The travel of heat and charge in solids

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Contents

Kinetic theory of gases

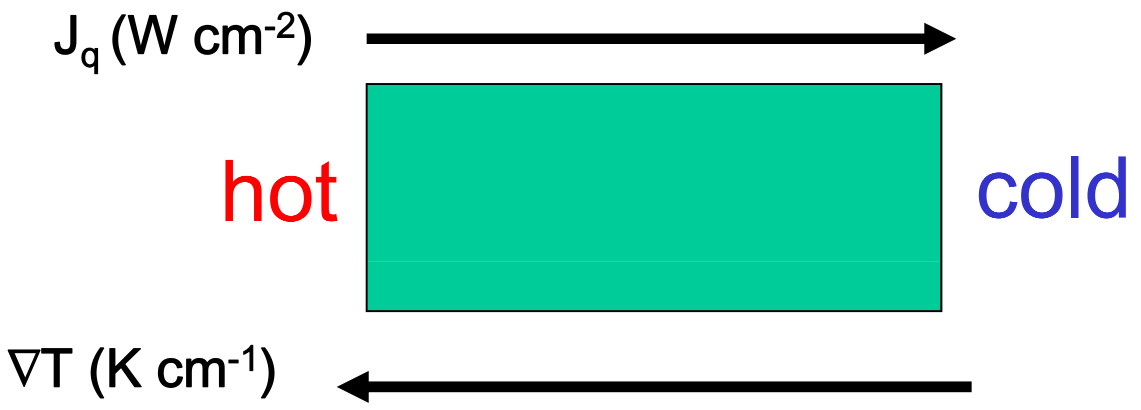
$$\kappa = 1/3\ C\ v\ l$$

Thermal conductivity

Heat capacity

velocity

mean-free-path



