From Fermi to non-Fermi liquids

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MaNEP SWITZERLAND

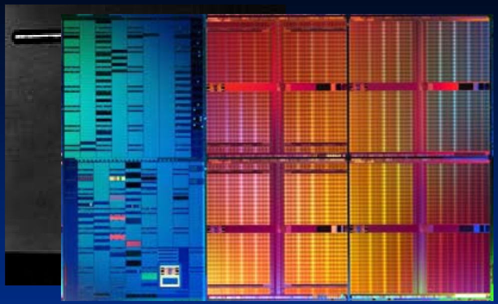
How to describe solids ?

- Understood: free electrons
- Real systems : Coulomb interaction

  $E \sim 10\,000$ K !

- Properties of realistic systems ?
- Free electron theory works quite well !

- Transistor



1956

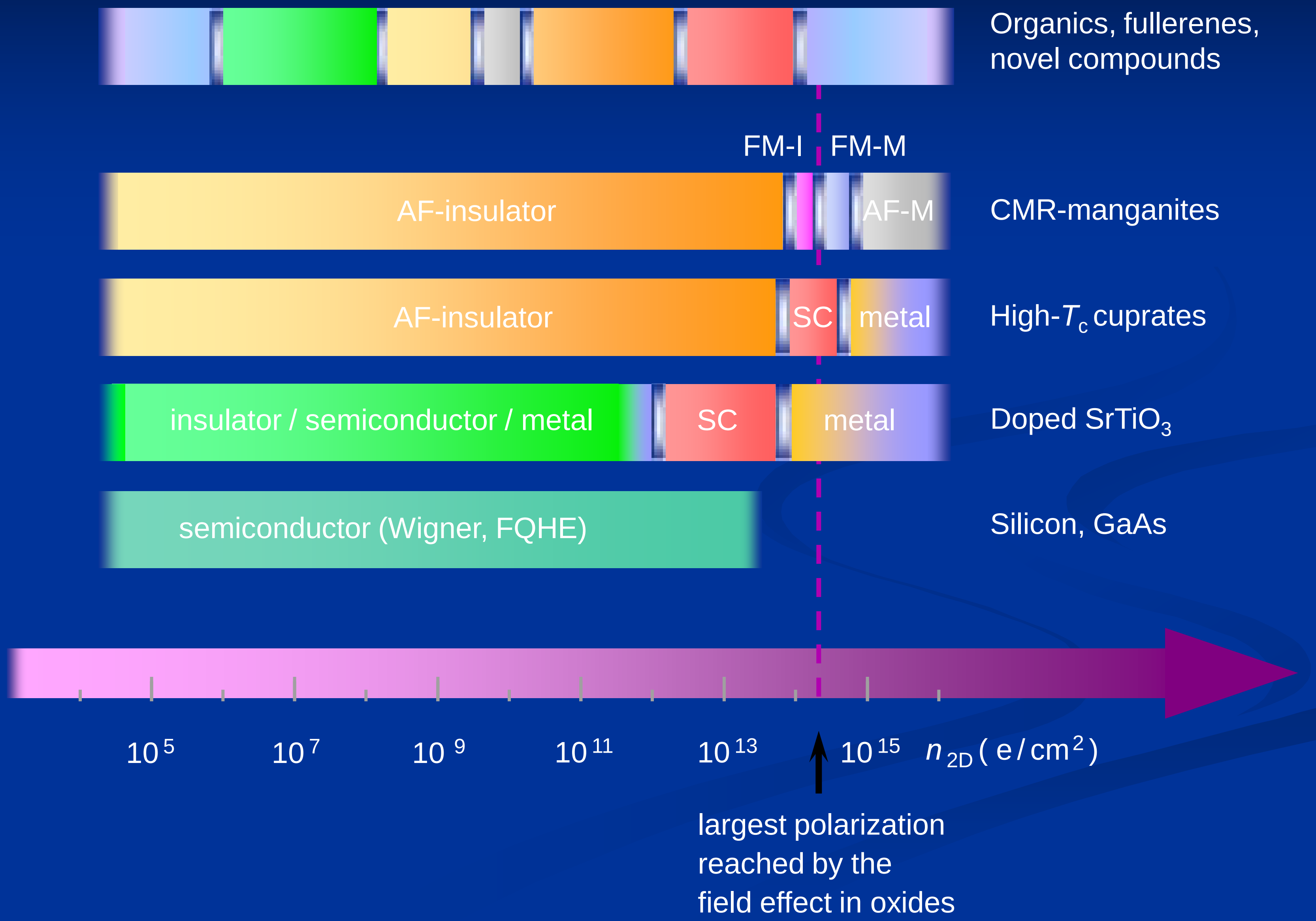
- Supraconductivité

(1913),1972, 1973,1987,2003

- Magnétorésistance géante

2007

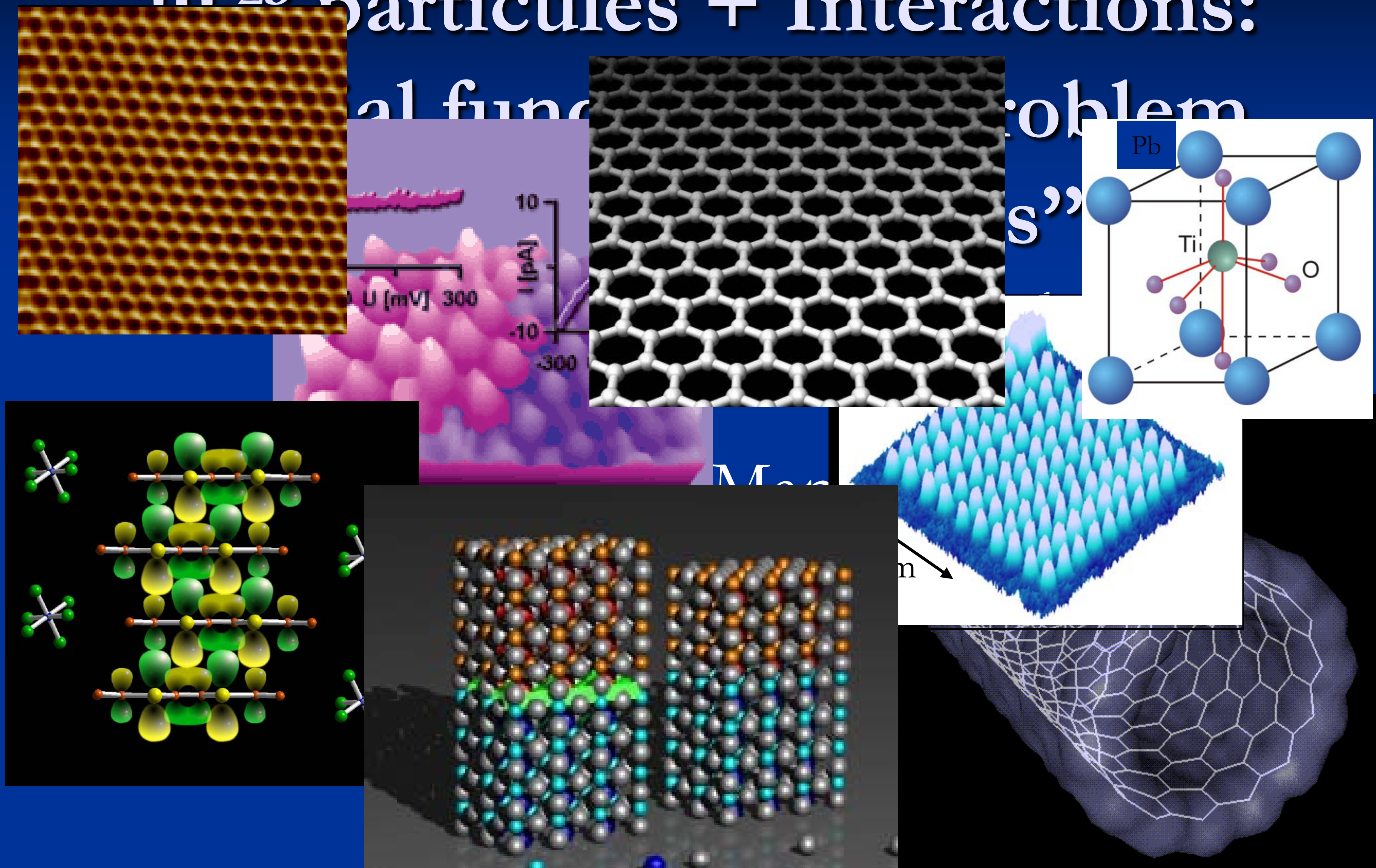
Densities



Organics, fullerenes, novel compounds
FM-I
FM-M
AF-insulator
AF-M
CMR-manganites
AF-insulator
SC
metal
High-T_c cuprates
insulator / semiconductor / metal
SC
metal
Doped $SrTiO_3$
semiconductor (Wigner, FQHE)
Silicon, GaAs
10^5
10^7
10^9
10^{11}
10^{13}
10^{15}
n_{2D} (e / cm^2)
largest polarization reached by the field effect in oxides

Need to understand interactions !

10^{23} particules + Interactions:



What’s up doc ?

- Reminder of free electrons
- Basics of Fermi liquids
- How to break Fermi liquids
    - Mott transition
    - Luttinger liquids
    - Deconfinement

Details in

http://dpmc.unige.ch/gr_giamarchi/Solides/solides.html

Details on the lectures and all the lecture material can be found on Dokeos

Notes on many body physics (in english)

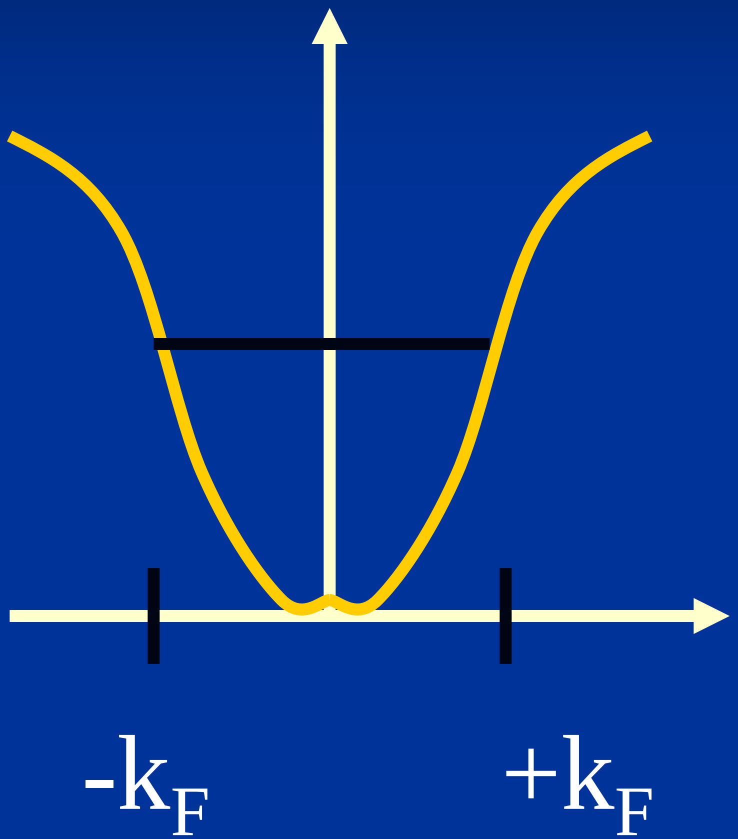
Shorter and less technical description [Notes of a course in the "Les Houches in Singapore" 2009 school]

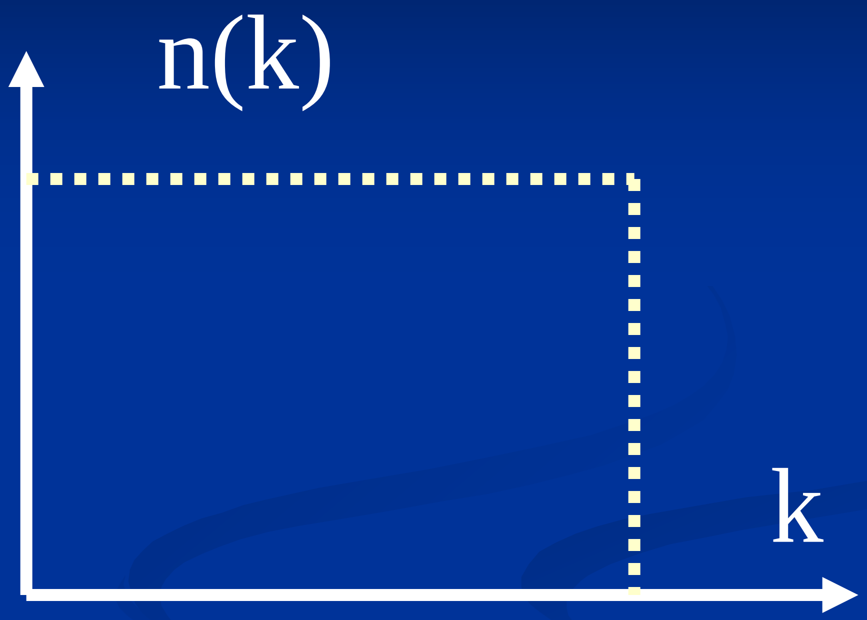
Some related lectures on many body physics are given here:

- **Notes on many body physics (in french)**
- **Notes of previous version of Solides III and IV (in french) by B. Giovannini and C. Berthod**

Free electrons

Free electrons : basics





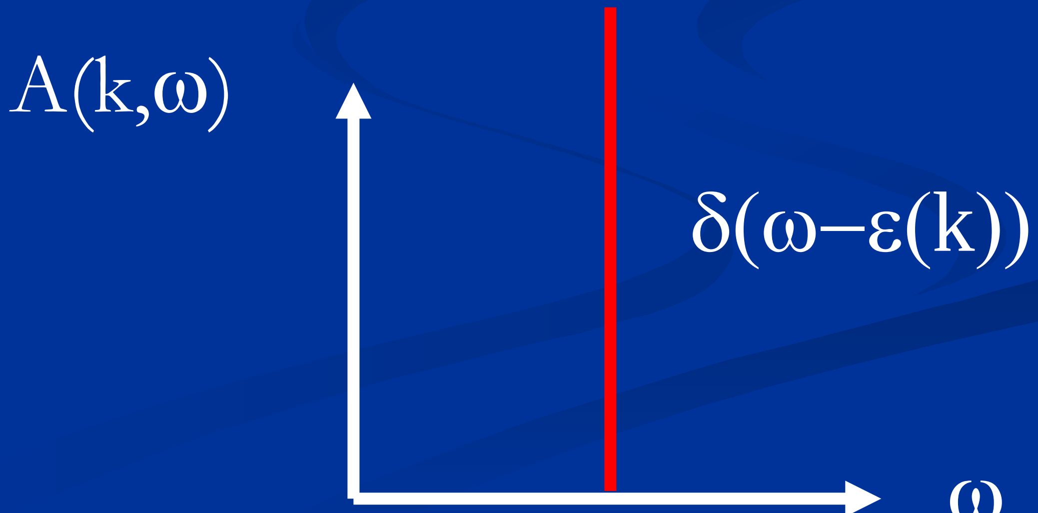
Individual excitations:
fermions (particles or holes)

Properties

- Thermodynamics

$$C_V \propto T \qquad \kappa = Cste$$

- Spectral function



How to probe ?

- Specific heat $C_V = \gamma T$

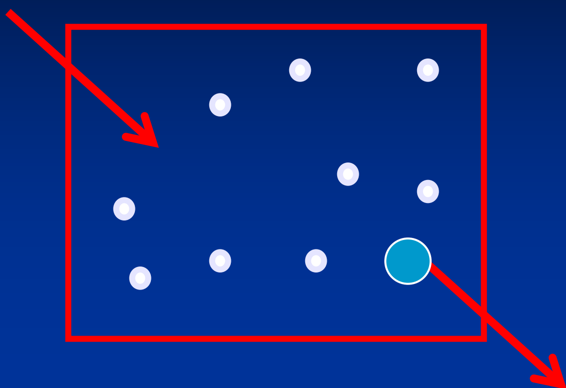
- Responses:

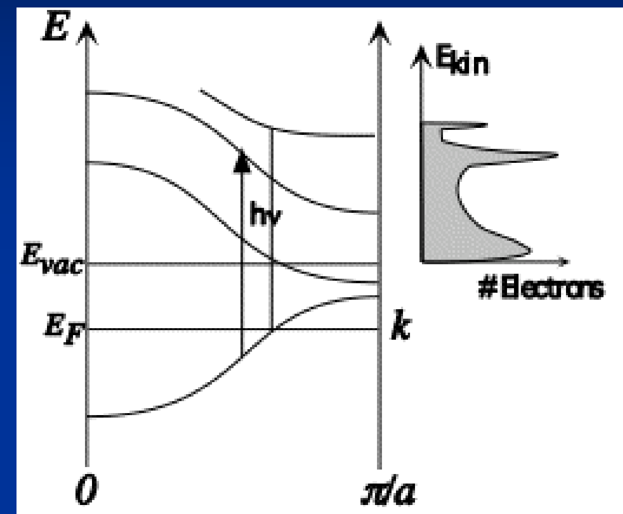
  NMR, Neutrons (spin correlations)

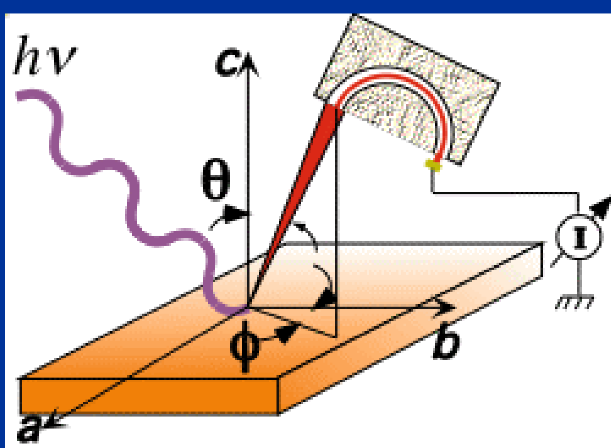
  Transport (charge excitations)

  Photoemission (single particles)

Photoemission





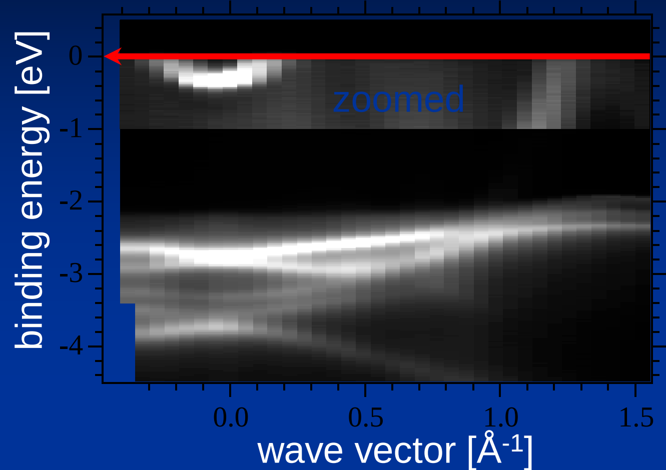


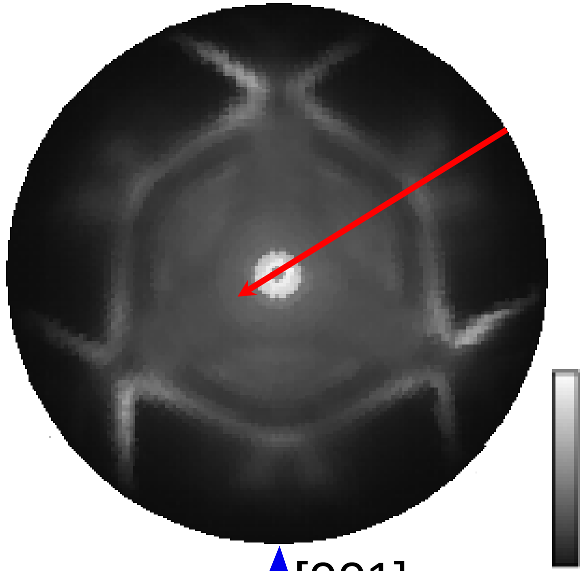
$$k_{//} = \sqrt{\frac{2mE_{kin}}{\hbar}}\sin\theta$$

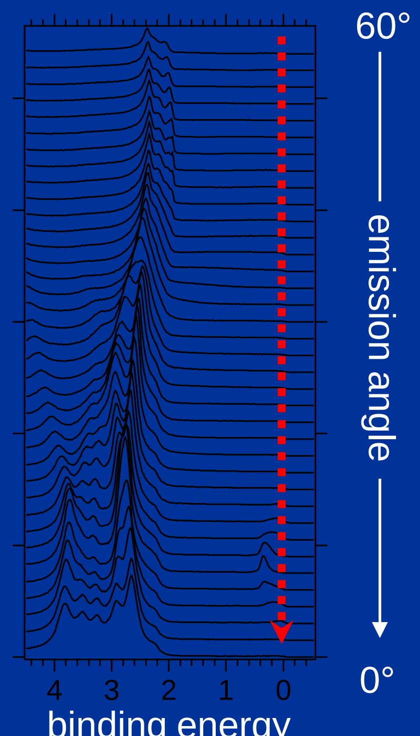
Measures Spectral function A(k,ω)

Bandmapping

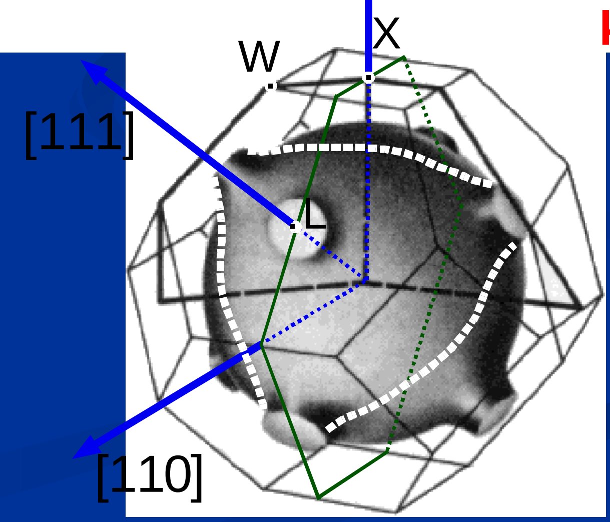
Fermi surface mapping







E(k)

