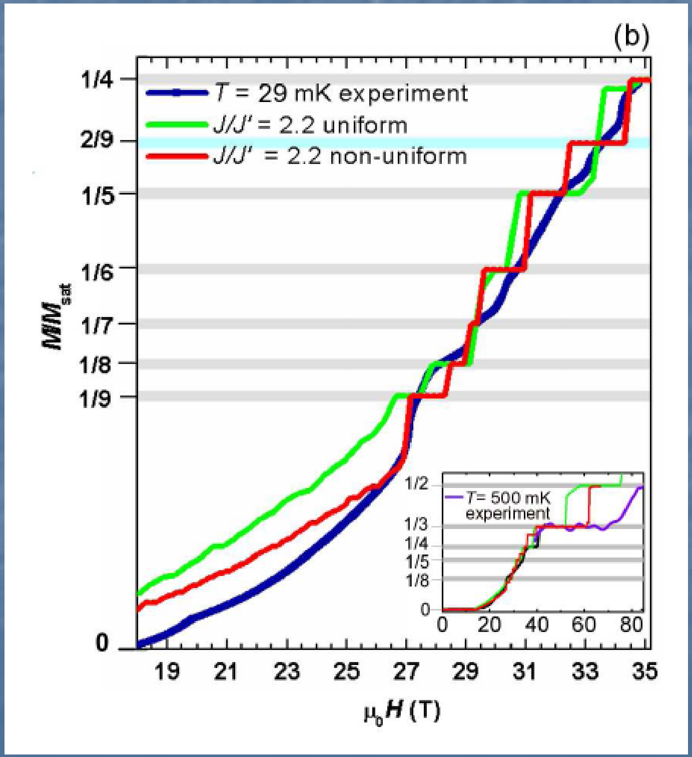
J/J'=1.5



Plateaux at 1/2 and 1/3

Chern-Simons revisited

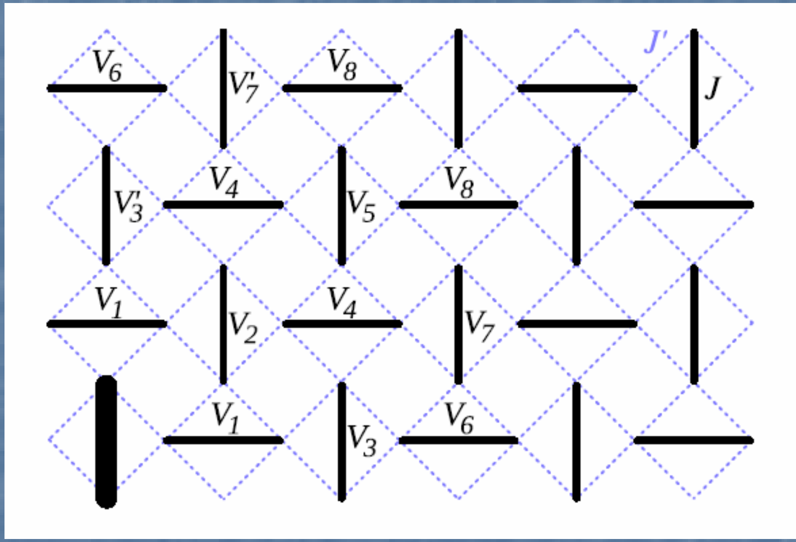
Sebastian et al, 2008



Plateaux at 1/p (p=2,…,9) and at 2/9

NB: J/J'=2.2

Why is it so difficult?