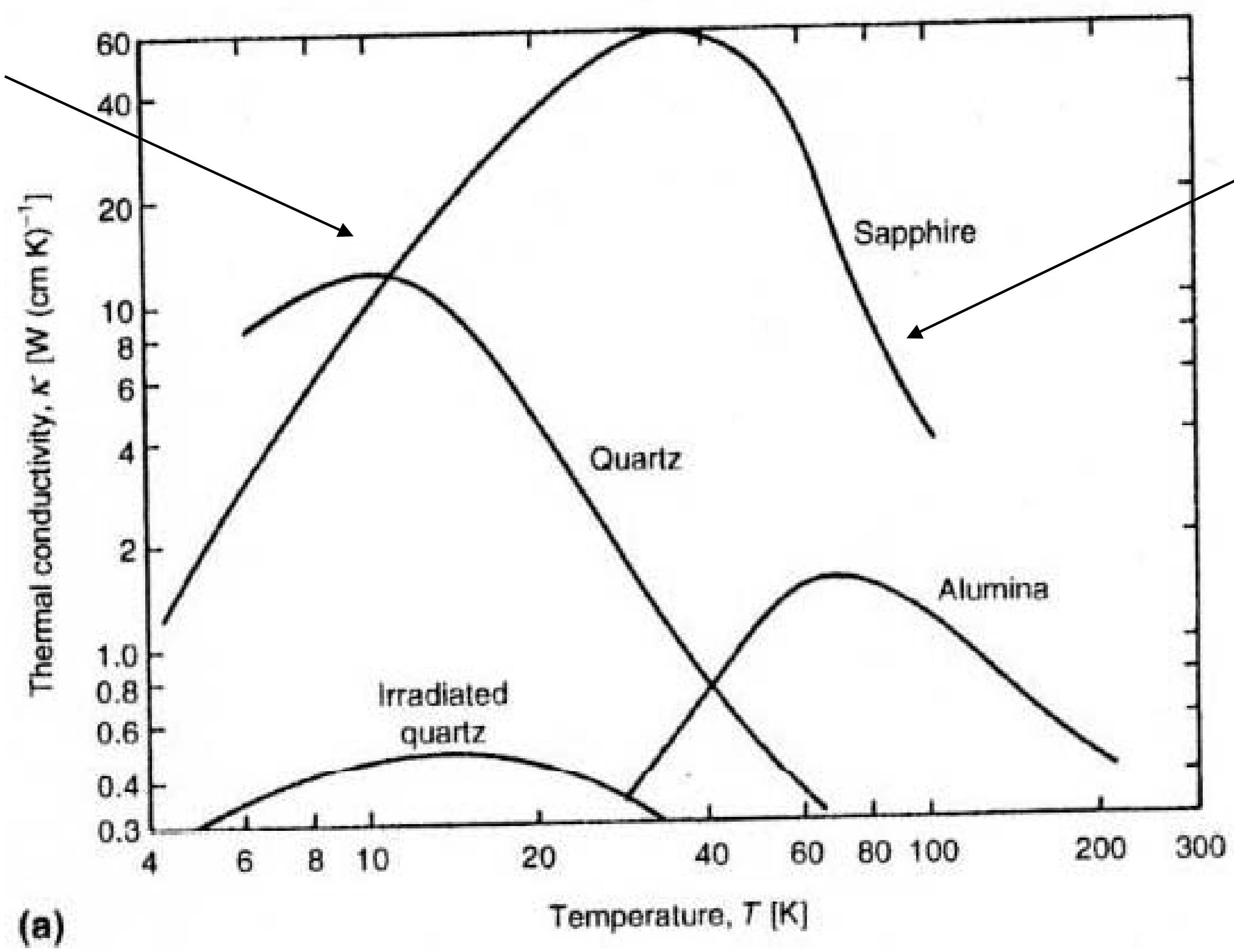
$\kappa = J_q/(-\nabla T)$

[$WK^{-1}cm^{-1}$]

Heat conduction in insulators

Phonon-defect scattering

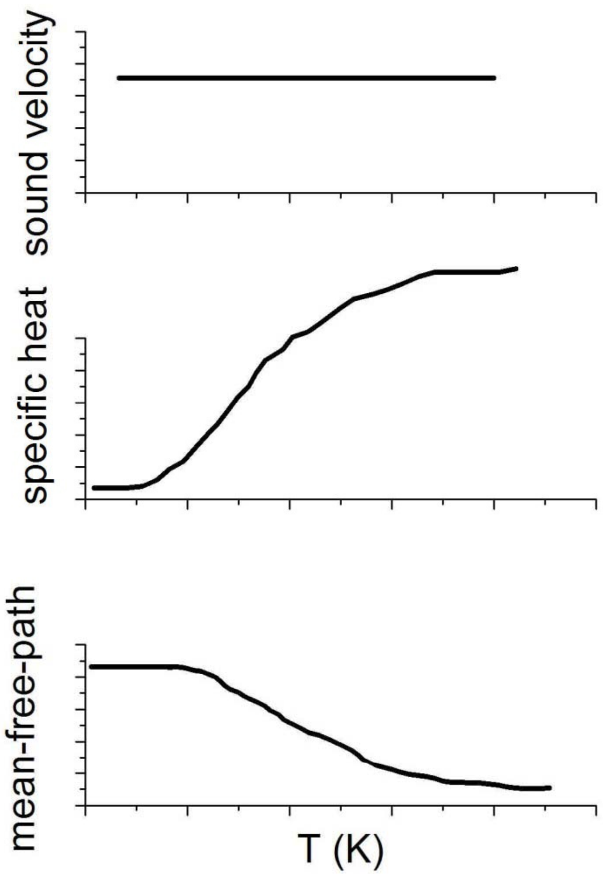
Phonon-phonon scattering

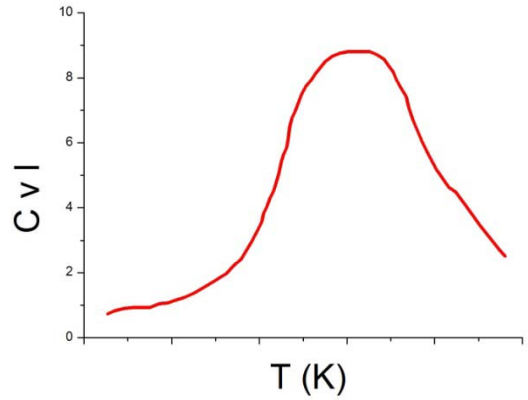


Only phonons carry heat!