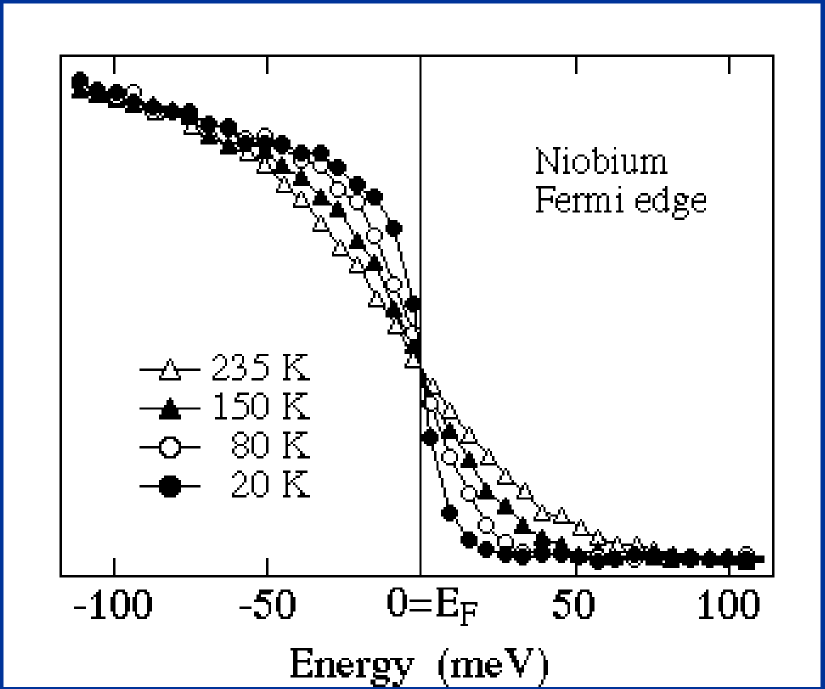


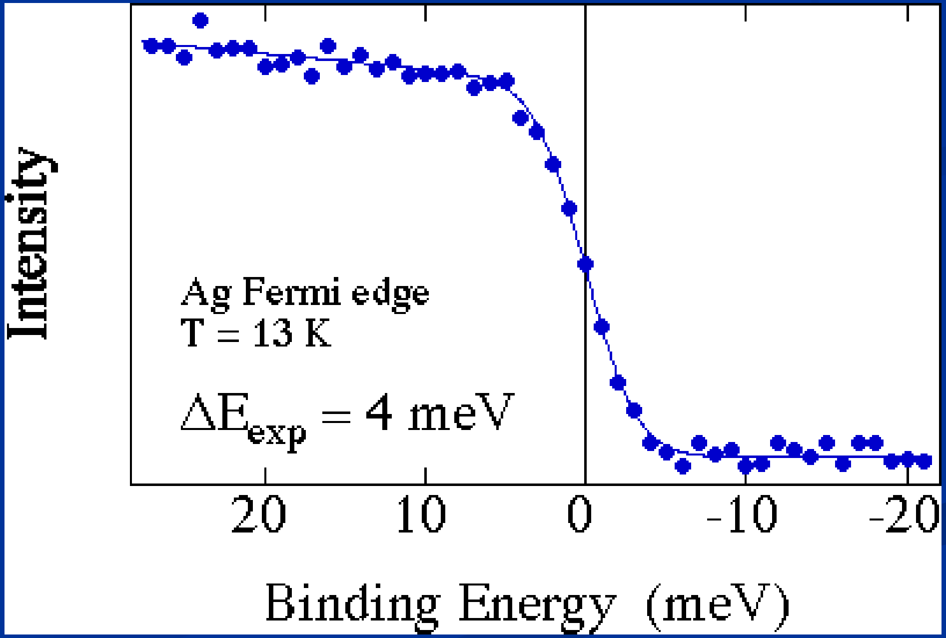
k(E_F)

Cu Fermi surface (Ashcroft/Mermin)

``Free electrons'' works

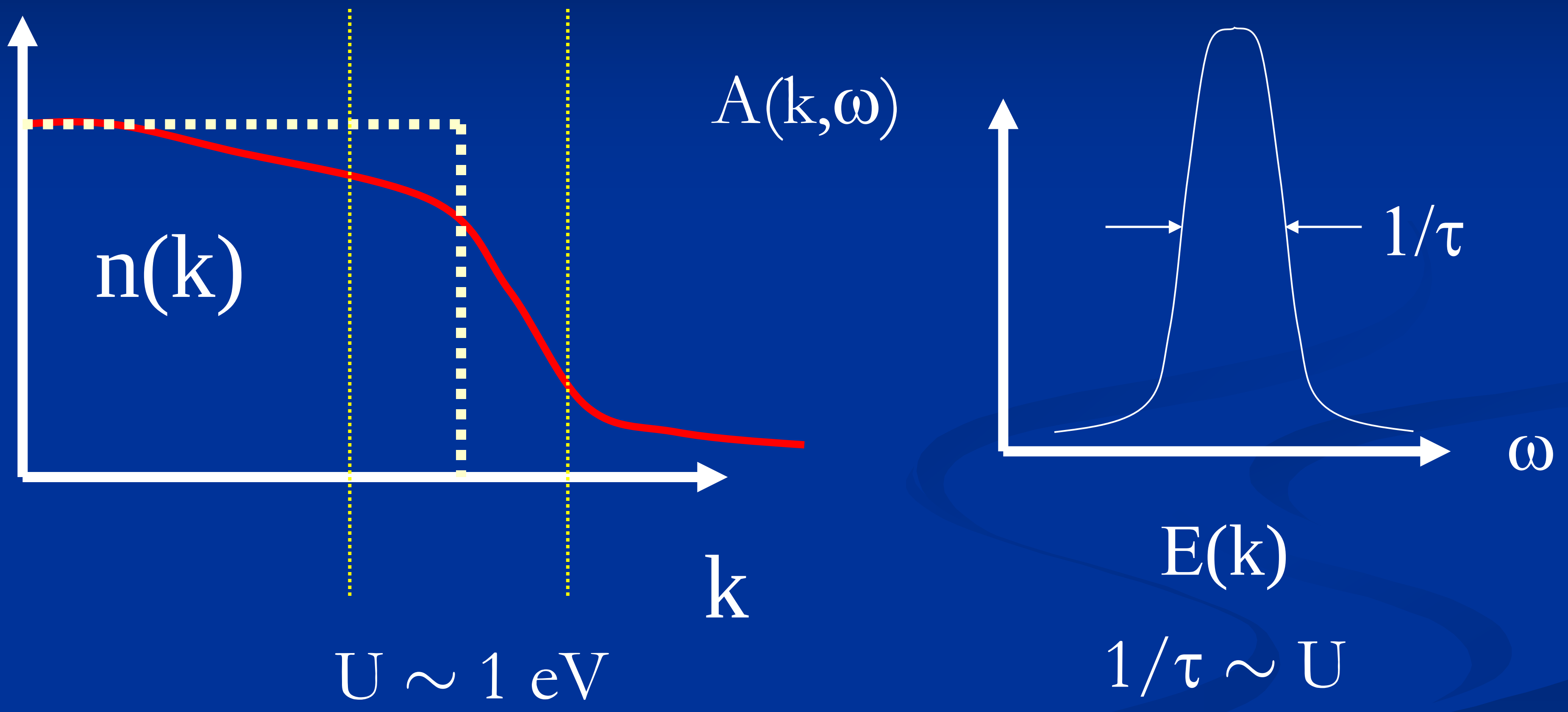
$C_V = \gamma T$ $\kappa = \text{cste}$ $\chi = \text{cste}$





(M. Grioni)

This is *crazy* !



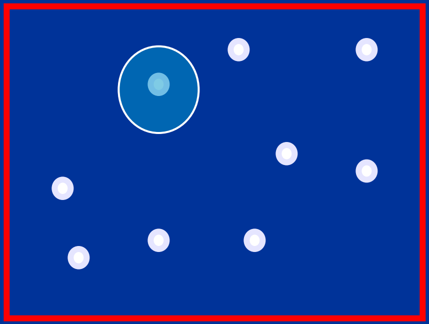
Solution: Fermi liquid theory

Fermi liquids

Fermi liquid 101

- Landau Fermi liquid

- Individual fermionic excitations exist (quasiparticles)



$$m \to m^*$$

More formal

$$A(k,\omega) = \frac{-1}{\pi}\mathrm{Im}\, G_{\mathrm{ret}}(k,\omega)$$

$$G(k, t_2 - t_1) = -i\,\theta(t_2 - t_1)\langle[c_{k,t_2}, c^{\dagger}_{k,t_1}]_+\rangle$$

$$G^0(k,\omega) = \frac{1}{\omega - \xi(k) + i\delta}$$

$$A^0(k,\omega) = \delta(\omega - \xi(k))$$

$G($

$$A(k,\omega) = -\frac{1}{\pi}\frac{\mathrm{Im}\,\Sigma(k,\omega)}{(\omega - \xi(k) - \mathrm{Re}\,\Sigma(k,\omega))^2 + (\mathrm{Im}\,\Sigma(k,\omega))^2}$$

$\mathrm{Re}\Sigma \rightarrow m^*, Z$

$\mathrm{Im}\Sigma \rightarrow 1/\tau$

$$A(k,\omega) = \delta(\omega - \xi(k) - \operatorname{Re}\Sigma(k,\omega))$$

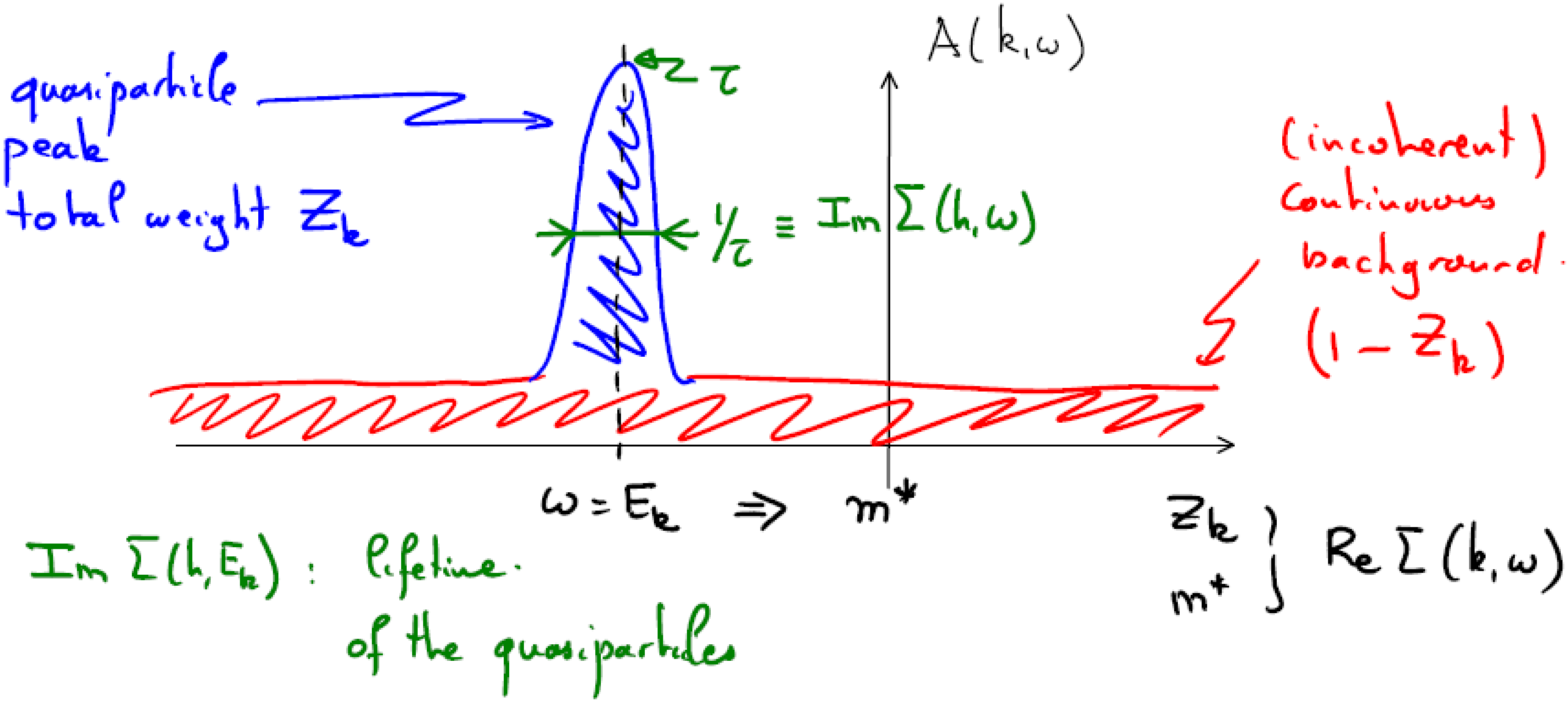
$$E(k) - \xi(k) - \operatorname{Re}\Sigma(k, \omega = E(k)) = 0$$

$$E(k) = 0 + \frac{k_{\mathrm{F}}}{m^*}(k - k_{\mathrm{F}})$$

$$\frac{m}{m^*} = \frac{1 + \frac{m}{k_{\mathrm{F}}} \left.\frac{\partial \operatorname{Re}\Sigma(k,\omega)}{\partial k}\right|_{\omega=E(k)}}{1 - \left.\frac{\partial \operatorname{Re}\Sigma(k,\omega)}{\partial \omega}\right|_{\omega=E(k)}}$$

$$A(k,\omega) = Z_k \delta(\omega - E(k))$$

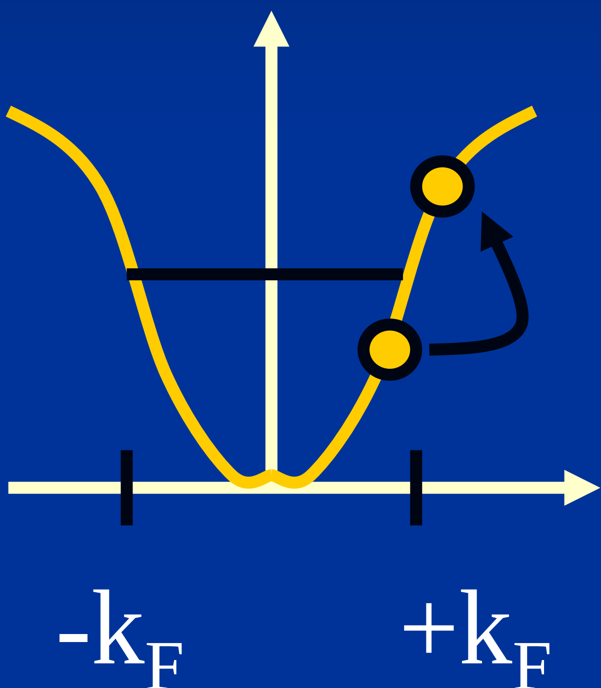
$$Z_k = \left[\frac{\partial}{\partial \omega}(\omega - \xi(k) - \operatorname{Re}\Sigma(k,\omega))\Big|_{\omega=E(k)} \right]^{-1} = \frac{1}{1 - \left.\frac{\partial \operatorname{Re}\Sigma(k,\omega)}{\partial \omega}\right|_{\omega=E(k)}}$$

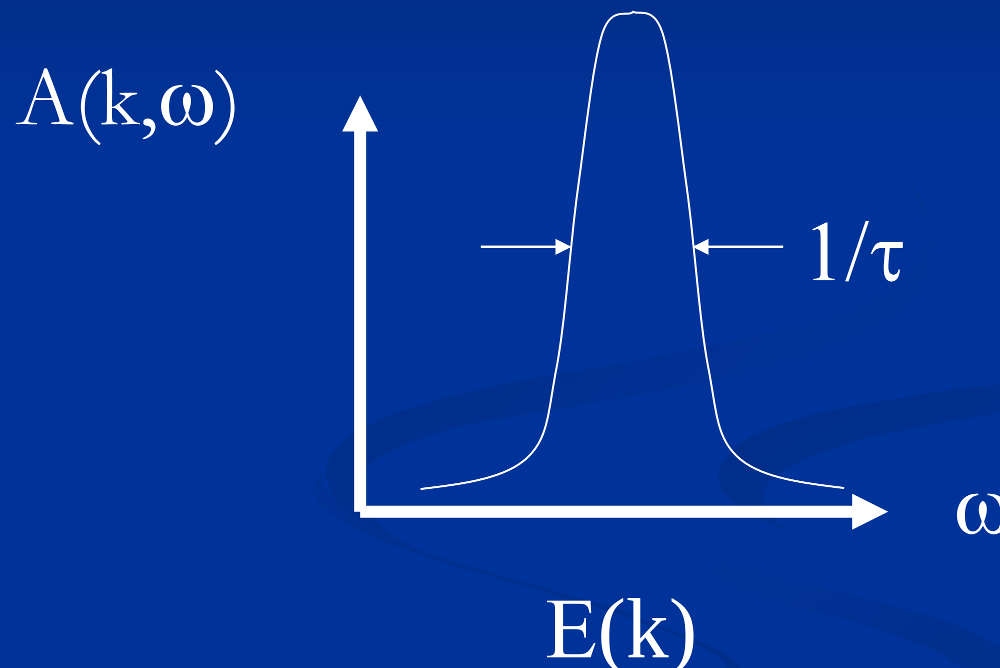


quasiparticle
peak
total weight Z_k
τ
$1/\tau \equiv \mathrm{Im}\,\Sigma(k,\omega)$
$A(k,\omega)$
(incoherent)
continuous
background
$(1 - Z_k)$
$\omega = E_k \Rightarrow m^*$
Z_k
m^*
$\mathrm{Re}\,\Sigma(k,\omega)$
$\mathrm{Im}\,\Sigma(k,E_k)$: lifetime
of the quasiparticles

Lifetime

- scattering between QP: lifetime





$$P \propto \int_{-\omega}^{0} d\omega_1 \int_{0}^{\omega+\omega_1} d\omega_2 = \frac{1}{2}\omega^2$$

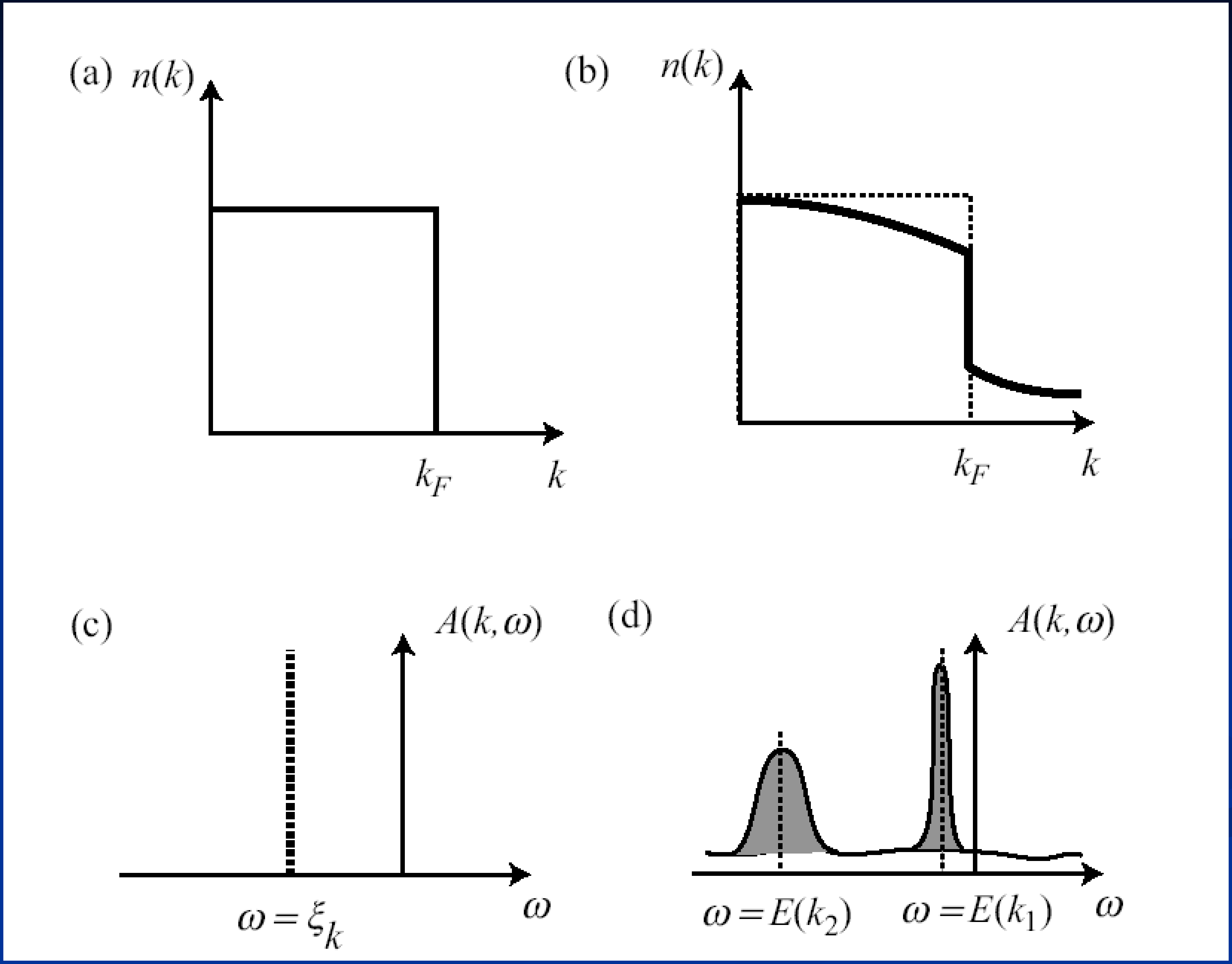
- Lifetime larger than average energy

$$\psi \propto e^{iE(k)t - t/\tau} \qquad 1/\tau \propto \omega^2$$

- QP are sharp (nearly free) excitations close to the Fermi surface

- Transport: $\rho \sim T^2$

- Only a fraction Z in QP states



(a)
$n(k)$
k_F
k
(b)
$n(k)$
k_F
k
(c)
$A(k,\omega)$
$\omega = \xi_k$
ω
(d)
$A(k,\omega)$
$\omega = E(k_2)$
$\omega = E(k_1)$
ω