High commensurability

⬇

Long-range triplet-triplet interactions

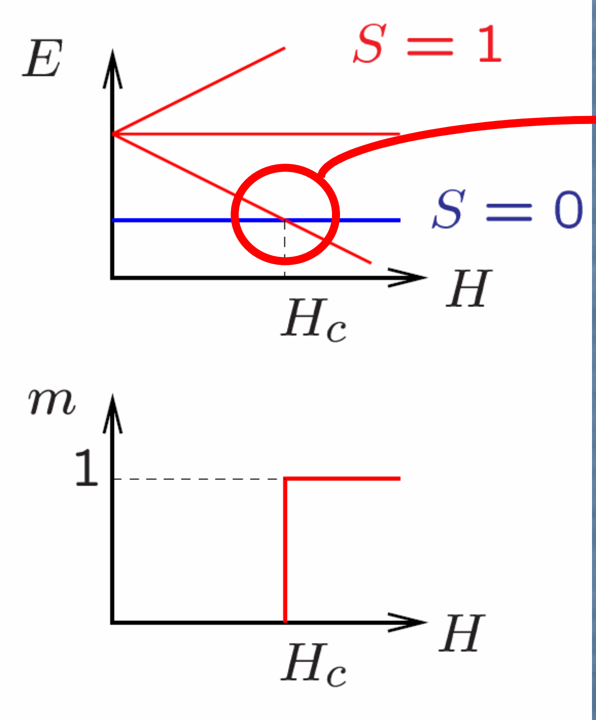
High-order perturbation theory

- Degenerate perturbation in J'/J to 15th order
  → effective model with $\simeq$ 15'000 terms
- Pade approximants → coefficients
- Map hard-core bosons onto spin-1/2
  → $b^+=S^-$, $b=S^+$, $n= \frac{1}{2} - S^z$ Matsubara-Matsuda
- Treat spins as classical vectors
  → Hartree approximation for bosons