Heat conduction in insulators

Phonons are both carriers and scatterers



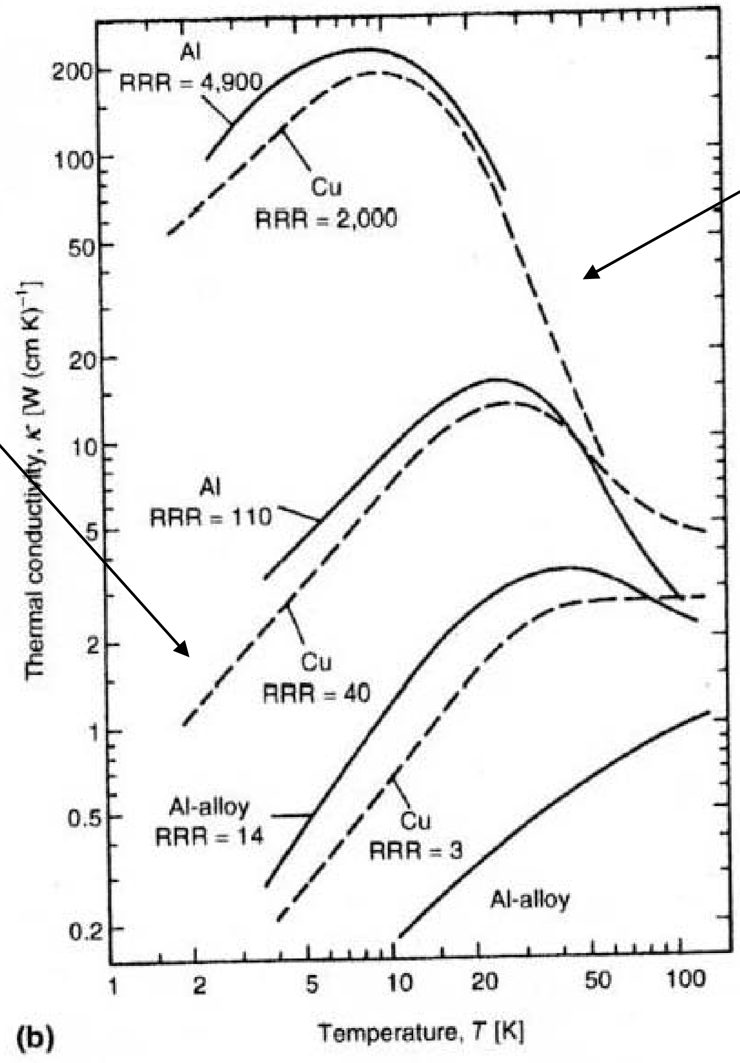


As T increases,

- More carriers !
- More scattering centers!