Collective modes

- Charge mode:

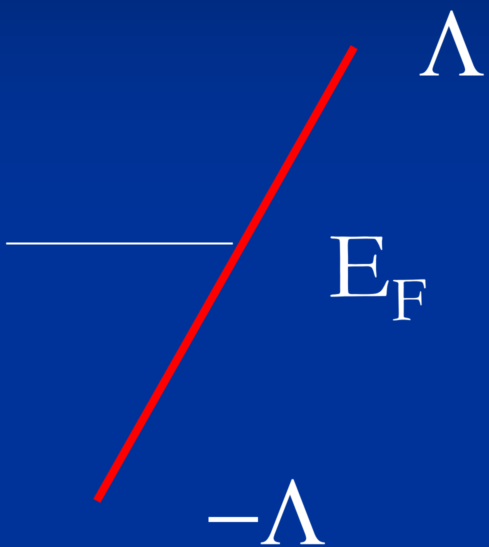
  Zero sound: $\omega = V_\rho q$

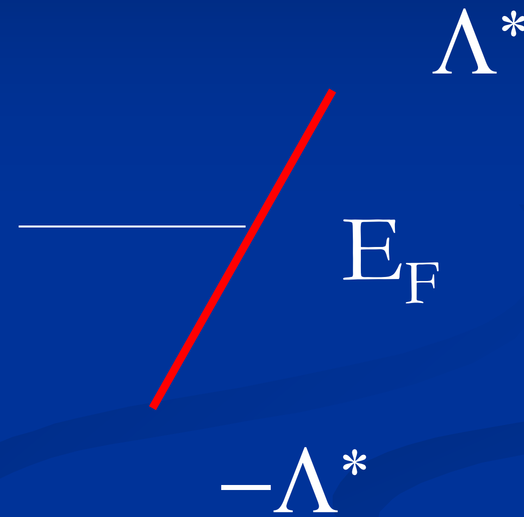
  Plasmon: $\omega = Cste$

- Spin mode:

  Spin wave: $\omega = V_s q$

RG interpretation of FL





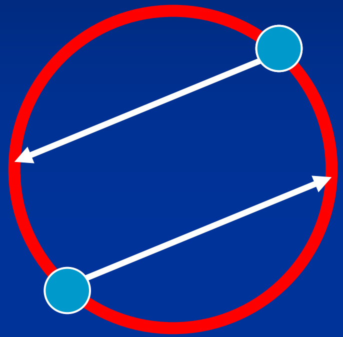
Λ: cutoff

U: interaction

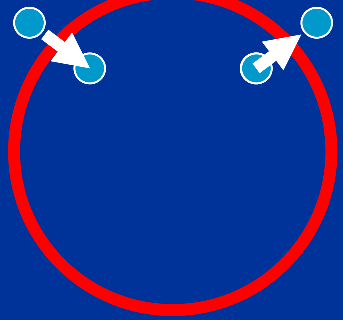
Λ^*: cutoff

U^*: interaction

Hamiltonian (Landau)



$$H_{\mathrm{int}} = \sum_{k,k',q} V(q) \quad c^{*}_{k+q}\, c^{*}_{k'-q} c_{k'} c_{k}$$



$$H_{FL} = \sum_{k,k'} f_{\hat{k},\hat{k}'} n(k) n(k')$$

Summary of Fermi liquid

- Thermodynamics:

$$C_V \propto T \qquad \qquad \kappa = Cste$$

- « Individual » fermionic excitations

(effective mass m*, weight Z)

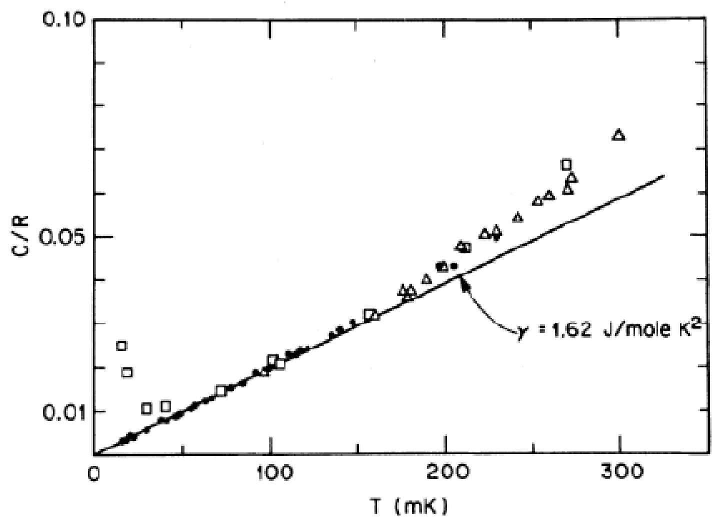
- Lifetime: transport etc. $\rho \propto T^2$

- Collective modes (charge and spin)

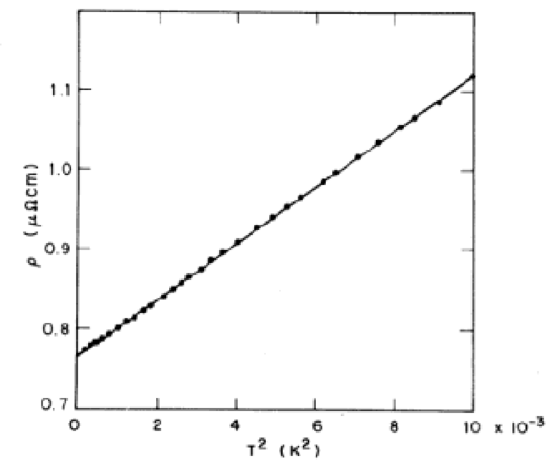
Fermi liquid theory

- Shown perturbatively in U
- Much more general and robust

Element	m*/m	χ/χ_0
Nb	2	1
3He	6	20
Heavy fermion	100	100

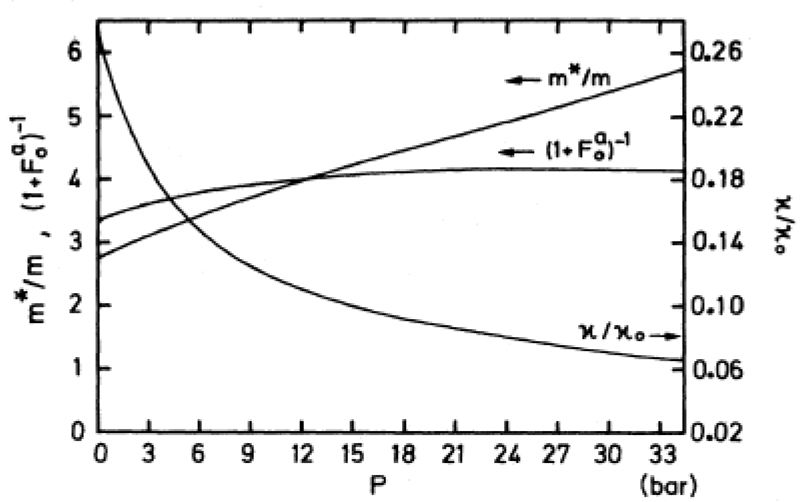


0.10
0.05
0.01
C/R
0
100
200
300
T (mK)
γ = 1.62 J/mole K²



1.1
1.0
0.9
0.8
0.7
ρ (μΩcm)
0
2
4
6
8
10 x 10⁻³
T² (K²)

CeAl$_3$



m*/m
(1+F₀ᵃ)⁻¹
χ/χ₀
m*/m , (1+F₀ᵃ)⁻¹
χ₀/χ
0
1
2
3
4
5
6
0.02
0.06
0.10
0.14
0.18
0.22
0.26
0
3
6
9
12
15
18
21
24
27
30
33
P
(bar)

He$_3$

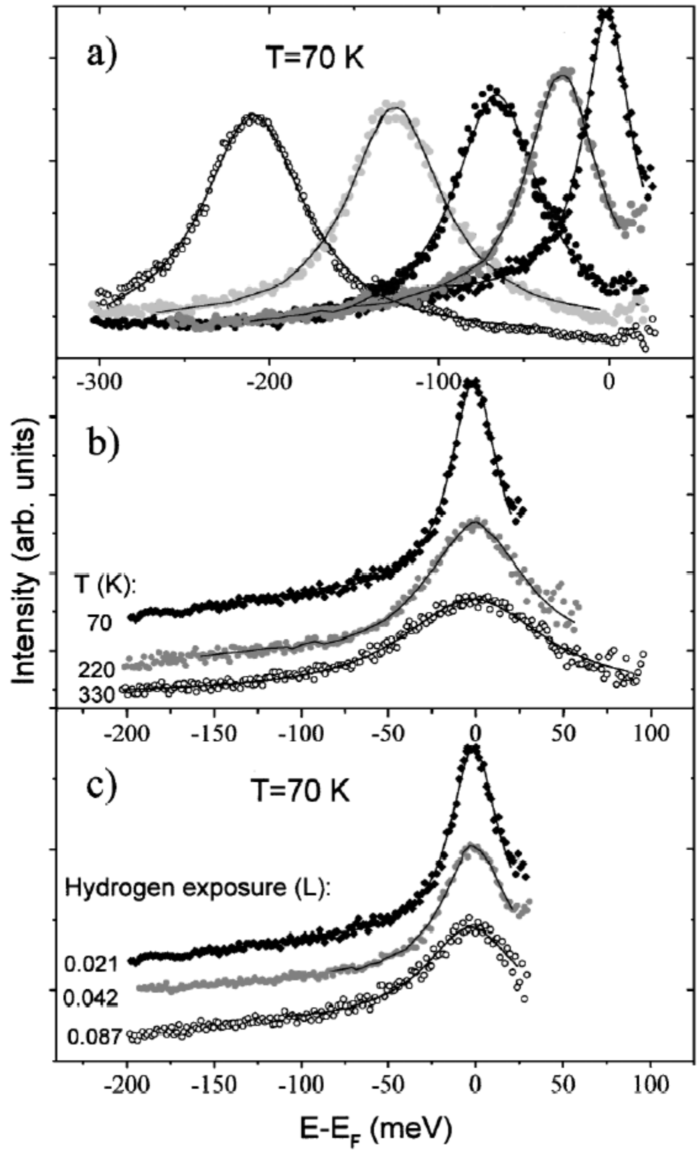


FIG. 2. Spectral intensity as a function of binding energy for constant emission angle, normalized to the experimentally determined Fermi cutoff. Data are symbols, while lines are fits to the Lorentzian peaks with a linear background. The dependence on the (a) binding energy, (b) temperature, and (c) hydrogen exposure is shown.

T. Valla et al. PRL 83 2085 (1999)

Mo(110) surface

Use of Fermi liquids

- Takes care of largest and most complicated processes
- Starting point to take into account perturbations (lattice, impurities, etc.)
- Simple instabilities and ordered states

How to destroy a Fermi liquid

- Strong or unusual interactions (localized electrons, BCS, …)

- Special fermi surfaces (nesting, singularities at E_F)

- Special dimensions (d=1, d=2 (?))

Nesting

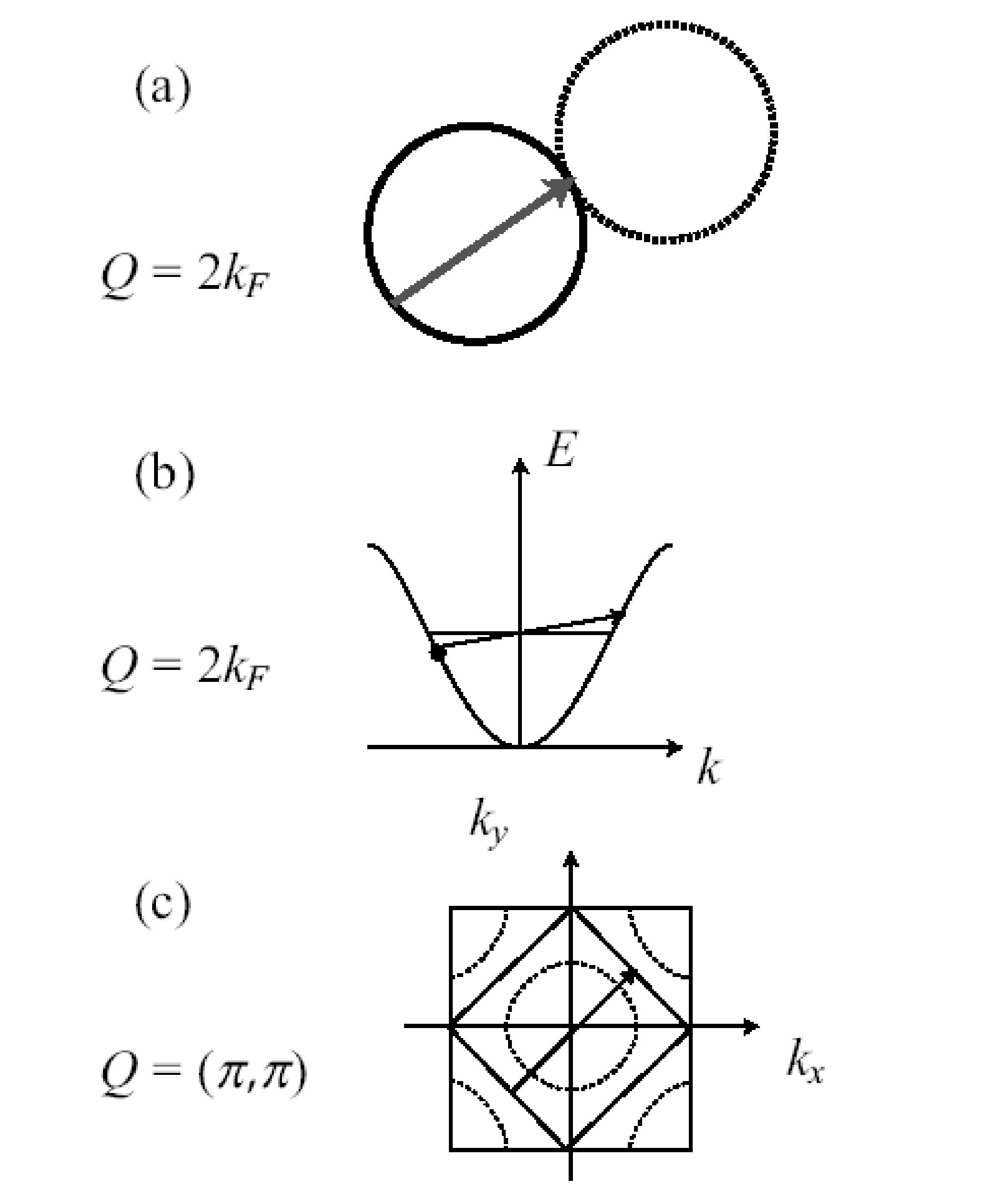
$$\chi(q,\omega) = \frac{1}{\Omega}\sum_k \frac{f_F(\xi_k) - f_F(\xi_{k+q})}{\omega + \xi(k) - \xi(k+q) + i\delta}$$

$$\xi(k+Q) = -\xi(k)$$

$$\mathrm{Re}\,\chi(Q,\omega=0) = -\int d\xi\, N(\xi)\frac{\tanh(\beta\xi/2)}{2\xi}$$

$$\chi(Q,\omega=0) \sim -N(\xi=0)\ \log(E/T)$$

Divergent !

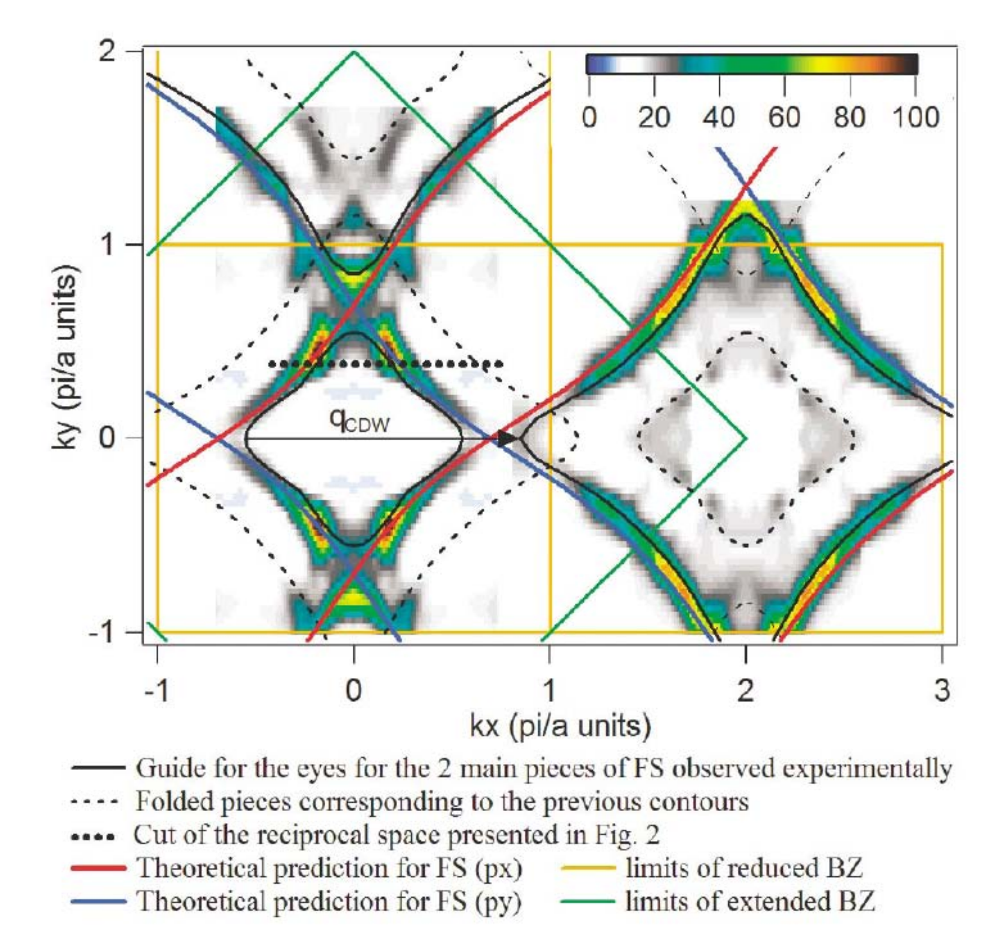


Difficult in high dimensions

Easy in d=1

Special surfaces

Tellurides: $CeTe_3$



V. Brouet et al. PRL 93 126405 (2004)

What happens ?

Naively : ordered state

$$\chi(q,\omega) = \frac{\chi^0(q,\omega)}{1 + U\chi^0(q,\omega)}$$

RPA

$\chi = \infty$ at T_c

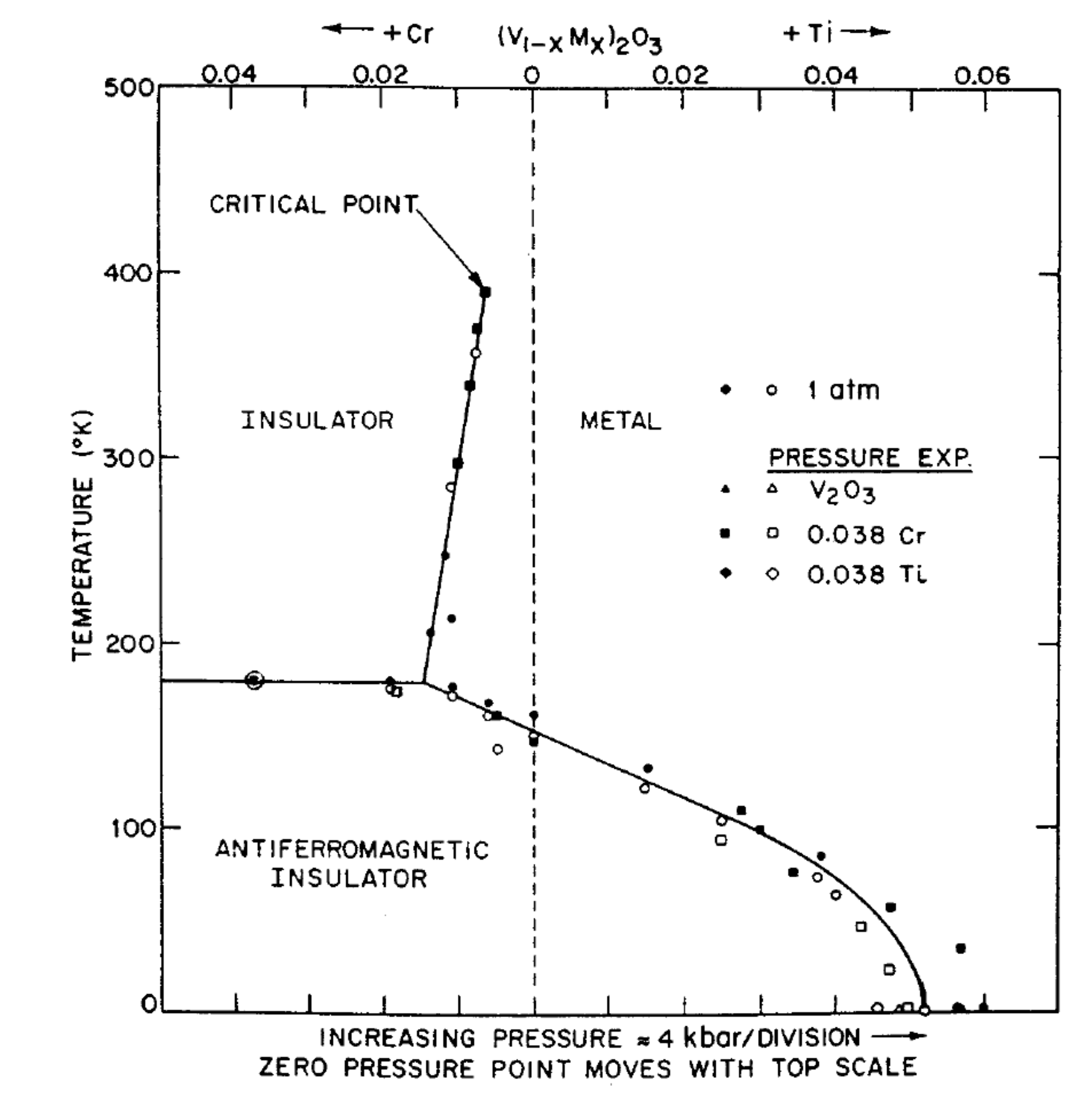
Instabilities compete

Fluctuations ?