J. Dorier, K. Schmidt, FM, PRL, in press

Triplets as hard-core bosons

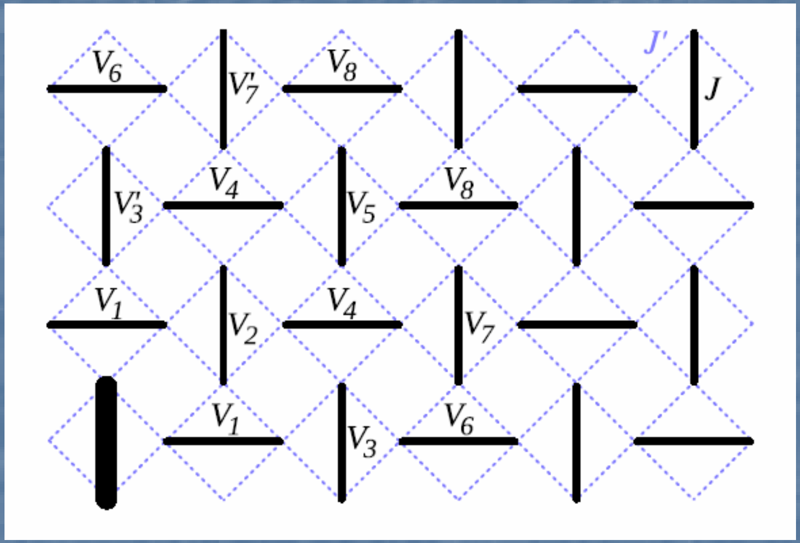
Isolated dimer

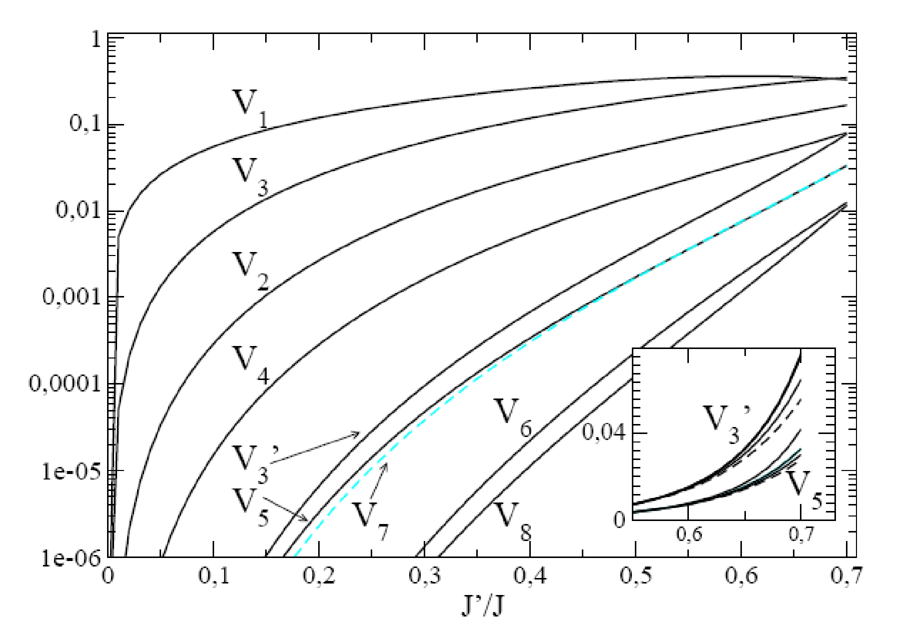


S=0: empty site

S_z=1: boson

High-order expansion



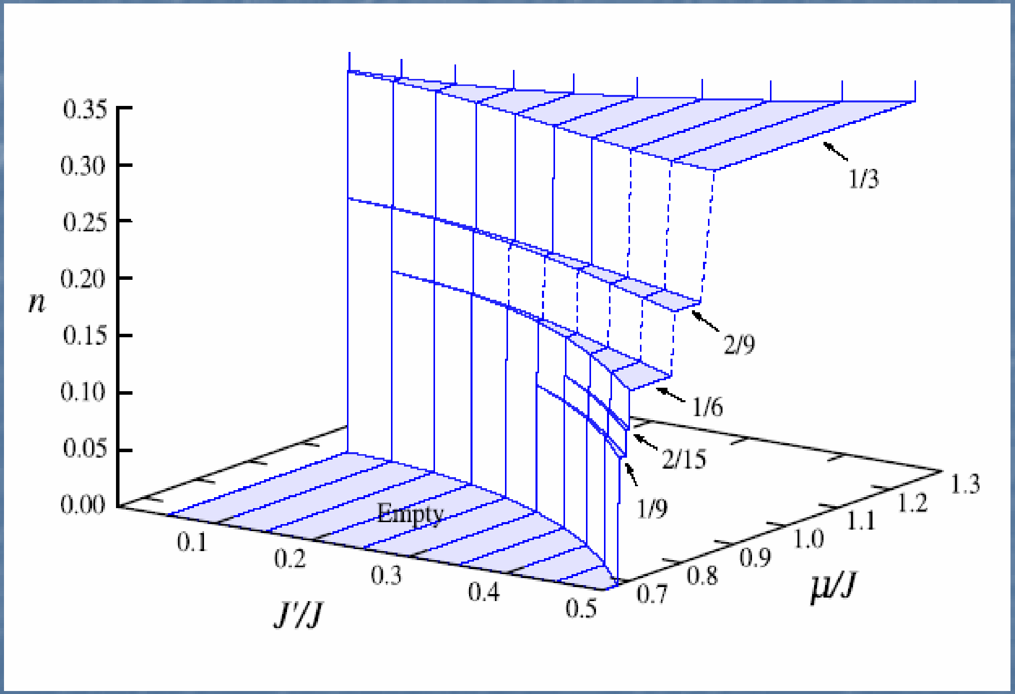


Extrapolation: reliable up to J'/J=0.5

Alternative: CORE (A. Abendschein, S. Capponi, PRL, in press)

