Magnetization plateaux: the intriguing case of SrCu$_2$(BO$_3$)$_2$

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Scope

- SrCu$_2$(BO$_3$)$_2$: early experiments
- Mechanism of plateau formation
- Evidence of new plateaux: brief review
- Theory of plateaux: early results
- High order perturbation theory
- Conclusions and perspectives
SrCu$_2$(BO$_3$)$_2$

Kageyama et al, PRL ‘99

Cu$^{2+}$ -> Spin 1/2

J $\approx$ 85 K

J'/J $\approx$ 0.63
Magnetization of SrCu$_2$(BO$_3$)$_2$

Kageyama et al
PRL '99

Plateaux
- M=0
- M=1/8
- M=1/4
- M=1/3
NMR at 1/8-plateau

Many different sites → Broken translation symmetry

K. Kodama, M. Takigawa, M. Horvatic, C. Berthier, H. Kageyama, Y. Ueda, S. Miyahara, F. Becca, FM, Science '02
Heisenberg model

\[ \mathcal{H} = \sum_{i,j} J_{ij} \vec{S}_i \cdot \vec{S}_j + H \sum_i S_i^z \]

- \text{SU}(2) symmetry

- \text{U}(1) spin rotation around z

- + spatial symmetries (translations and point group)
‘Natural’ plateaux

No broken translational symmetry

Trimerized chain, Hida, JPSP ’94; Odd-leg ladders, Cabra et al, PRL ’97

M. Uchida et al, PRB 2002

Spin-1 dimers
‘Classical’ plateaux

Classical GS stabilized in a finite field range by quantum fluctuations (order by disorder)

Chubukov, 1990

Triangular lattice
S=1/2

Review: Honecker et al, 2004
‘Quantum’ plateaux

GS without classical analog

FM, EPJB ’98
K. Totsuka, PRB ’98

J.- B. Fouet et al, PRB 2006
**Shastry-Sutherland model**

**Ground-state** Product of singlets on J-bonds (Shastry, Sutherland, ’81)

**Triplets** Almost immobile and repulsive (Miyahara et al, ’99)

**Plateaux** (Miyahara et al, ’00)
Basic mechanism

- **Triplet Hopping**
  - $t = \frac{J}{4}$
  - $t = 0$

- **Triplet Repulsion**
  - $V = \frac{J}{4}$
  - $V = \frac{J}{2}$

- Frustration
- Kinetic energy
- Repulsion
- Metal-insulator transition
- Magnetization plateau
Recent experiments

- Takigawa et al, PRL 2008, NMR
  → Translation symmetry broken above 1/8 plateau
- Levy et al, EPL 2008, torque
  → New phase above 1/8
- Sebastian et al, unpublished, torque
  → Plateaux at 1/p (p=2,...,9) and 2/9
- Takigawa et al, unpublished, NMR
  → Plateaux at 1/8+\(\varepsilon\), 1/6, 1/4

→ Theorists: Calculate magnetization plateaux!
Exact diagonalizations

Miyahara and Ueda, 1999, 2000

16 sites
20 sites
24 sites

Huge finite-size effects
Perturbation theory in $J'/J$

Momoi and Totsuka, 2000

3rd order perturbation theory

Repulsion to 3rd neighbour

Plateaux at 1/2 and 1/3
Phenomenological approach

Triplet-triplet repulsion $\propto e^{-r/r_0}$ beyond 3rd neighbour

No kinetic energy $\Downarrow$

Many plateaux

Not predictive

Miyahara and Ueda, 2000
Chern-Simons theory

G. Misguich, T. Jolicoeur, S. Girvin, PRL 2001

$J/J' = 1.5$

Plateaux at $1/2$ and $1/3$
Chern-Simons revisited

Sebastian et al, 2008

Plateaux at $1/p$ ($p=2,...,9$) and at $2/9$

NB: $J/J' = 2.2$
Why is it so difficult?

- High commensurability
- Long-range triplet-triplet interactions
High-order perturbation theory

- Degenerate perturbation in $J'/J$ to 15th order
  $\to$ effective model with $\simeq 15'000$ terms
- Pade approximants $\to$ coefficients
- Map hard-core bosons onto spin-1/2
  $\to b^+ = S^-, b = S^+, n = \frac{1}{2} - S^z$ Matsubara-Matsuda
- Treat spins as classical vectors
  $\to$ Hartree approximation for bosons

J. Dorier, K. Schmidt, FM, PRL, in press
Triplets as hard-core bosons

Isolated dimer

S=0: empty site
S_z=1: boson
High-order expansion

Extrapolation: reliable up to $J'/J=0.5$

Alternative: CORE (A. Abendschein, S. Capponi, PRL, in press)

Very good agreement up to $J'/J=0.5$
Magnetization plateaux of Shastry-Sutherland model

J. Dorier, K. Schmidt, FM, PRL, in press
Effective hard-core boson model (boson $\equiv$ triplet) but canonical transformation

$$U^\dagger S_{1r}^z U \simeq \frac{1}{2} n_r - \frac{J'}{2J} n_{r-x} + \frac{J'}{2J} n_{r+x}$$

$$U^\dagger S_{2r}^z U \simeq \frac{1}{2} n_r + \frac{J'}{2J} n_{r-x} - \frac{J'}{2J} n_{r+x}$$
Comparison with experiments

- **1/6 plateau**: very robust in our calculation, possibly observed in several experiments
- **1/8 plateau**: not present in our calculation
  - residual interactions?
  - magnetization not precisely determined?
- **2/15 plateau** = 1/8 + ε?
(2/g_c)M (µ_B/Cu)

Field (T)

NMR line position
NMR intensity in the mixed phase

Pulsed field at 1.5 K (+ adiabatic cooling),

by torque @ GHMFL,
F. Lévy et al., EPL 81, 67004 (2008) and unpublished

by torque @ NHMFL,
S. Sebastian et al., arXiv:0707.2075
Conclusions/Perspectives

- Magnetization plateaux in SrCu$_2$(BO$_3$)$_2$
  - Very challenging, both for theorists and for experimentalists
- Common trends: more plateaux than previously assumed
- Strong prediction of theory: plateau at 1/6
- Further work:
  - Exp$: Better calibration of magnetization
  - Theory: Include residual interactions
  - Both: Understand intermediate phases
deconvoluted spectra:

\[ T = 0.43 \text{ K} \]

- Internal field (T):
  - 30.7 T
  - 29.5 T
  - 28.9 T
  - 27.9 T
  - 33.9 T
  - 33.3 T
  - 32.5 T
  - 31.7 T

- Central line - normalized NMR intensity (a. u.):
  - 1/8
  - 1/8 + \varepsilon
  - "1/8"
  - "1/4"
  - "1/6 ?"

M. Takigawa et al., June 2008, unpublished