How to *really* break FL

- Lower dimension (fluctuations)
- Increase interactions
- Strange degrees of freedom

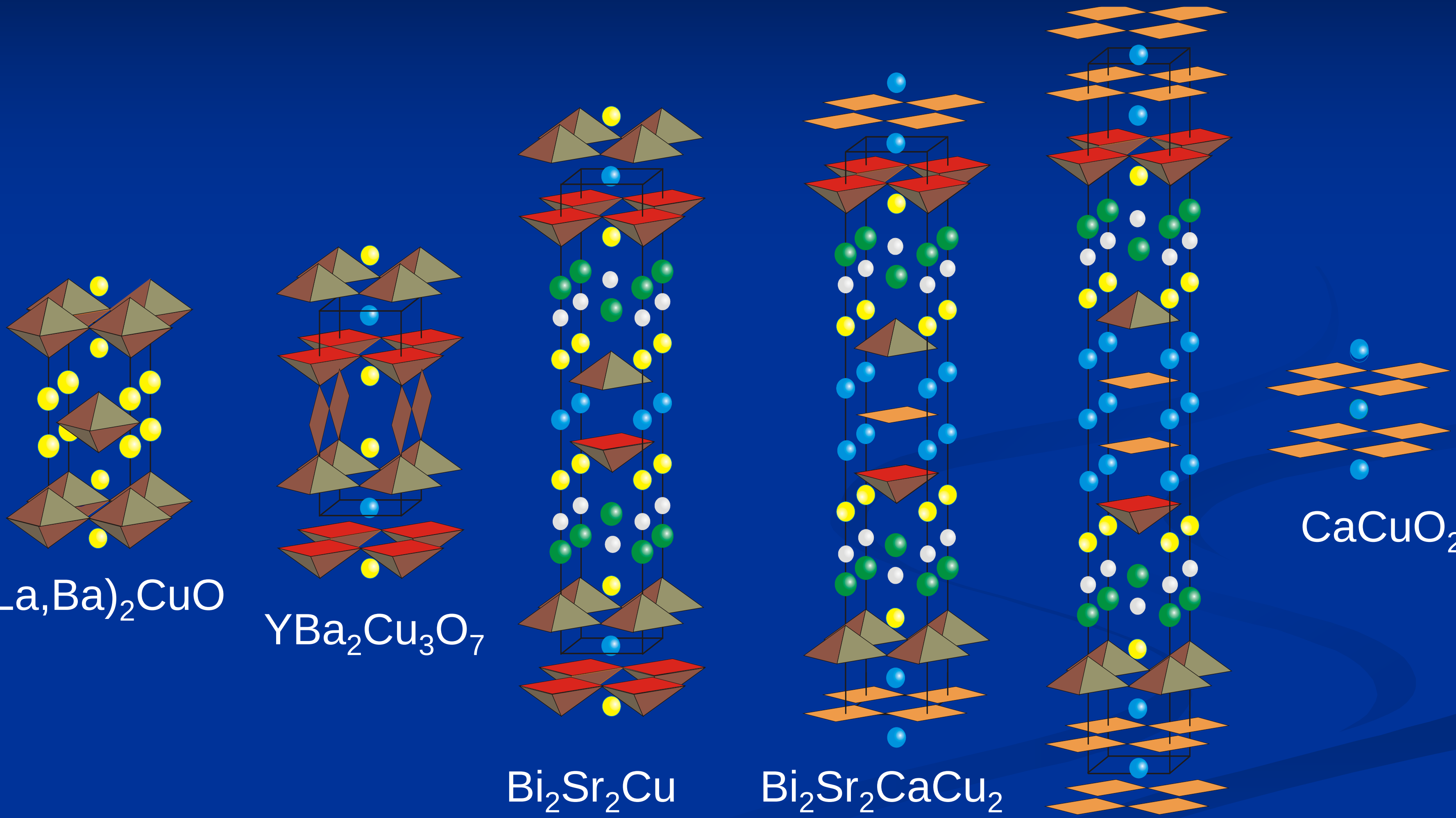
Mott transition



← +Cr
$(V_{1-x}M_x)_2O_3$
+Ti →
0.04
0.02
0
0.02
0.04
0.06
500
400
300
200
100
0
TEMPERATURE (°K)
CRITICAL POINT
INSULATOR
METAL
ANTIFERROMAGNETIC INSULATOR
1 atm
PRESSURE EXP.
V_2O_3
0.038 Cr
0.038 Ti
INCREASING PRESSURE ≈ 4 kbar/DIVISION →
ZERO PRESSURE POINT MOVES WITH TOP SCALE

	Mott insulator				MIT		Metals near MIT					
Compounds	Type		Spin order	T_N(K)	Control parameter	order	ρ	χ	γ	Spin order	Spatial dimension of anisotropy	Remark
V_2O_{3-y}	d^2	MH	AF	~180	FC BC T	1	T^2 for BC	ePp~10 for BC ~30–40 for FC ($y=0.013$)	e ≤40 for BC and FC	SDW for FC P for BC	3 corundum	strong mass enhancement
$NiS_{2-x}Se_x$	d^8 $S=1$	CT	AF	40–80	BC T	weakly 1	T^2	ePp ~5 [1]	e ~20	AF	3 pyrite	$\rho \sim T^{1.5}$ at AF-P transition R_H enhanced
RNiO$_3$	d^7 $S=1/2$	CT	AF	130– 240	BC T	1	T^2	ePp 5	e 14	P	3 perovskite	
$NiS_{1-x}Se_x$	d^8 $S=1$	CT	AF	260	(FC)BC T(260 at $x=0$)	1	T^2	ePp 1.6–2.4	e ~6–7	P	3 NiAs	
$Ca_{1-x}Sr_xVO_3$	—	—	—	—	—	—	T^2	ePp ~2	e ~9	P	3 perovskite	
$La_{1-x}Sr_xTiO_3$	d^1 $S=1/2$	MH	AF	140	FC $x_c=0.05$	C	T^2	ePp ≤5	e ≤17	P	3 perovskite	typical mass enhancement by FC
$La_{1-x}Sr_xVO_3$	d^2 $S=1$	MH	AF	~150	FC $x_c=0.2$	C	$\sim T^{1.5}$	ePp ~9.5 [2]	e *	P	3 perovskite	
$La_{2-x}Sr_xCuO_4$	d^9 $S=1/2$	CT	AF	300	FC $x_c \sim 0.05$–0.06	C	$\sim T$	text	≤14	P SC	2 K_2NiF_4	high-T_c superconductor $T_c \leq 40$ K
$Nd_{2-x}Ce_xCuO_4$	d^9 $S=1/2$	CT	AF	~240	FC $x_c \sim 0.14$	C	$\sim T^2$	*	*	P SC	2	high-T_c superconductor $T_c \leq 24$ K
$YBa_2Cu_3O_{7-y}$	d^9 $S=1/2$	CT	AF	~400	FC $y_c \sim 0.6$	C	$\sim T$	text (pseudo gap)	≤18	P SC	2 perovskite-like	high-T_c superconductor $T_c \leq 90$ K
$Bi_2Sr_2Ca_{1-x}R_xCu_2O_{8+\delta}$	d^9 $s=1/2$	CT	*	*	FC	C	$\sim T$	≤0.2 [3]	*	P SC	2	high-T_c superconductor $T_c \leq 90$ K
$La_{1-x}Sr_xCuO_{2.5}$	d^9 $S=1/2$	CT	AF	110	FC $x_c \sim 0.15$	C	$\sim T^2$	~1	≤4 [4]	P	1 (or 3)	near critical point between AF and gapped phase, spin-ladder system
$Sr_{14-x}Ca_xCu_{24}O_{41}$	d^9 $s=1/2$	CT	gap	—	FC $x_c \sim 13.5$	C	$\sim T^2$	~2	*	P SC	1 (or 2)	superconductor under pressure at $x \sim 13.5$ ($T_c \sim 10$ K), spin-ladder system

High Tc



$Bi_2Sr_2Ca_2Cu_3O_{10}$

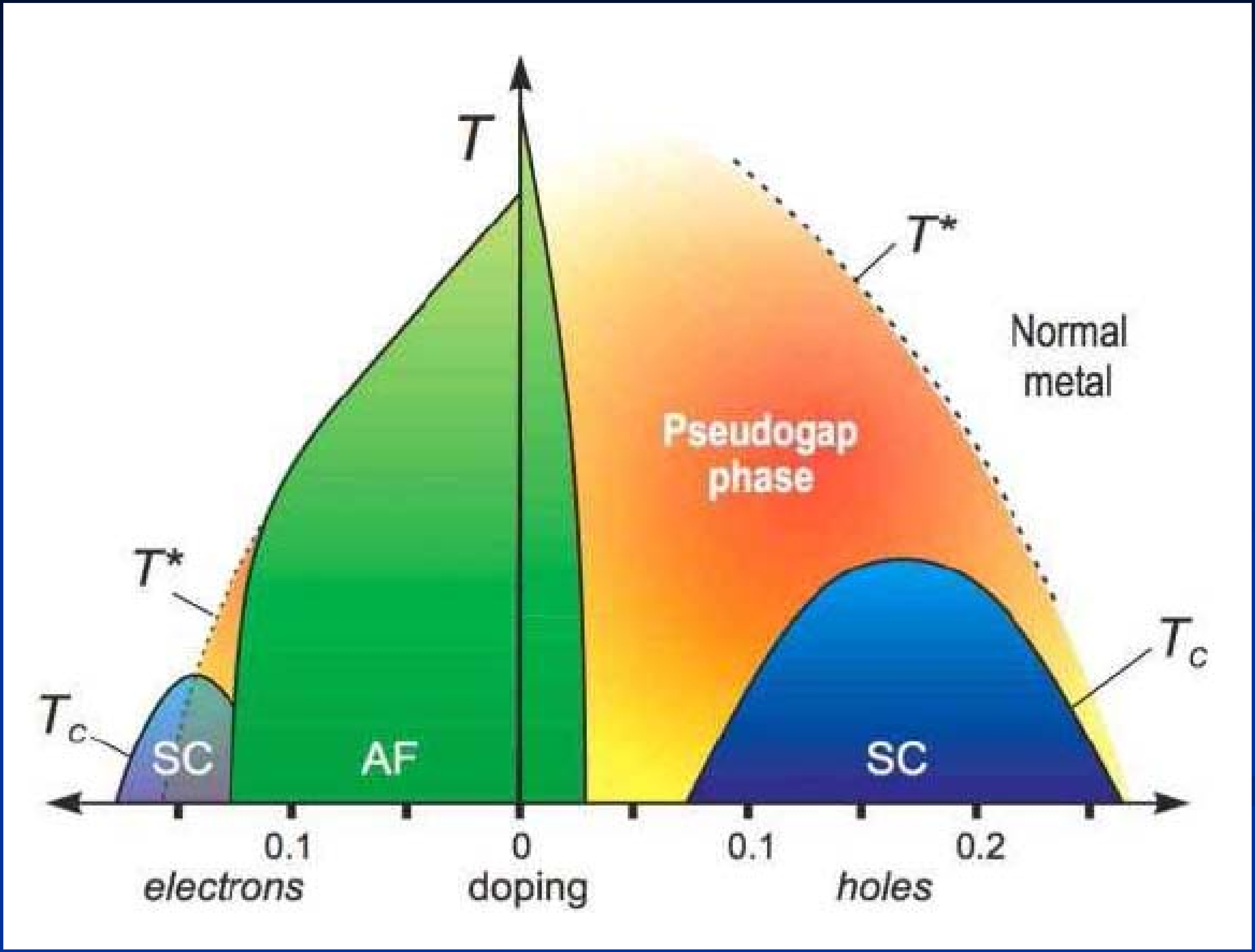
(La,Ba)$_2$CuO
4

$YBa_2Cu_3O_7$

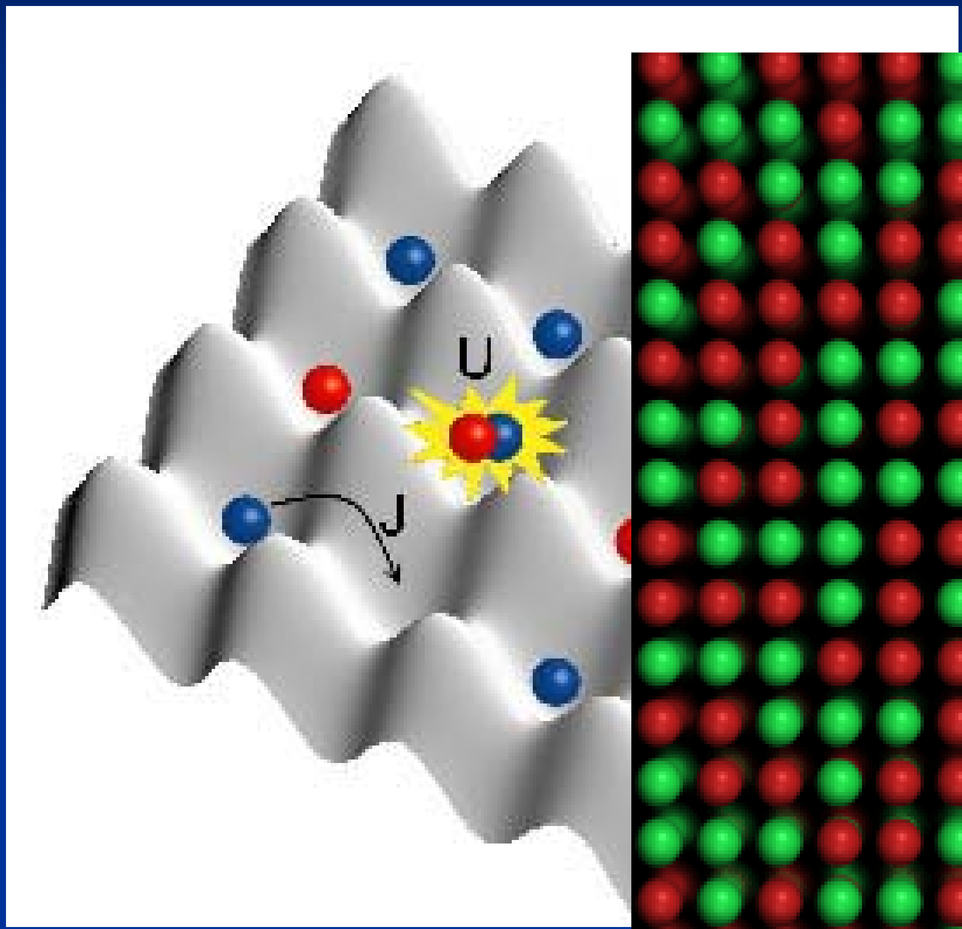
Bi_2Sr_2Cu
O_6

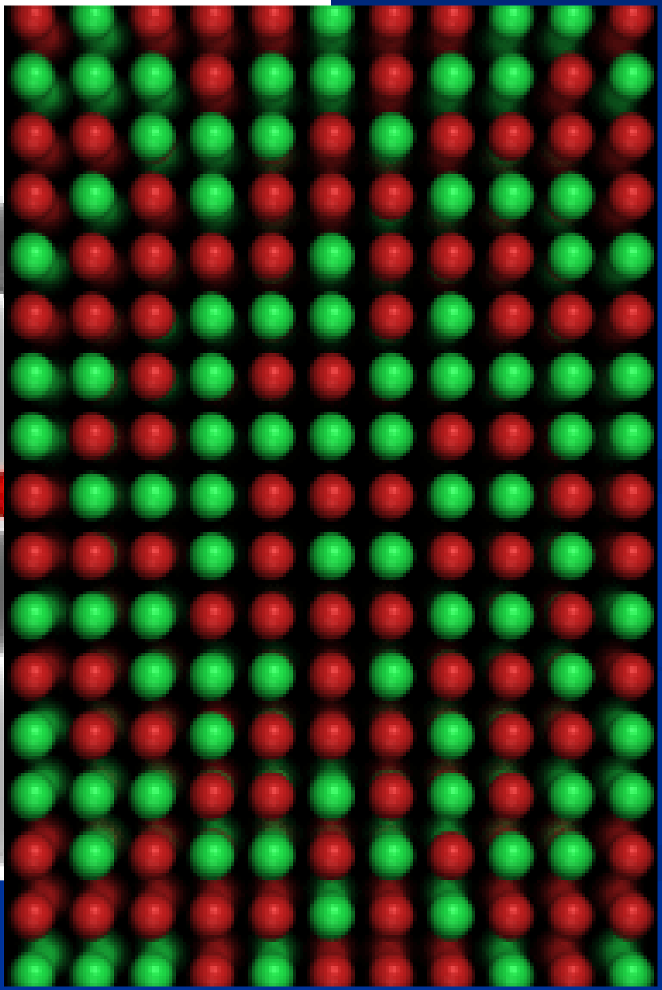
$Bi_2Sr_2CaCu_2$
O_8

$CaCuO_2$


T
T*
Normal
metal
Pseudogap
phase
T*
T_c
T_c
SC
AF
SC
0.1
0
0.1
0.2
electrons
doping
holes

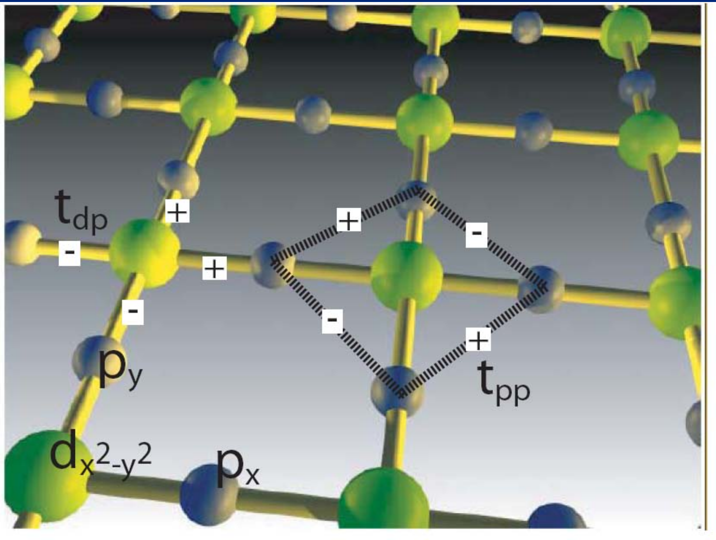
Mott insulators

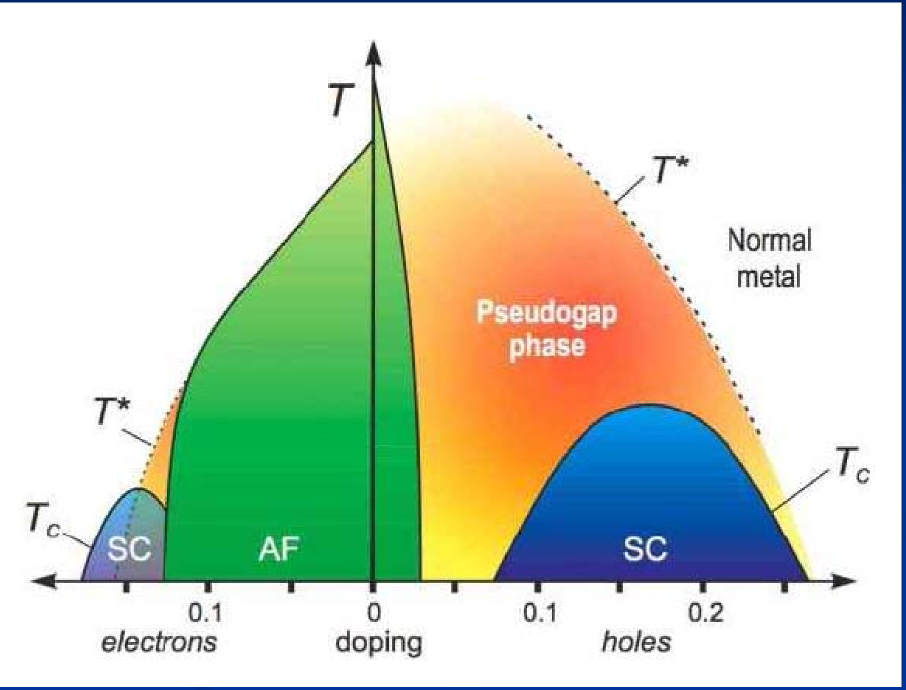




Doped Mott insulators ??