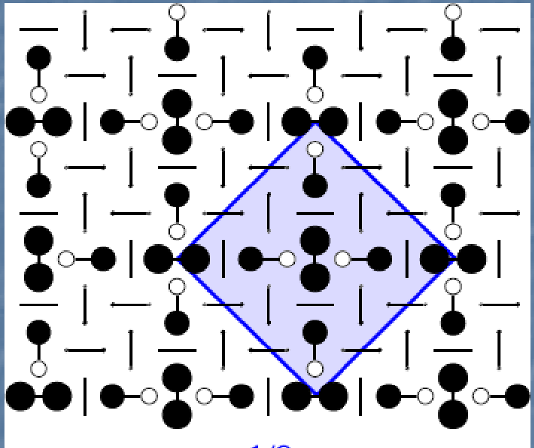
Very good agreement up to J'/J=0.5

Magnetization plateaux of Shastry-Sutherland model

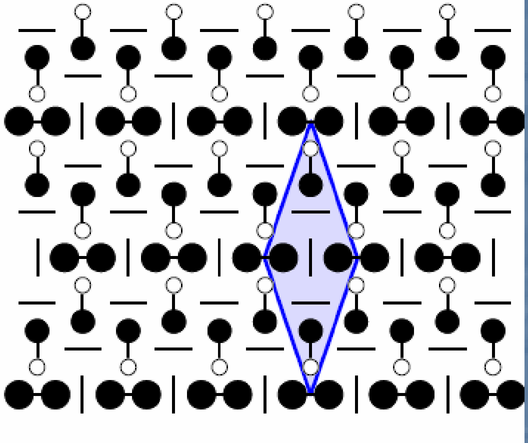


J. Dorier, K. Schmidt, FM, PRL, in press

Typical plateau structures



1/9



1/6

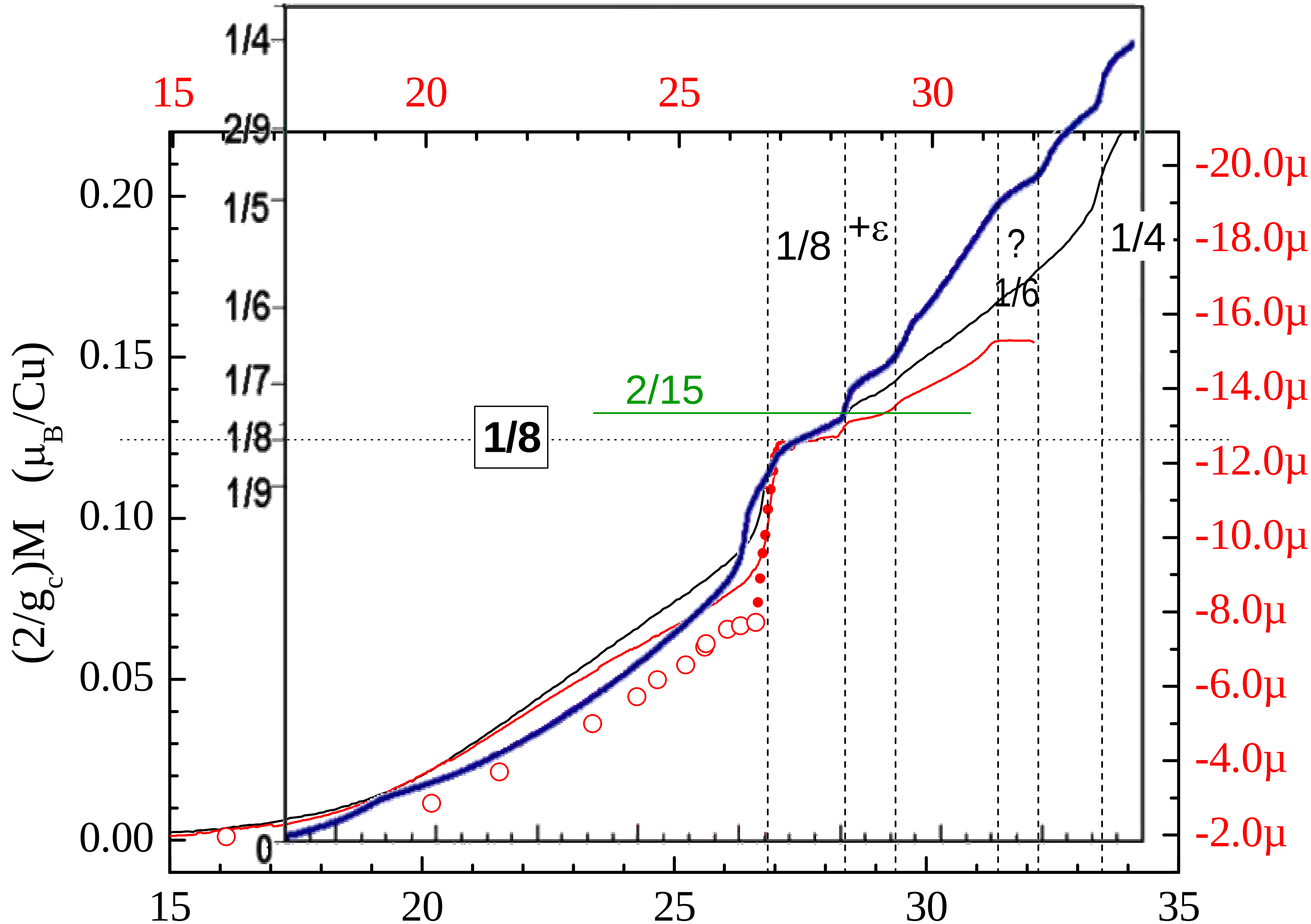
Effective hard-core boson model (boson ≡ triplet)
but canonical transformation

$$U^\dagger S^z_{1r} U \simeq \frac{1}{2} n_r - \frac{J'}{2J} n_{r-x} + \frac{J'}{2J} n_{r+x}$$

$$U^\dagger S^z_{2r} U \simeq \frac{1}{2} n_r + \frac{J'}{2J} n_{r-x} - \frac{J'}{2J} n_{r+x}$$

Comparison with experiments

- 1/6 plateau: very robust in our calculation, possibly observed in several experiments
- 1/8 plateau: not present in our calculation
  → residual interactions?
  → magnetization not precisely determined?
- 2/15 plateau = $1/8 + \varepsilon$?