Find a simplified model

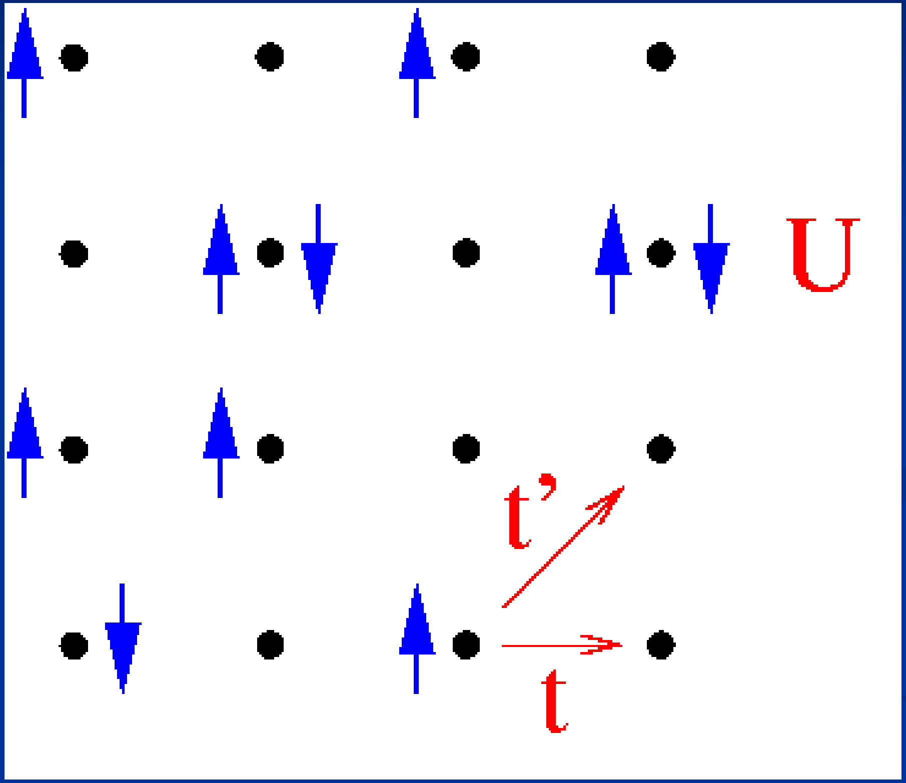




Simplest model containing:

- Bands (filling)
- Interactions

Hubbard model (1963)

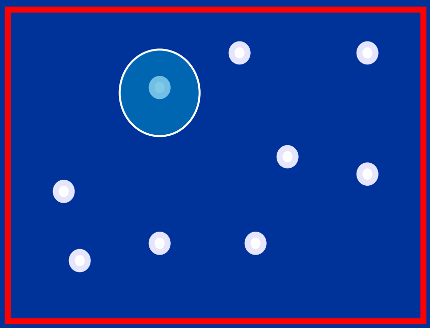


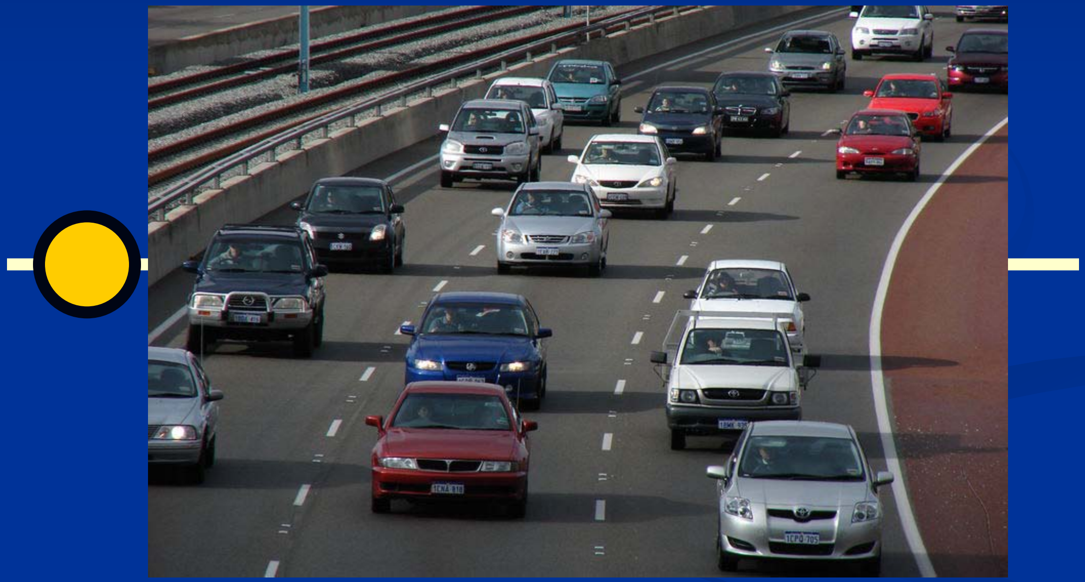
$$H = -t \sum_{<i,j>,\sigma} (c^{\dagger}_{i,\sigma} c_{j,\sigma} + h.c.) + U \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$

One dimensional systems: Luttinger liquid

One dimension is specially interesting

- No individual excitation can exist (only collective ones)





- Strong quantum fluctuations

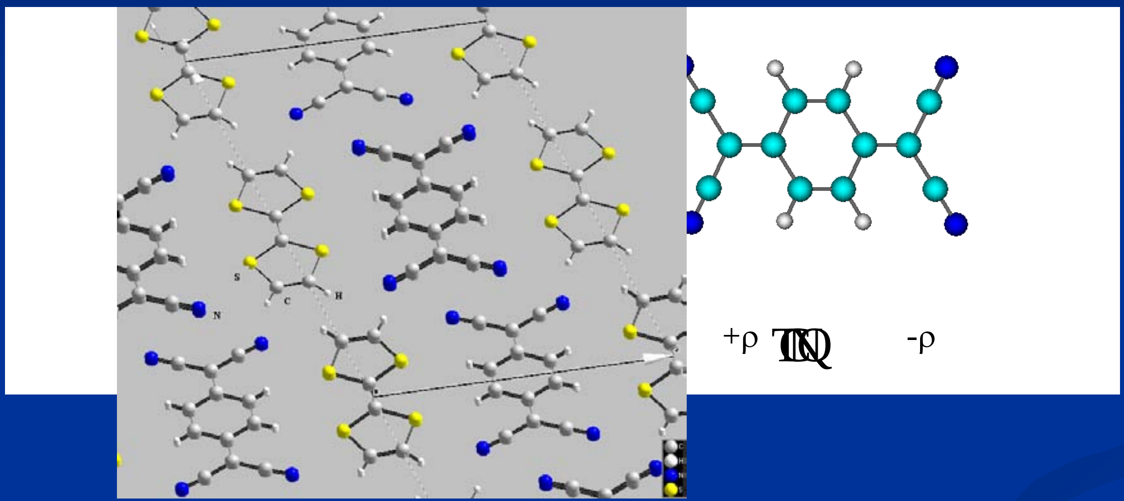
$$\psi \to \psi e^{i\theta}$$

Difficult to order

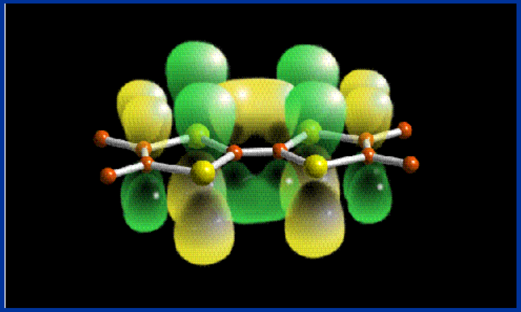
Interesting…
but…
does it exist ?

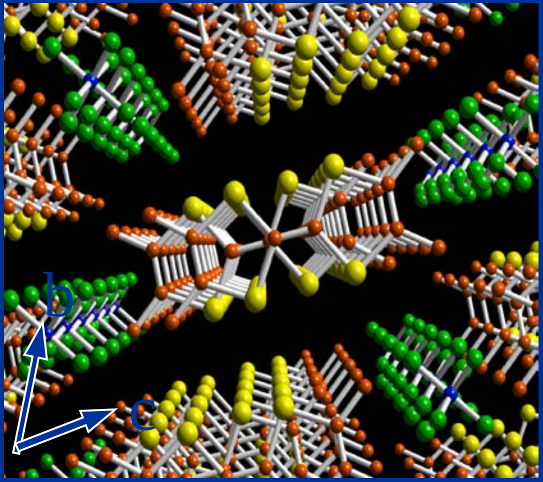
Organic conductors

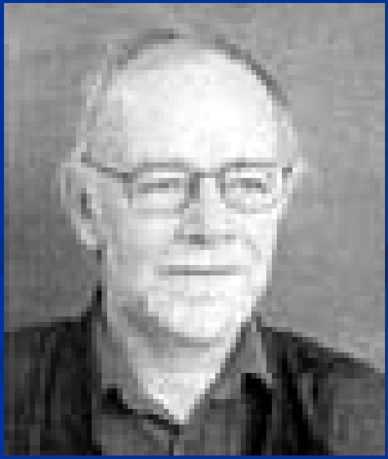
1973: F.Wudl, D.Cowan, A.Garito, A.Heeger



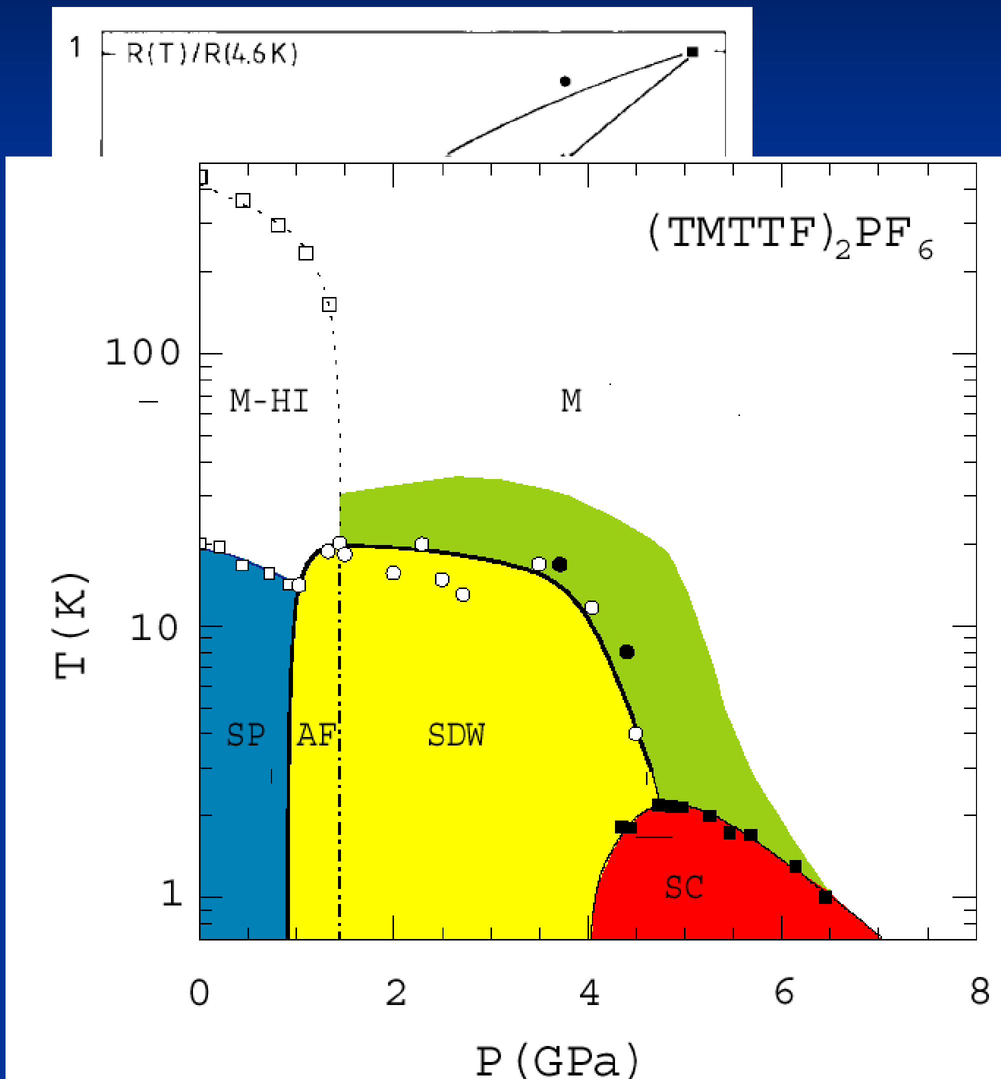
1979: K. Bechgaard





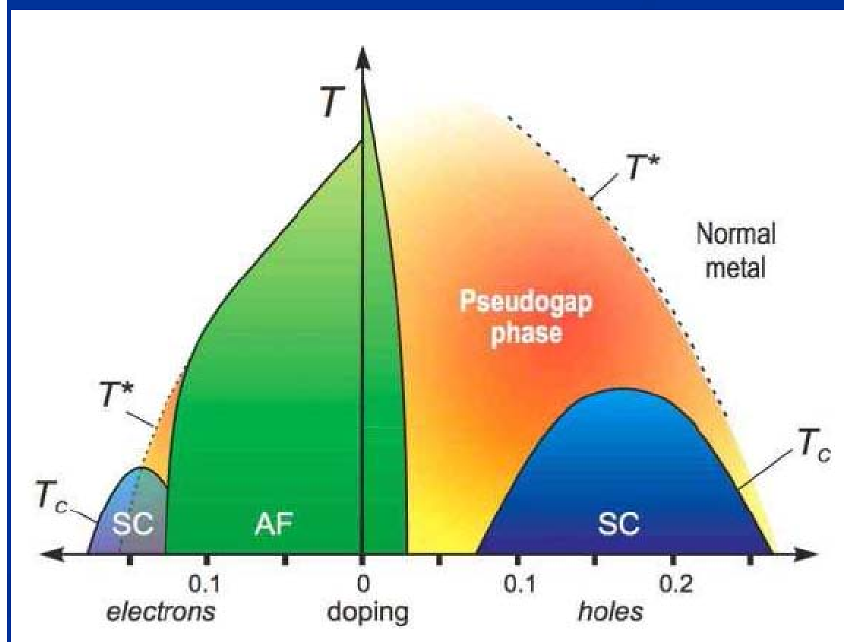


Superconductivity !



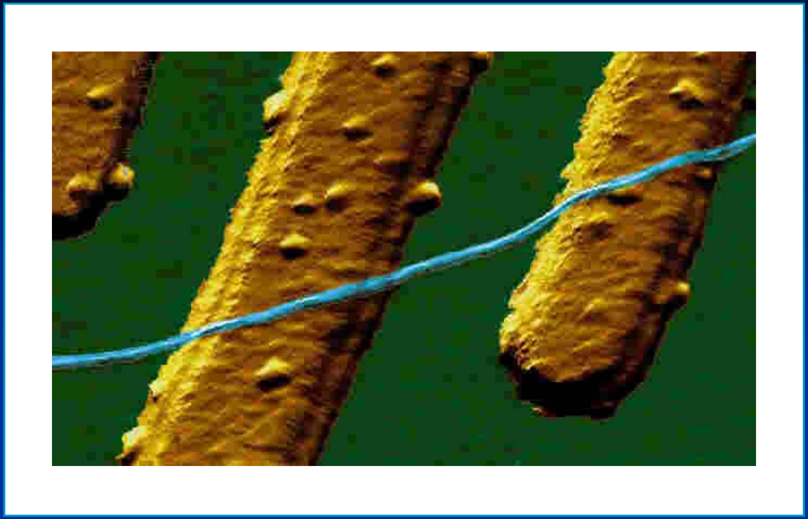
erome, M,Ribault,

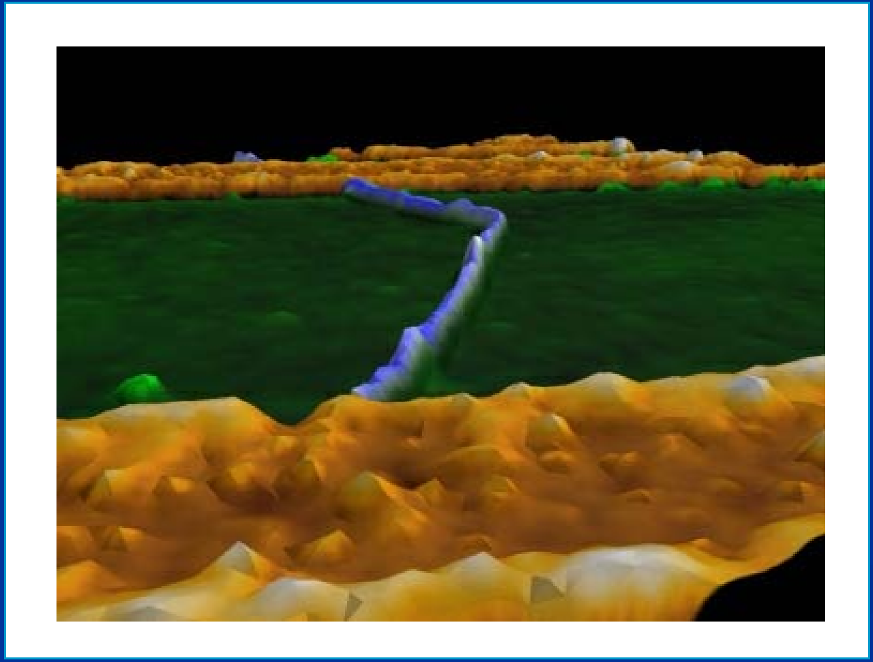
zaud and K.Bechgaard (1980)

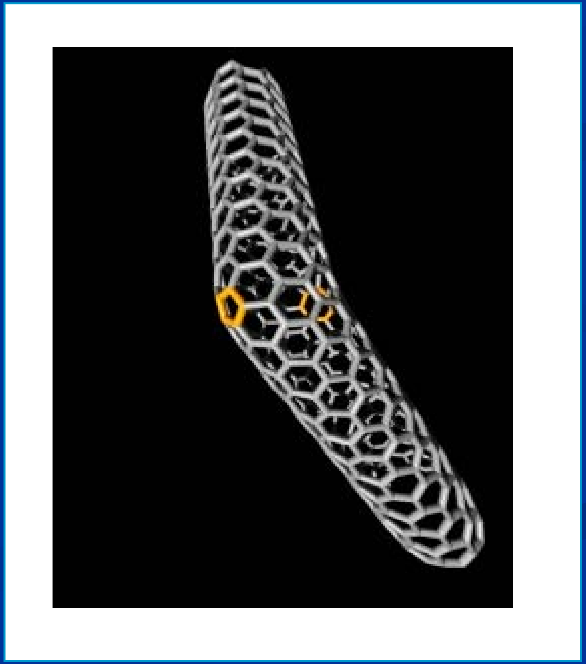


CARBON NANOTUBES

Cees Dekker



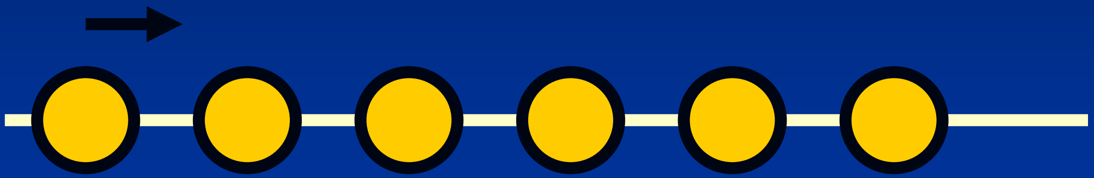




How to make a theory

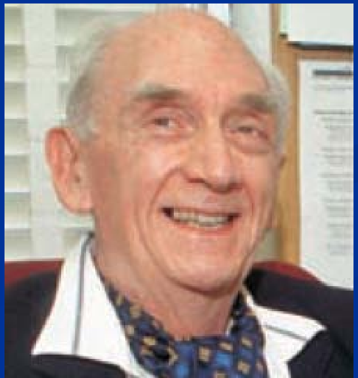
Tomonaga

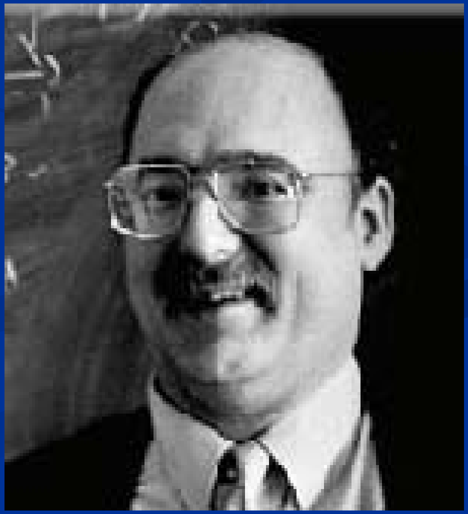
Luttinger

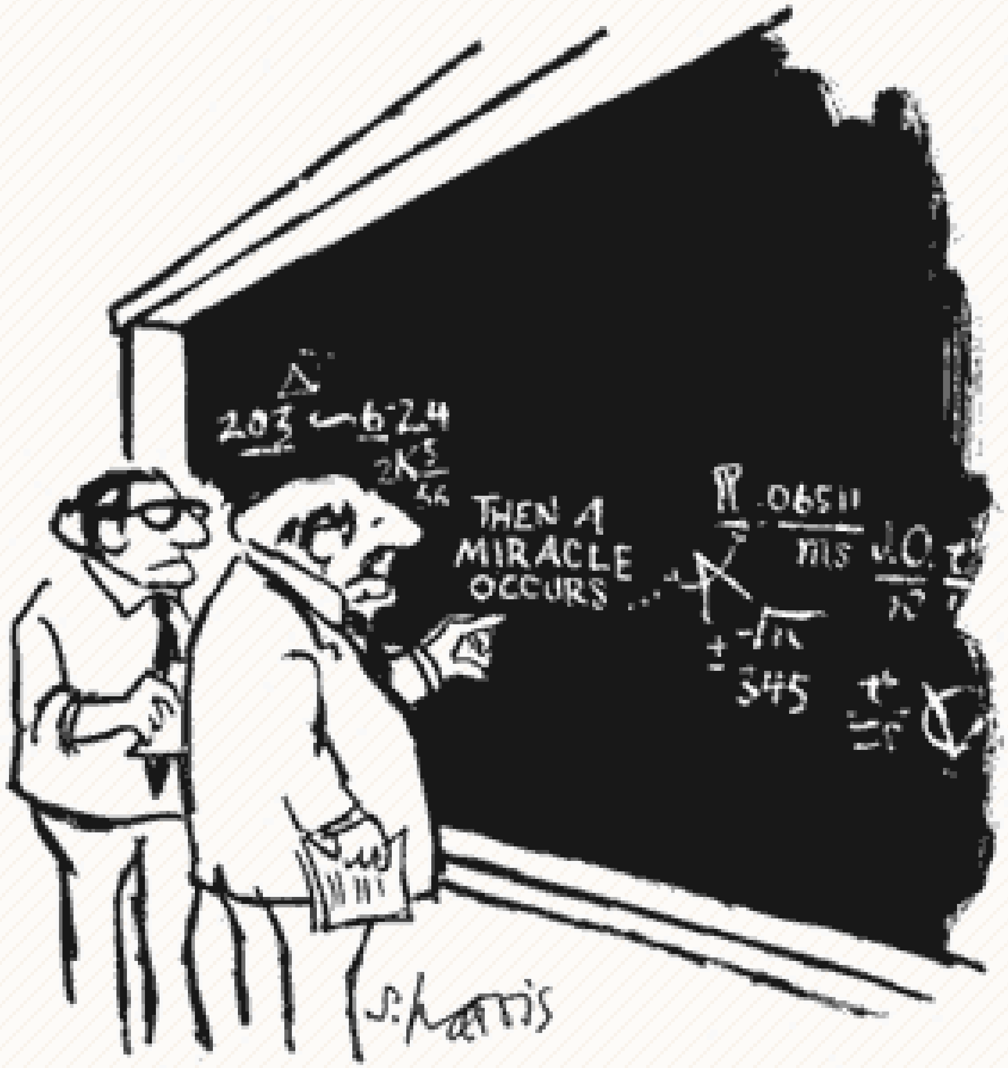


Good excitations: collective ones

60': A. Larkin, I. Dzialoshinskii, L. Gorkov, Bichkov
E. Lieb, D. Mathis
70': A. Luther, V.J. Emery
80': F.D.M. Haldane





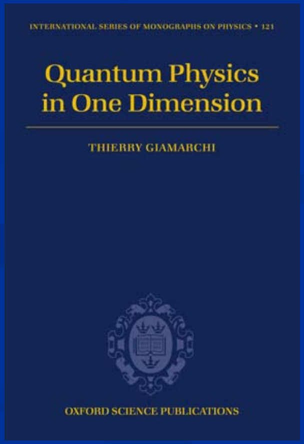


"I THINK YOU SHOULD BE MORE EXPLICIT HERE IN STEP TWO."

Details in:

TG, cond-mat/0605472 (Salerno lectures)

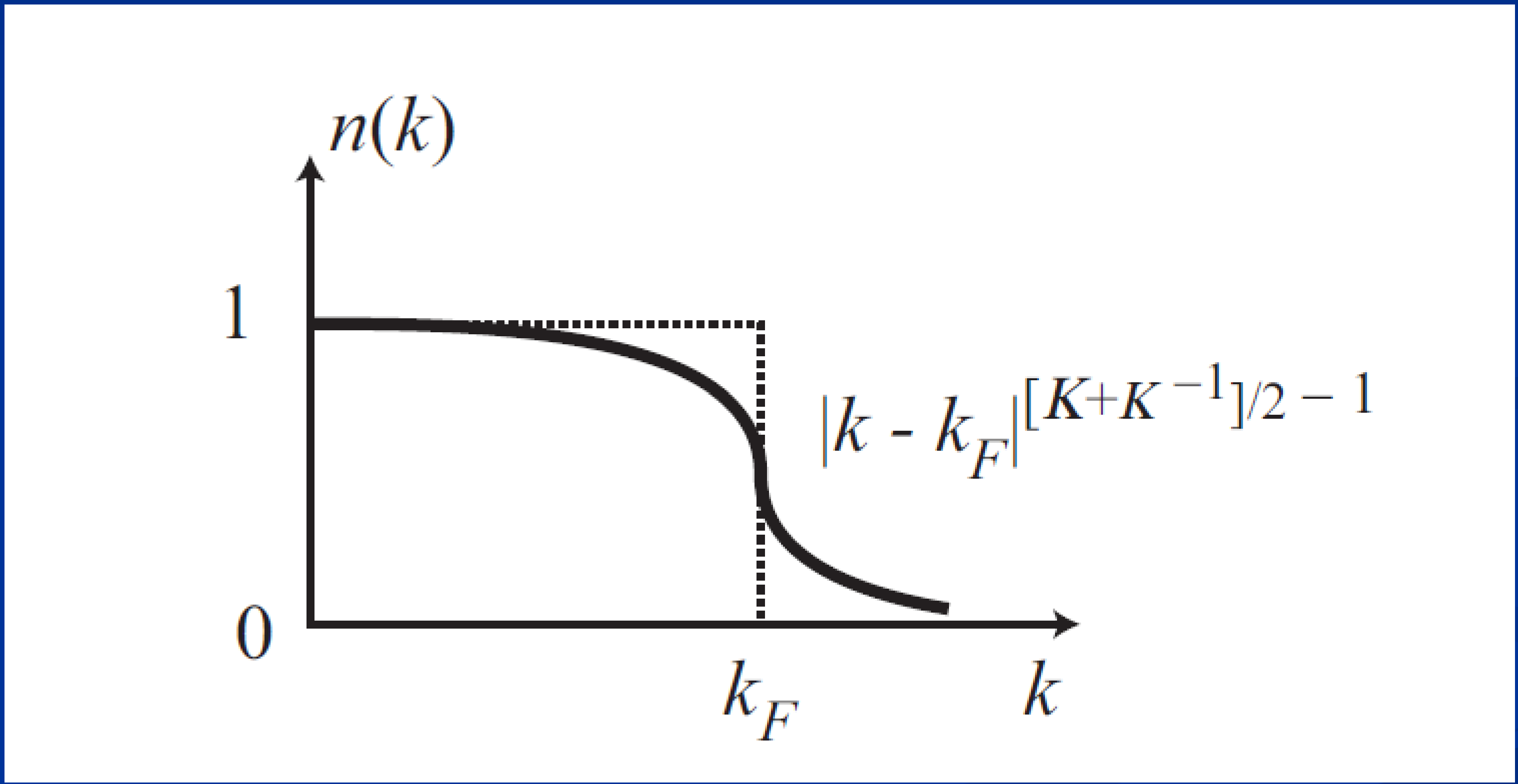
TG, Quantum physics in one dimension, Oxford (2004)



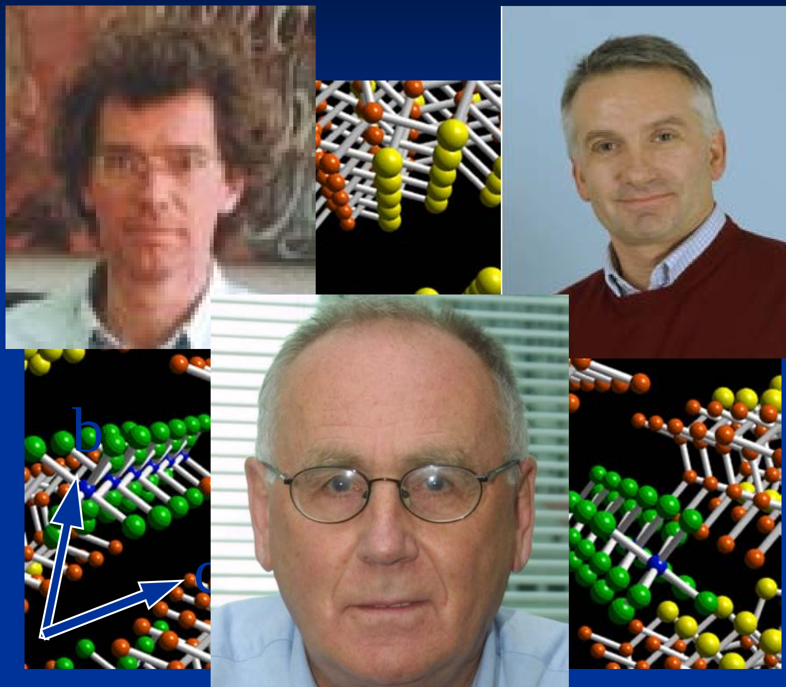
Universal description: Luttinger liquid

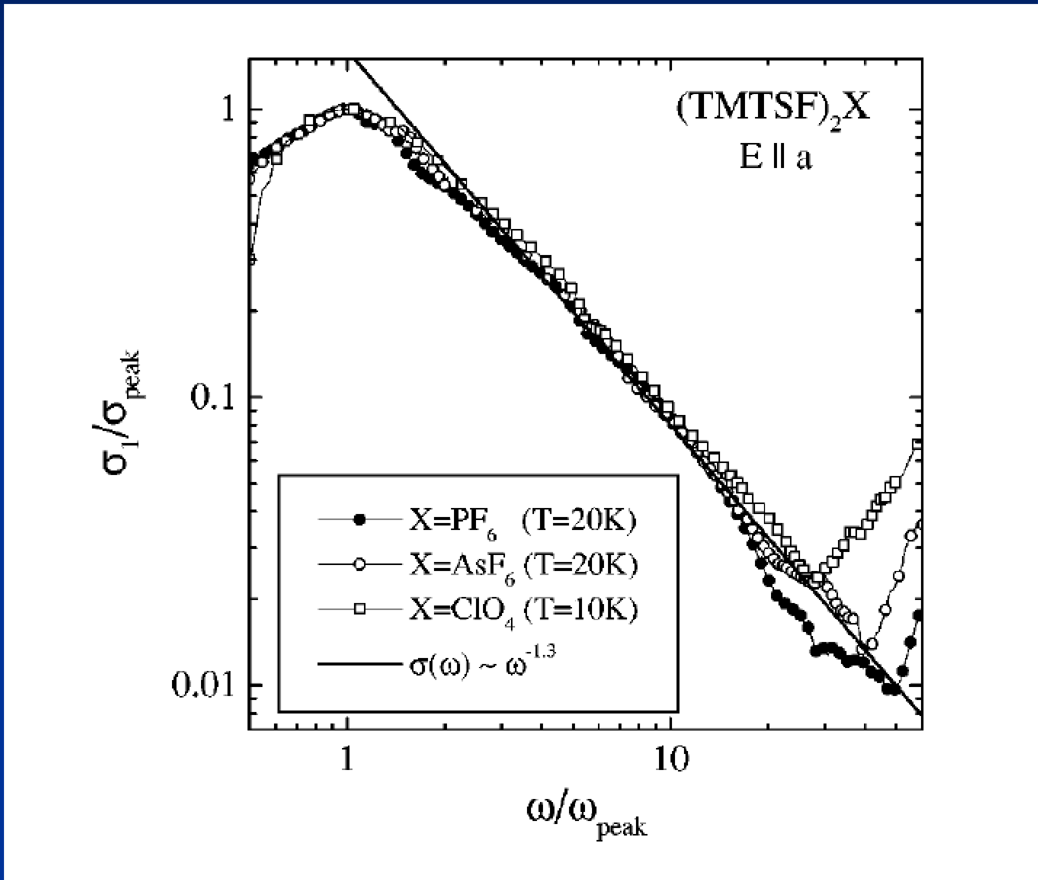
- Low energy effective description
- All effects described by two parameters: u and K (Luttinger liquid parameters)
- Power law correlation functions

Occupation factor



Organic conductors



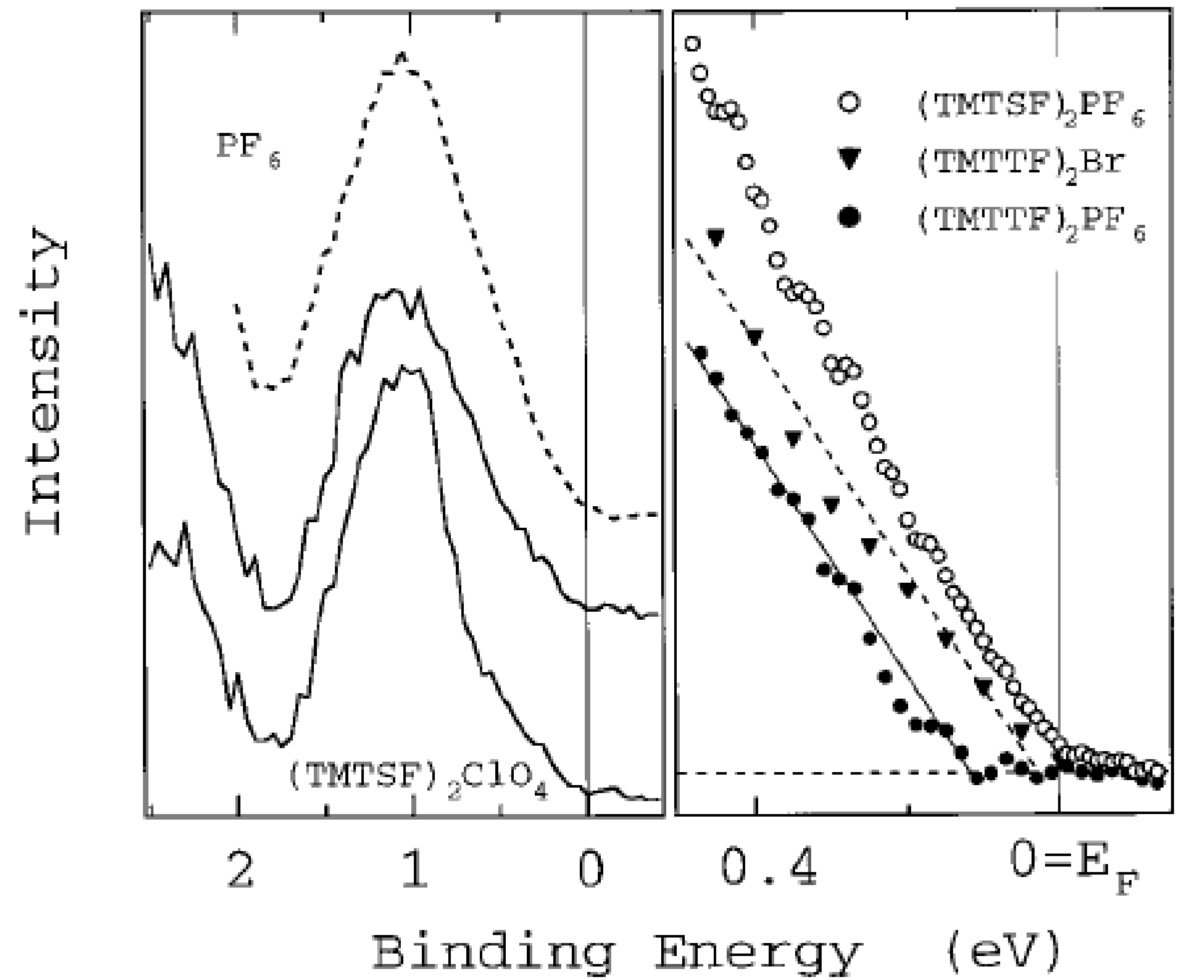


$$\sigma(\omega) \sim \omega^{\nu}$$

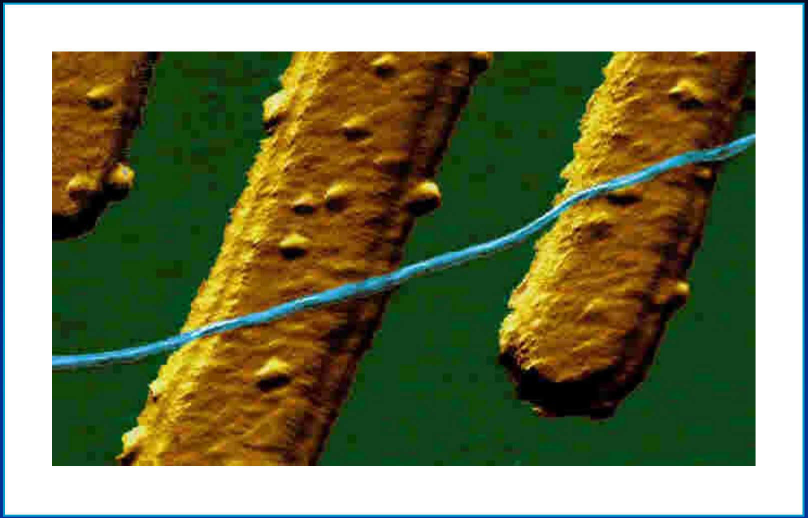
TG PRB (91) :
Physica B 230 (1996)

A. Schwartz et al. PRB 58 1261 (1998)

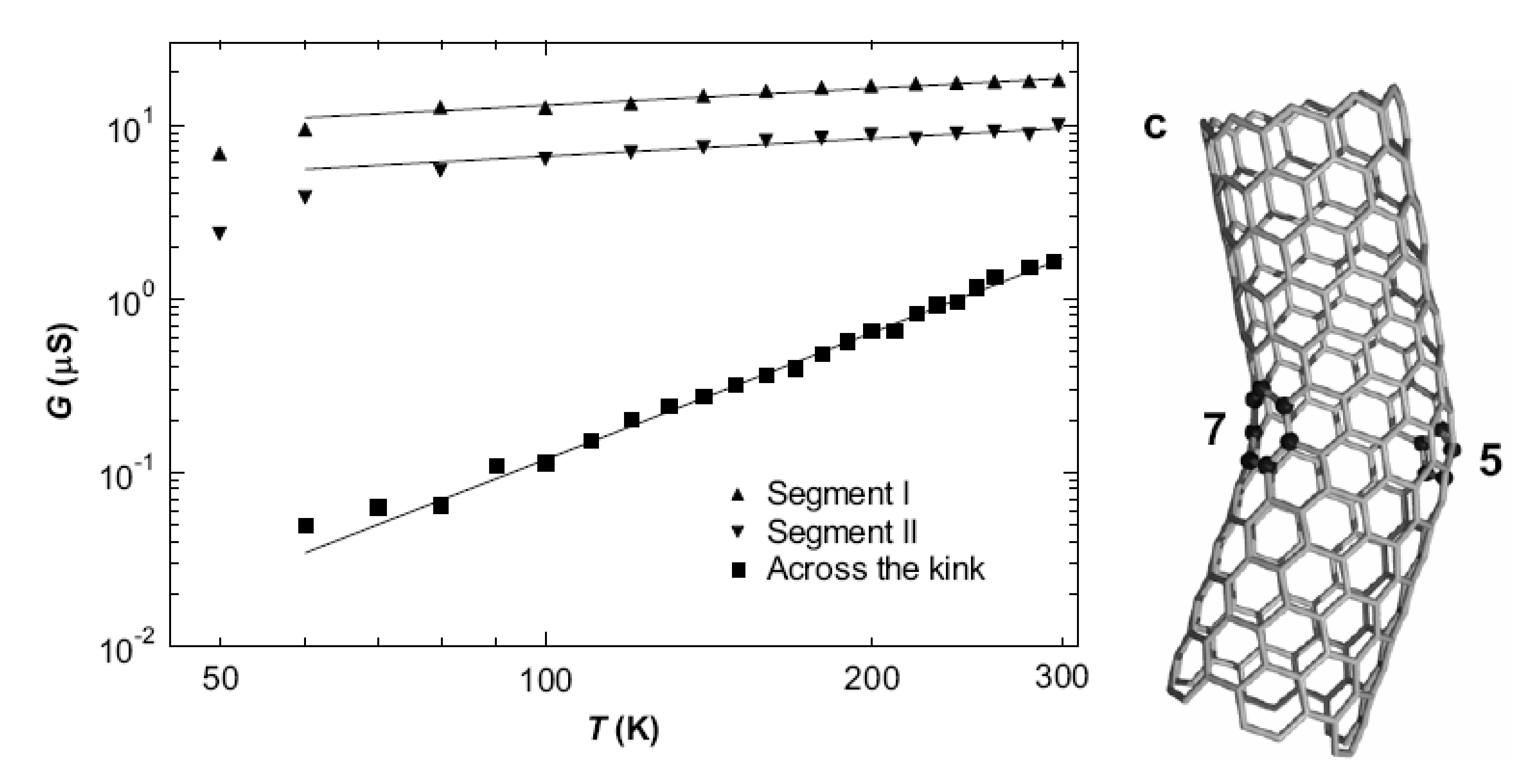
First observation of LL !!



V. Vescoli et al. EPJB 13 503 (2000)

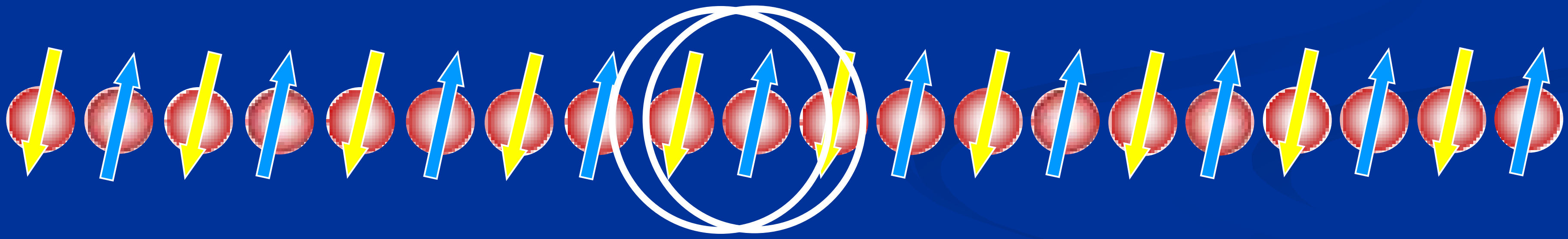


Z. Yao et al. Nature 402 273 (1999)



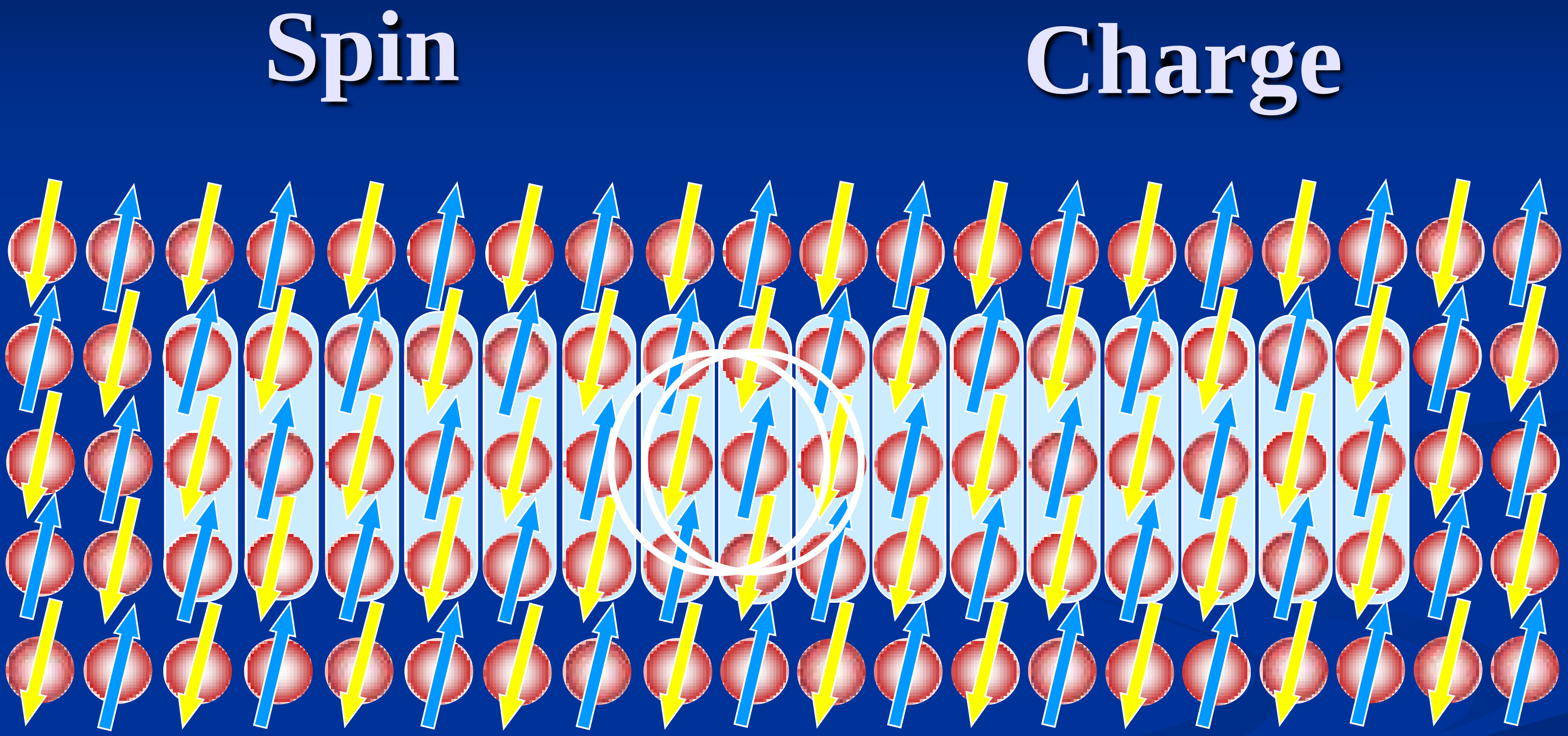
Fractional excitations

Spin Charge



Spinon Holon

Spin-Charge Separation higher D ?