○ NMR lineposition
• NMR intensity in the mixed phase

—— by torque @ GHMFL,
F. Lévy *et al.*, EPL 81, 67004 (2008) and unpublished

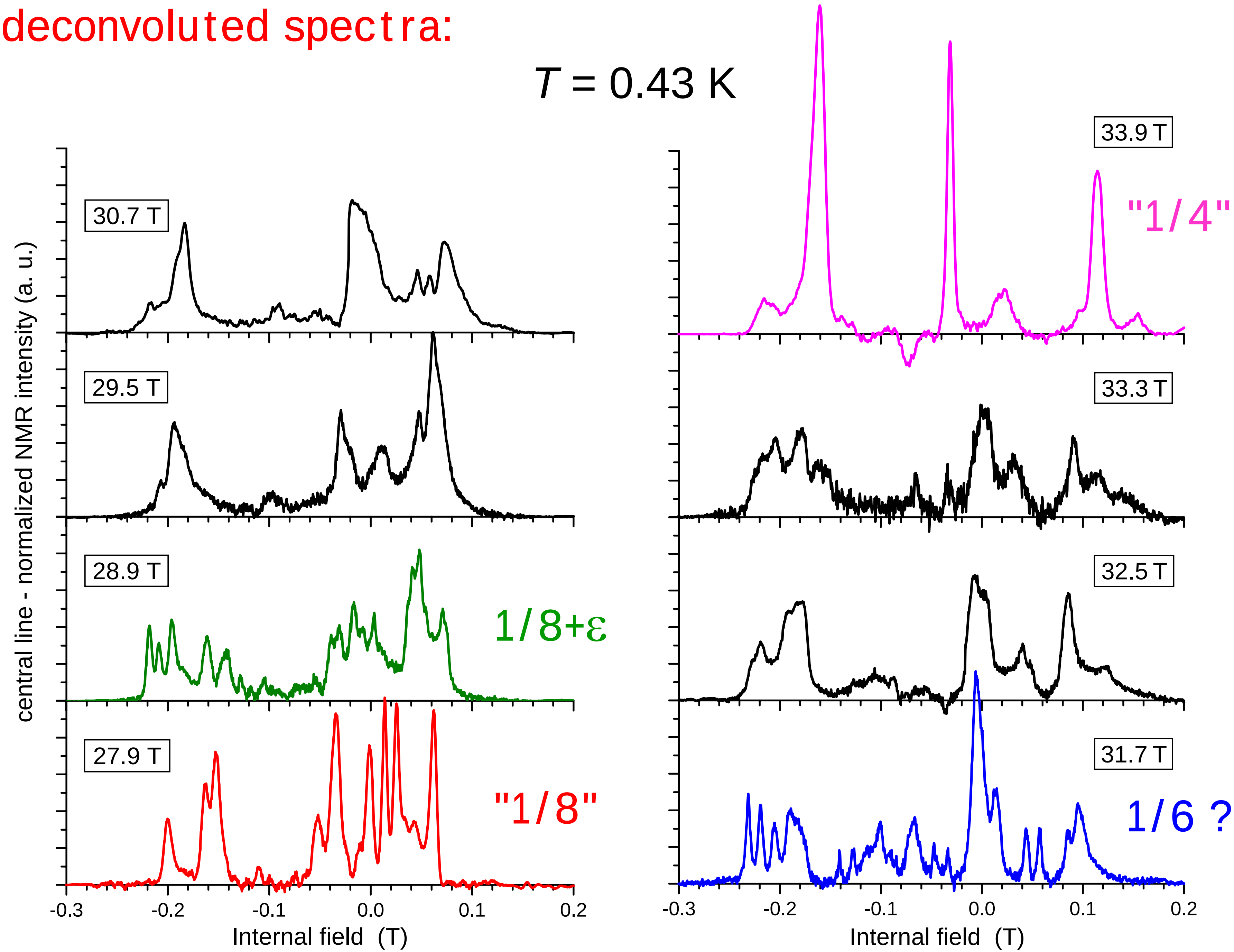
—— pulsed field at 1.5 K (+ adiabatic cooling),
K. Onizuka *et al.*, J. Phys. Soc. Jpn. **69**, 1016 (2000).

—— by torque @ NHMFL,
S. Sebastian *et al.*, arXiv:0707.2075

Conclusions/Perspectives

- Magnetization plateaux in $SrCu_2(BO_3)_2$
  → Very challenging, both for theorists and for experimentalists
- Common trends: more plateaux than previously assumed
- Strong prediction of theory: plateau at 1/6
- Further work:
  → Exp^t: Better calibration of magnetization
  → Theory: Include residual interactions
  → Both: Understand intermediate phases

deconvoluted spectra:



M. Takigawa *et al.*, June 2008, unpublished