Energy increases with spin-charge separation

Confinement of spin-charge: « quasiparticle »

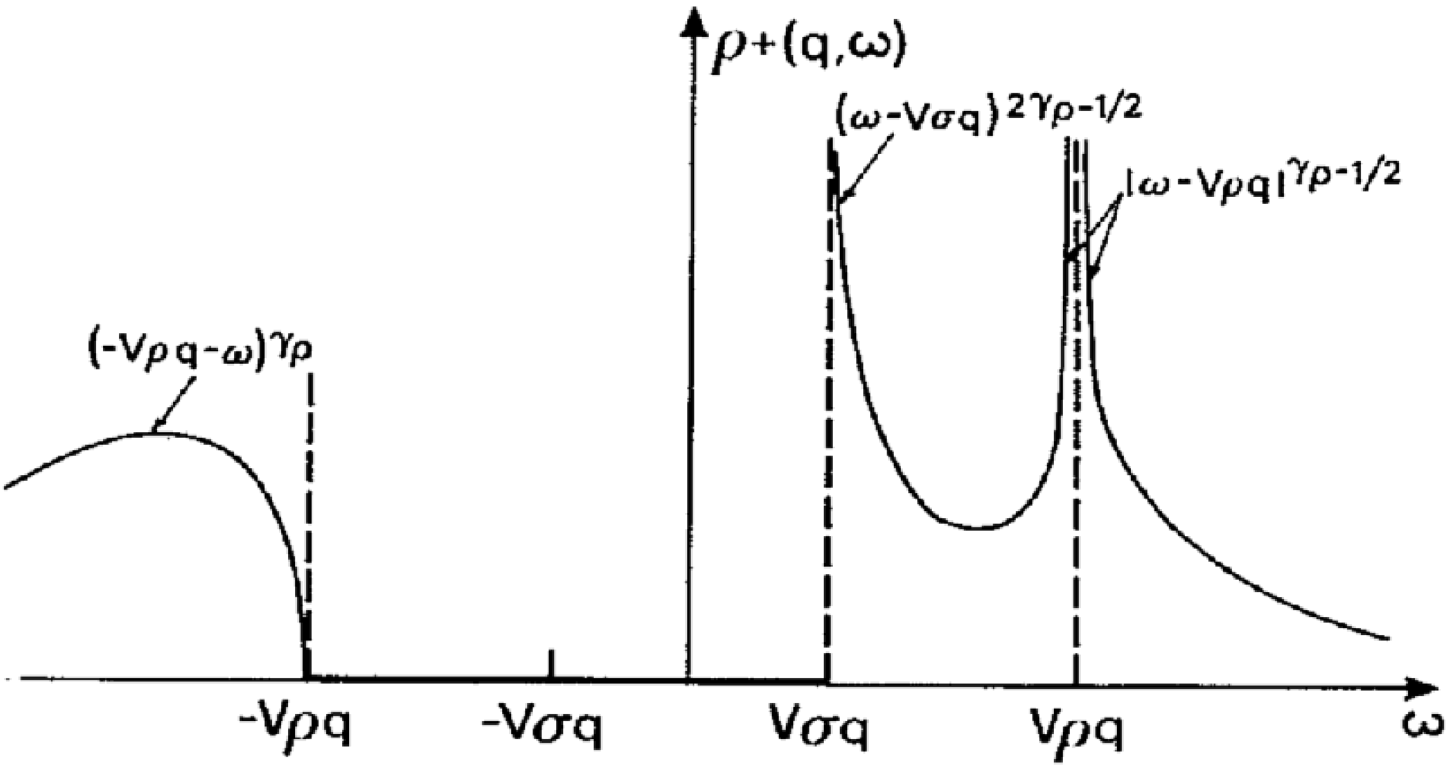
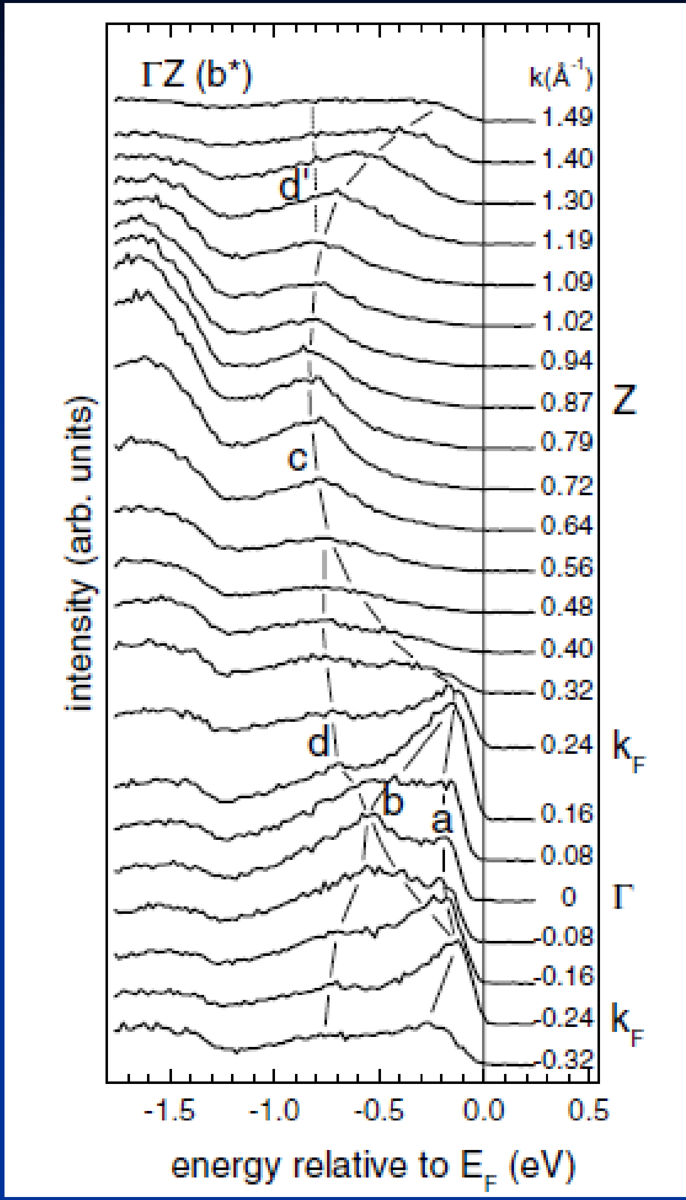
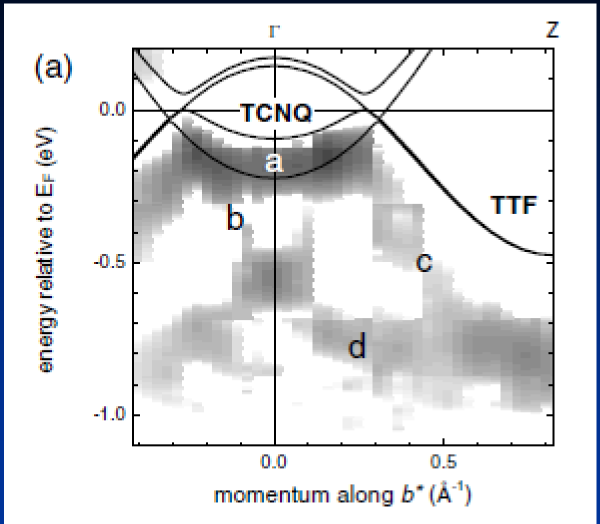
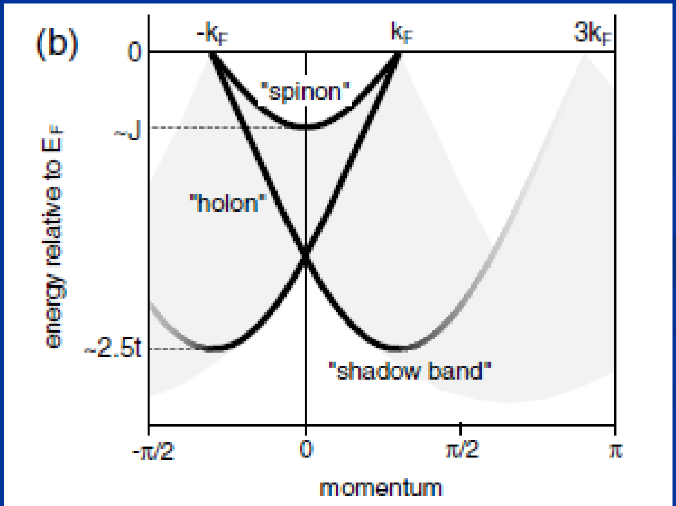


FIG. 3. Spectral function $\rho_+(q,\omega)$ for the spin-$\frac{1}{2}$ Luttinger liquid for $q > 0$.

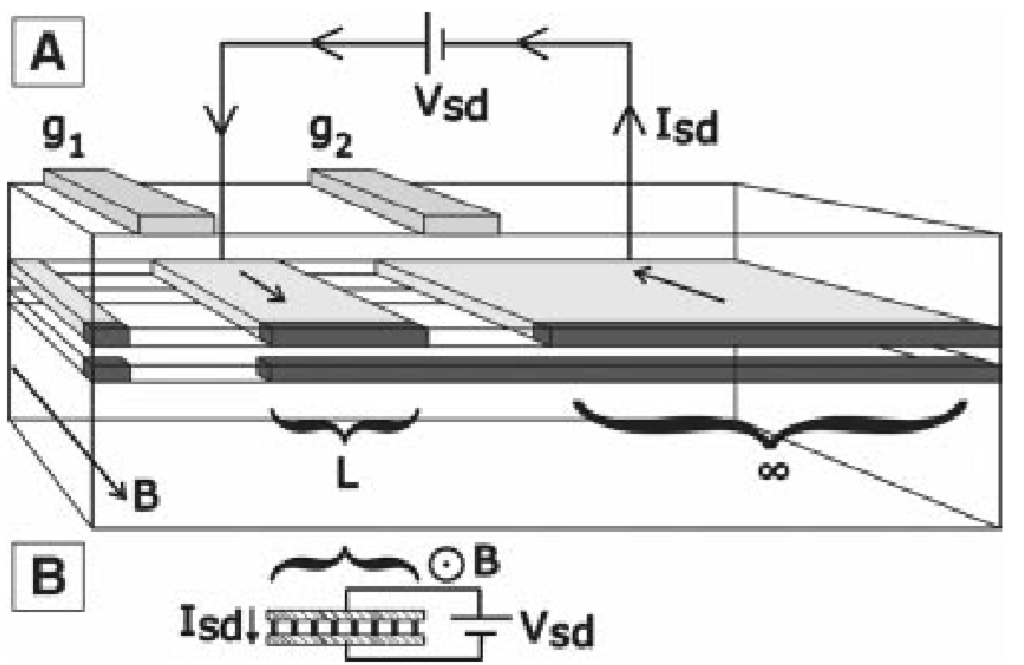
J. Voit

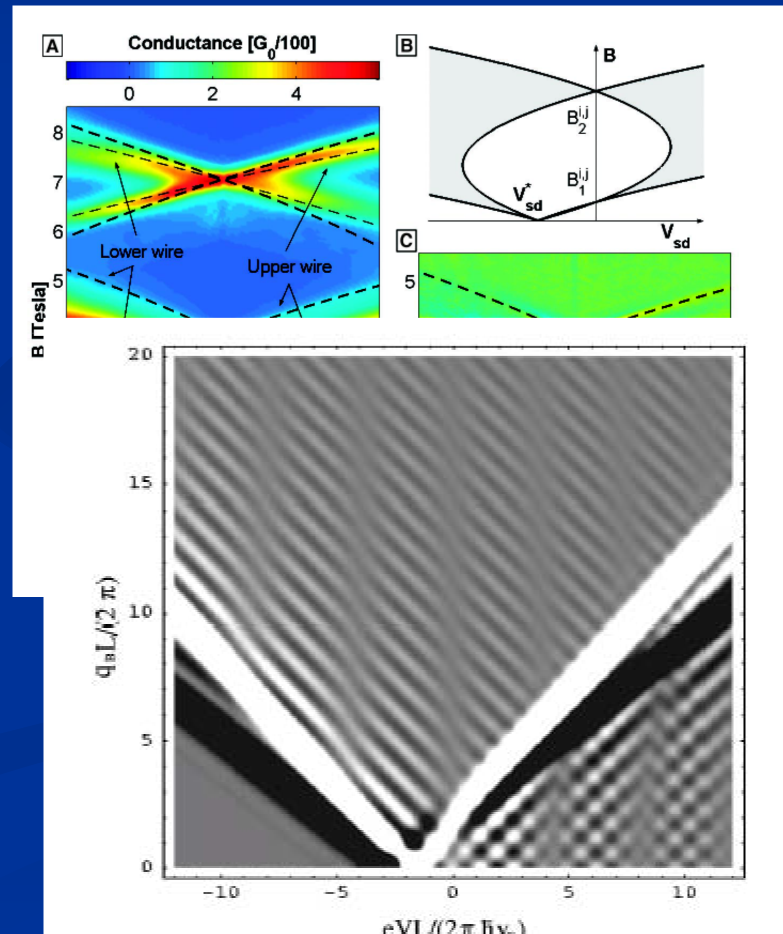






R. Claessens et al. PRL 88 096402 (2002)



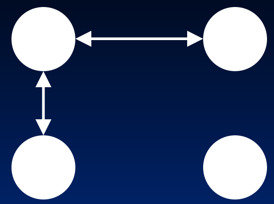


O.M Ausslander et al., Science **298** 1354 (2001)

Y. Tserkovnyak et al., PRL **89** 136805 (2002)

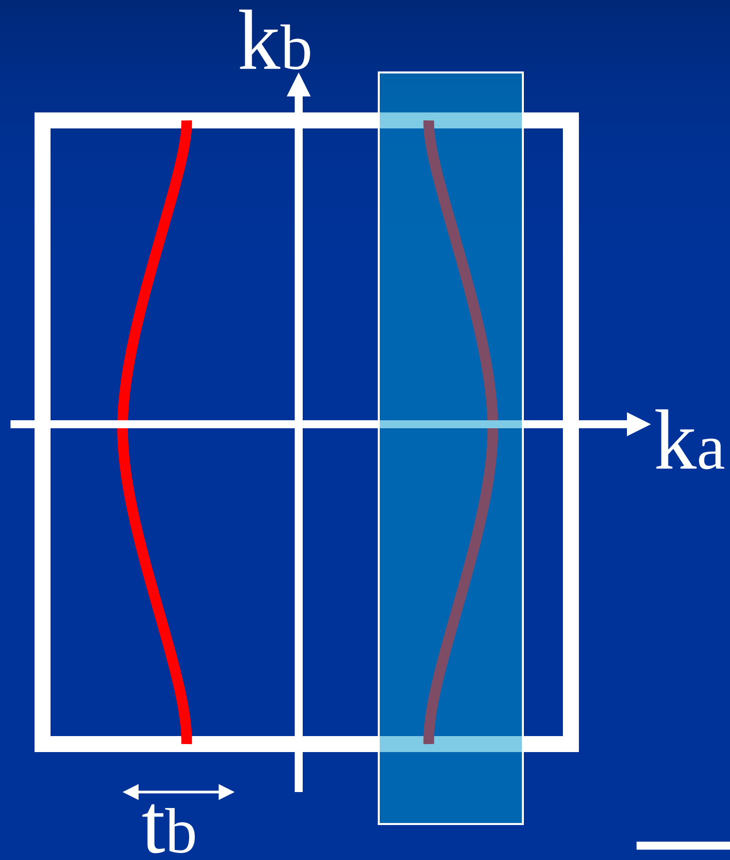
Y. Tserkovnyak et al., PRB **68** 125312 (2003)

Example: deconfinement (from 1D to 3D)



$$t_a > t_b > t_c$$

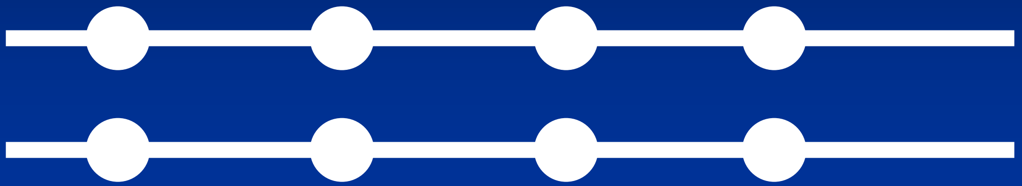
$$3000K, 300K, 20K$$



- High Energy (T,ω): 1D
- Low Energy (T,ω) : 2D,3D

Dimensional crossover

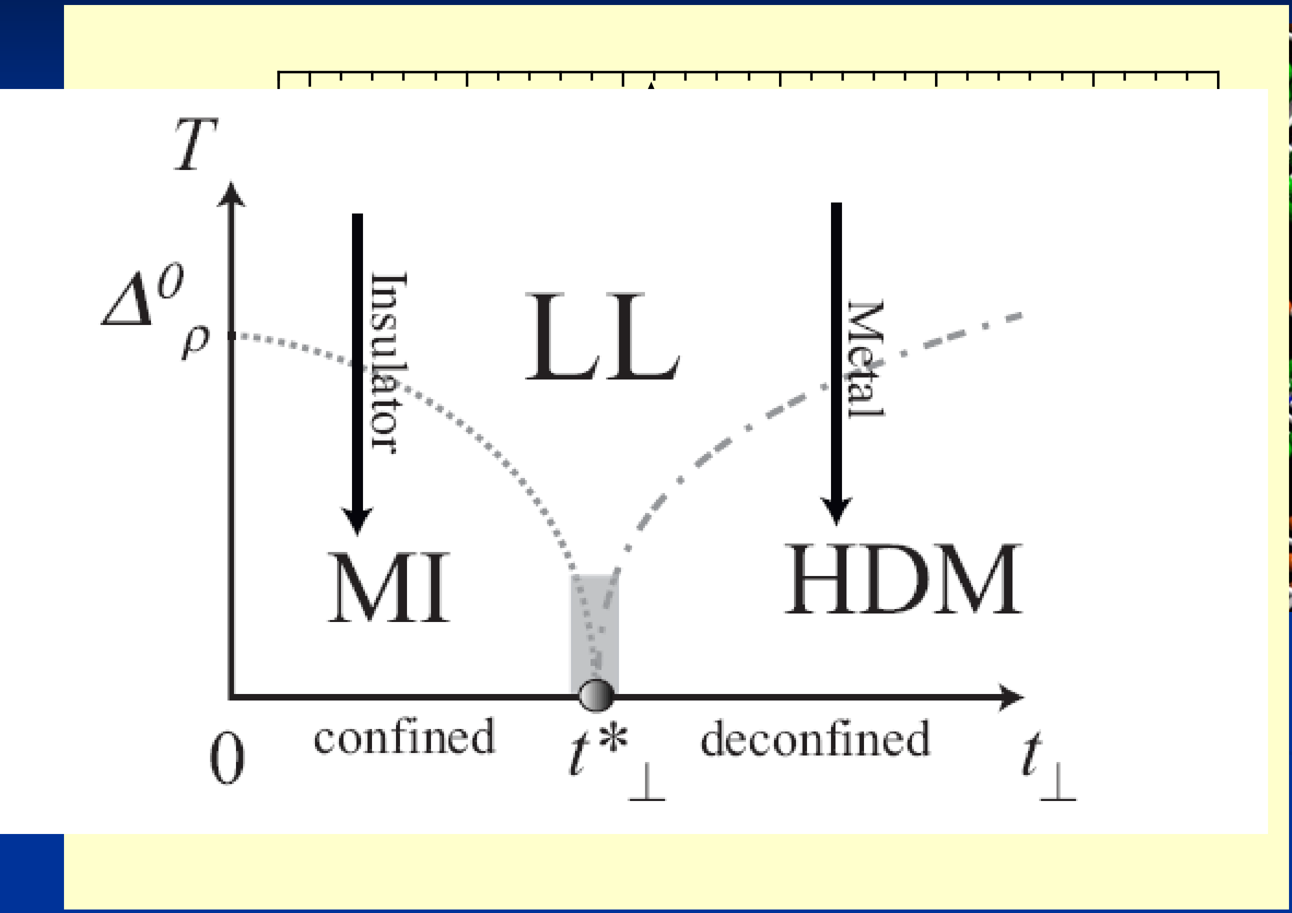
Mott insulators: confinement

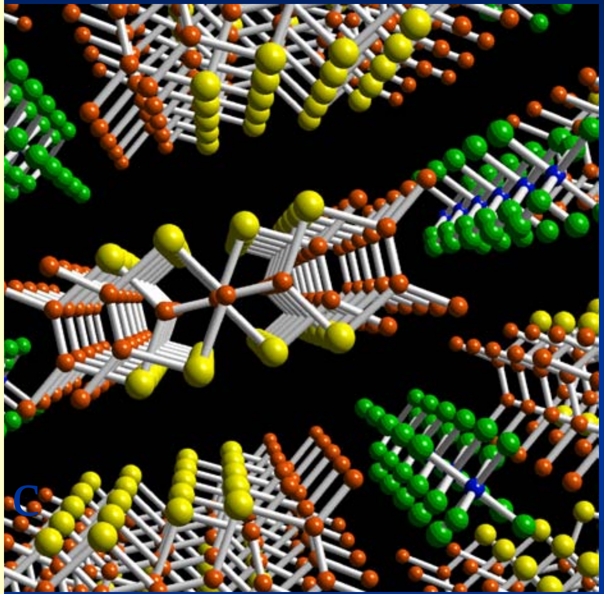


- 1 chain : Mott insulator U >0
- 3d : Mott insulator U > Uc

Competition Mott insulator/Interchain hopping

Deconfinement





TG Chemical Review 104 5037 (2004)

P. Auban-Senzier, D. Jérome, C. Carcel and J.M. Fabre J de Physique IV, (2004)
A. Pashkin, M. Dressel, M. Hanfland, C. A. Kuntscher, arxiv/0909.4795
D. Jaccard et al., J. Phys. C, 13 L89 (2001)

Coupled spinless chains

$$H = -t\sum_i (c_{i+1}^\dagger c_i + \mathrm{h.c.}) + V\sum_i n_i n_{i+1}$$

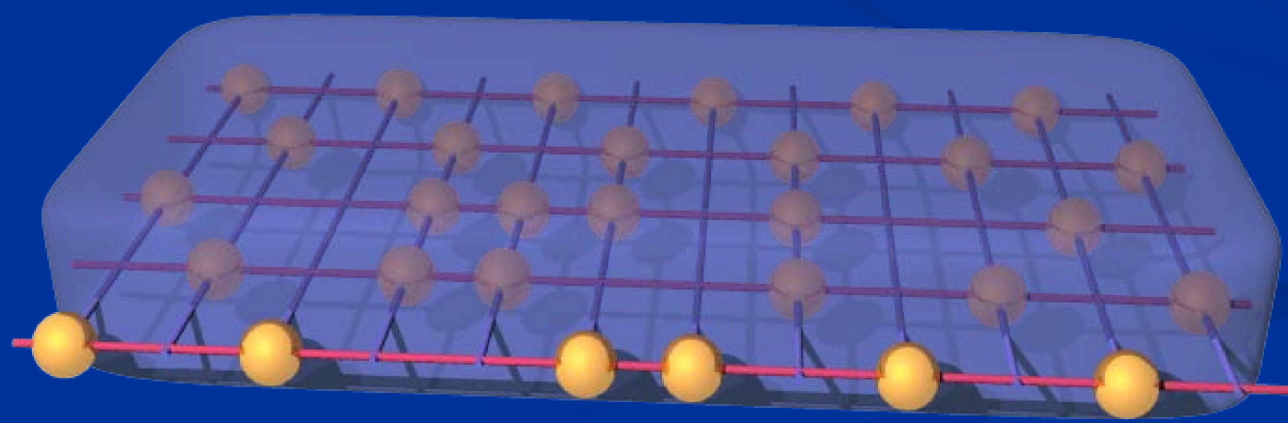
- Mott insulator V > 2t
- Metal (Luttinger liquid) V < 2t

$$H' = -t_\perp \sum_i \sum_{\alpha,\beta} c_{i,\alpha}^\dagger c_{i,\beta}$$

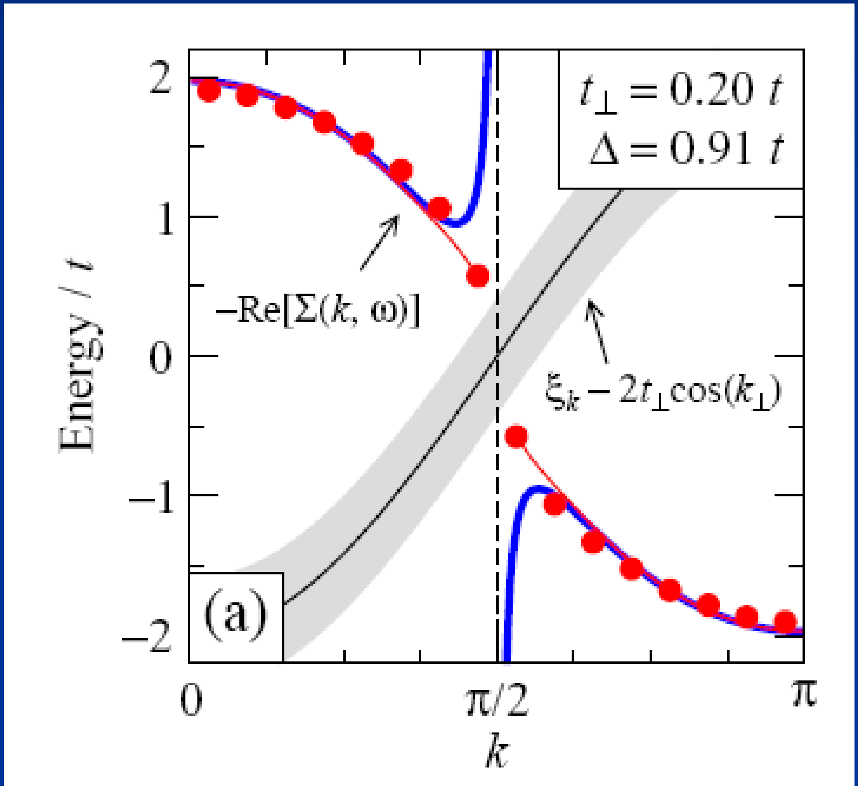
C. Berthod, TG, S. Biermann, A. Georges, PRL 97 136401 (06)

Determination of the Fermi surface

S. Biermann, A. Georges, A. Lichtenstein, TG, PRL 87 276405 (2001)
C. Berthod et al. PRL 97, 136401 (2006)



Fermi surface shape



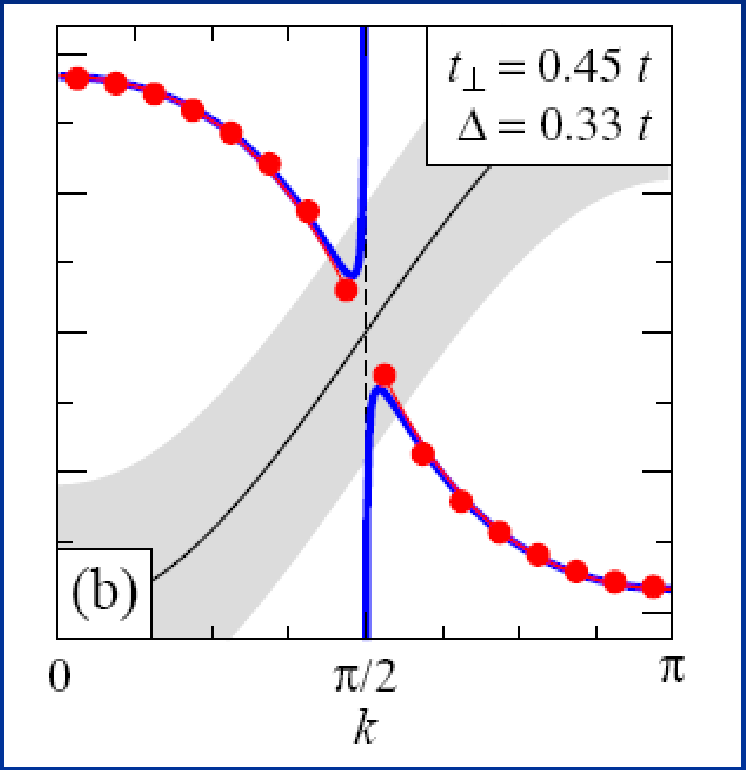
No solution

Insulator

No Fermi surface

Insulator: zero of the green function

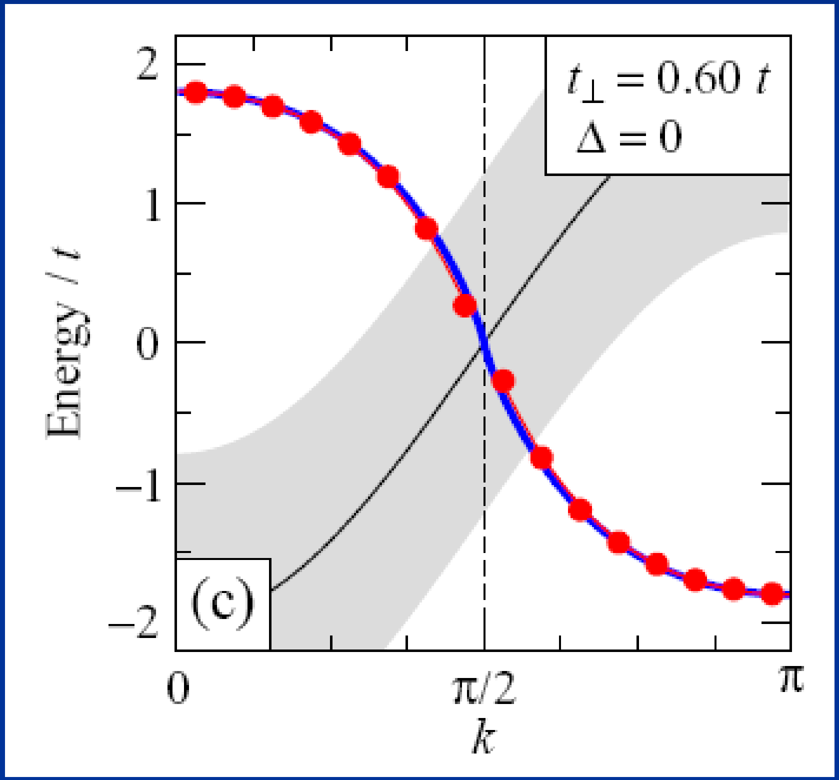
Pockets

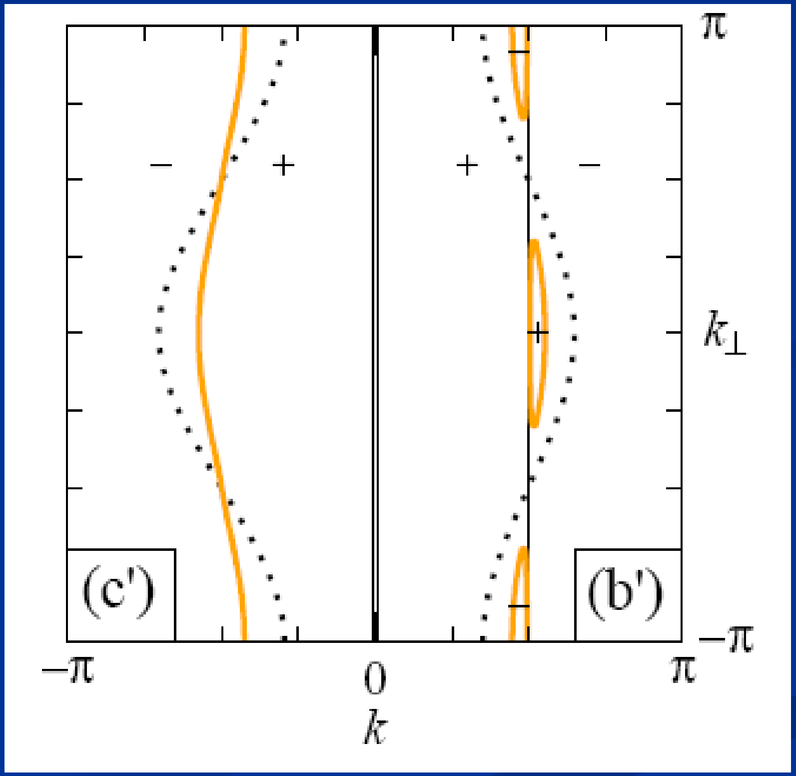


Two solutions

Pockets

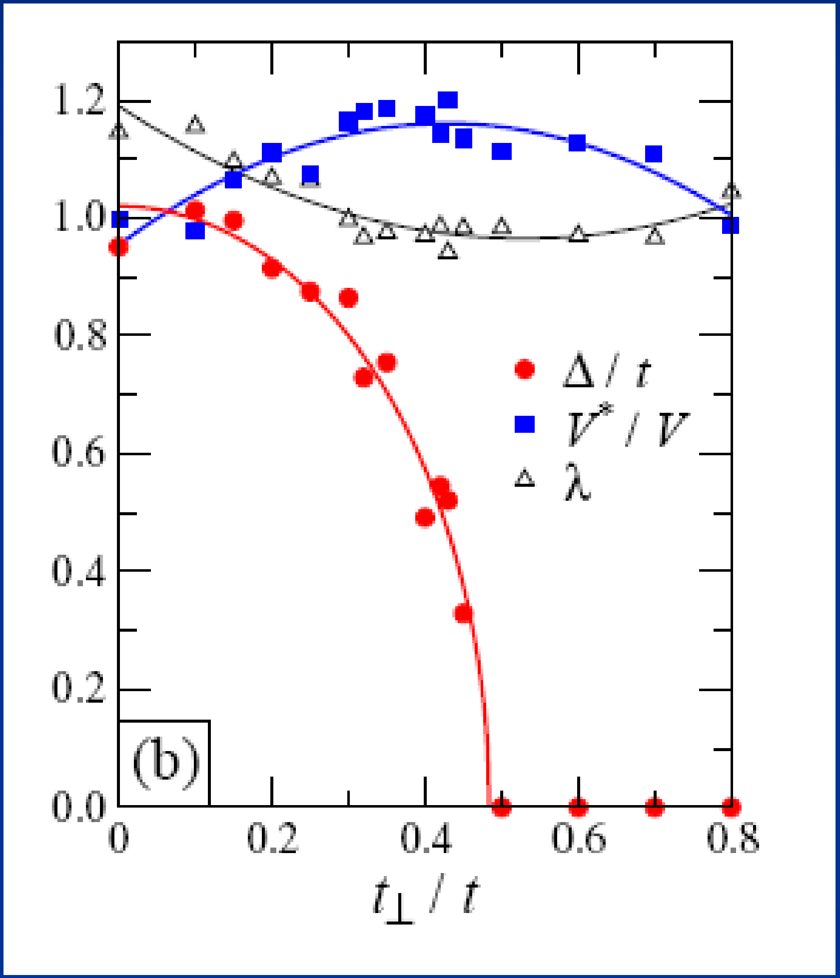
Full Fermi surface





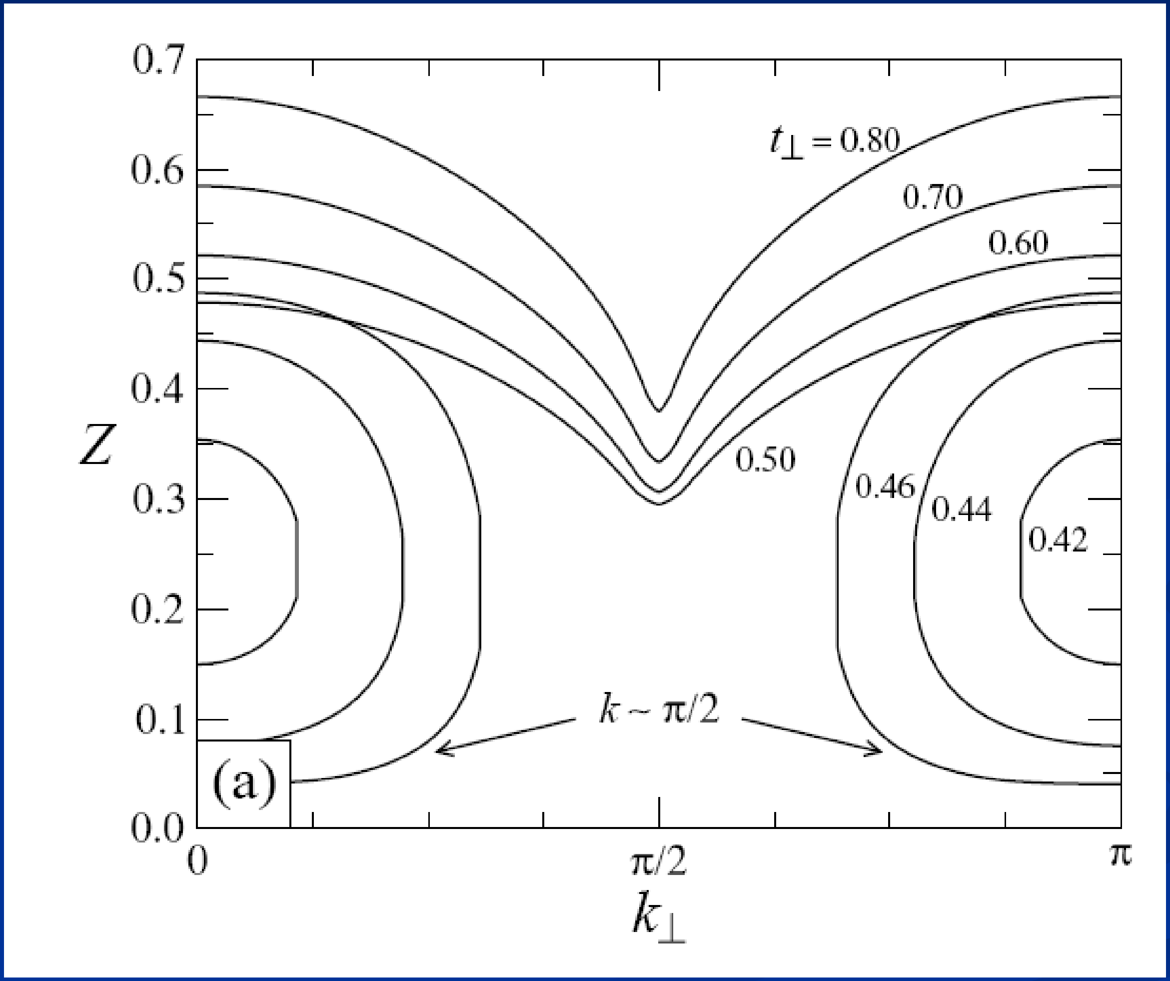
Luttinger theorem satisfied: zeros and poles

Evolution of self-energy



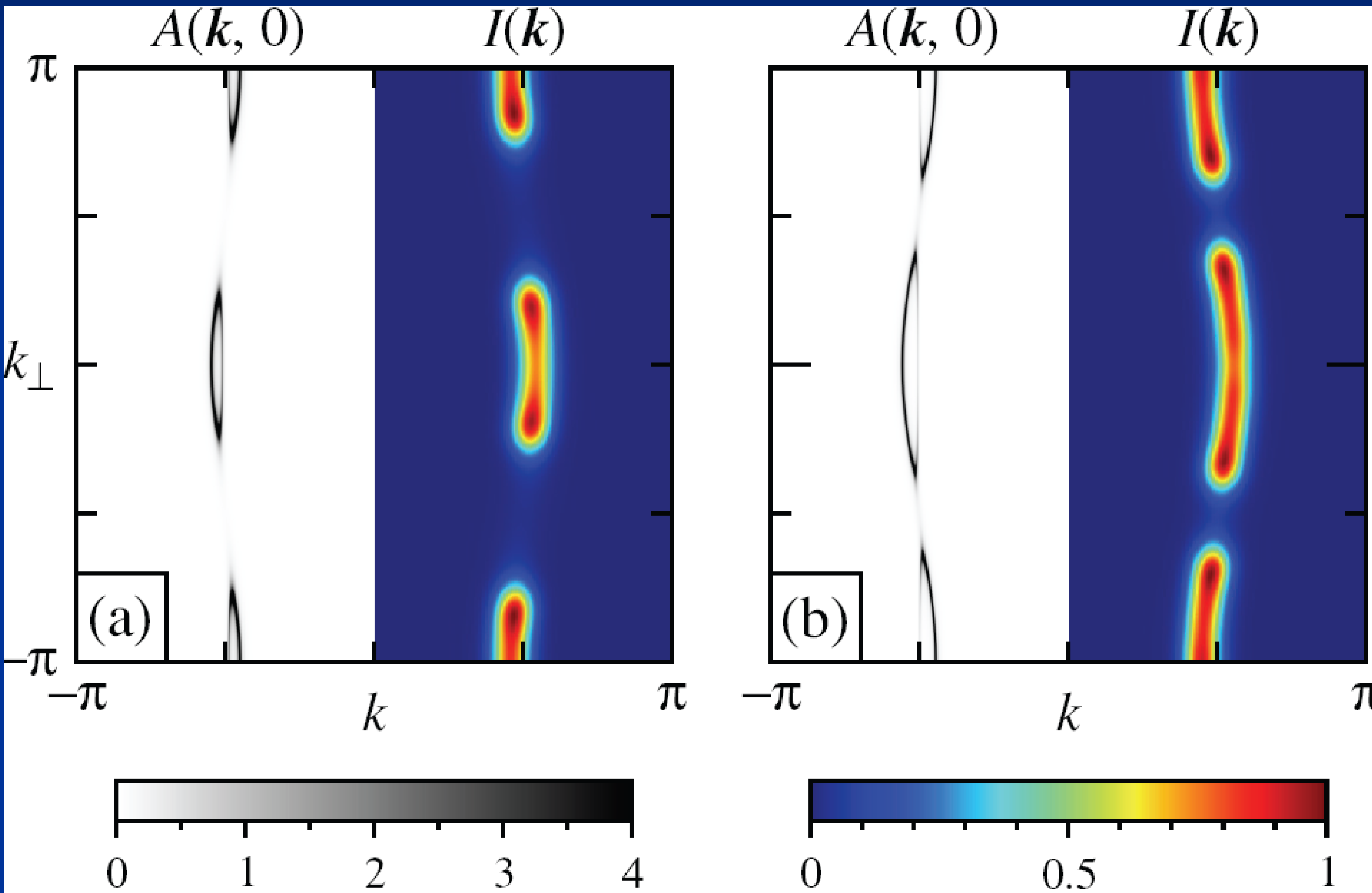
gap closes at t_{c2}

Hot spots



Important to have proper self energy

ARPES



Conclusions

Two well established electronic liquids

- « d=3 » : Fermi liquid; Excitations are « like » free fermions
- d=1 (and extensions)
  Luttinger liquid; Only collective excitations; non universal exponents

Other types of non fermi liquids (d=2)
????????????