Heat conduction in metals

Electron-defect scattering



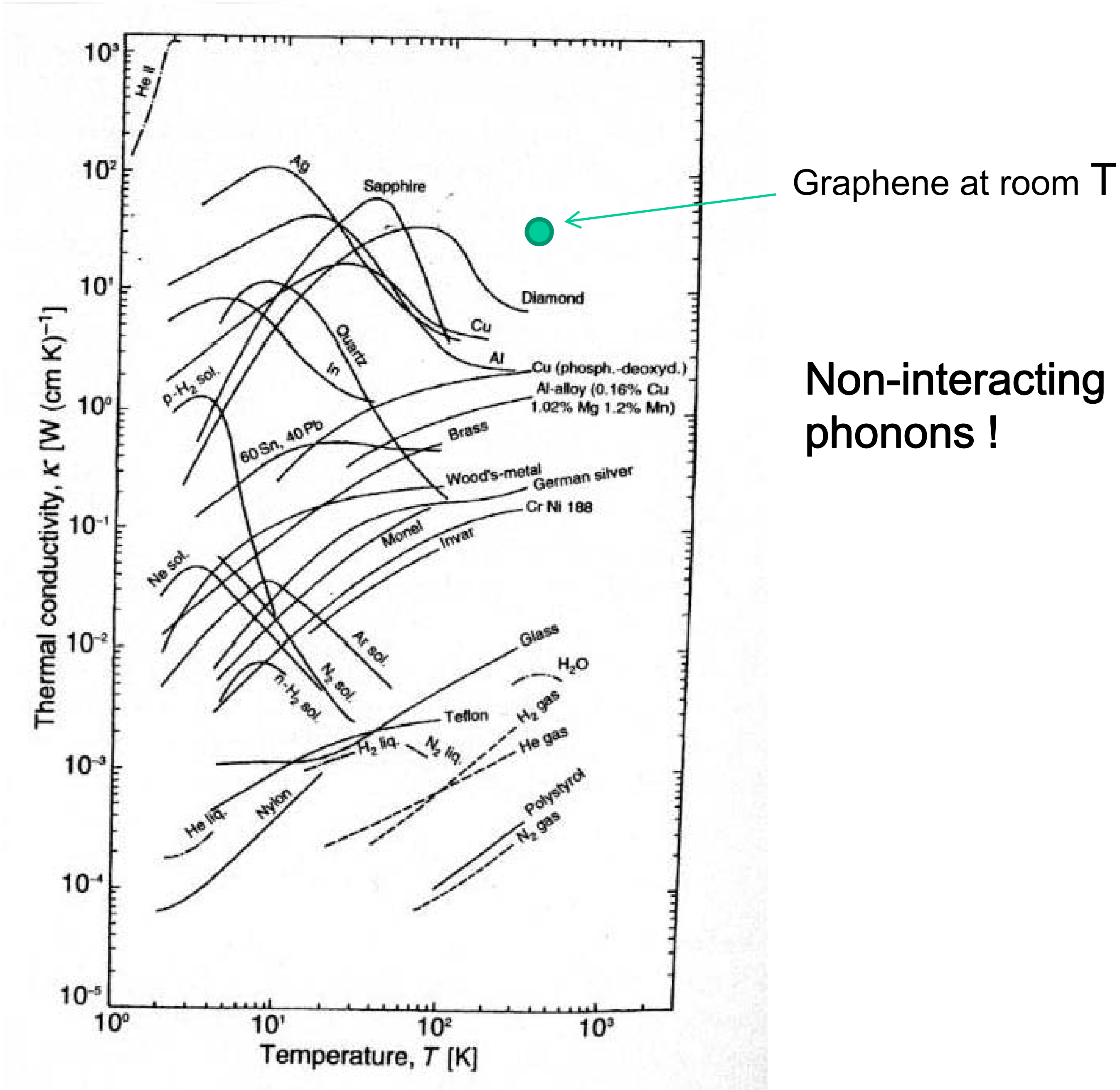
Electron-phonon scattering

Carriers:*electrons & phonons*

Scatterers: *phonons, deffects (and electrons)*

Electrons are dominant carriers of heat!

The best conductor at room temperature!



Non-interacting phonons !

In the zero temperature limit

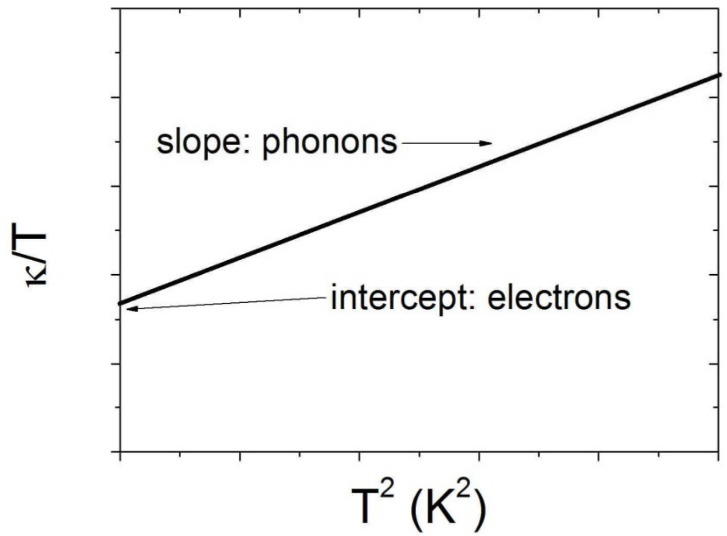
- Mean-free-path attains its maximum value

and then:

$\kappa_{ph} \propto T^3$ (phonons are bosons)

$\kappa_e \propto T$ (electrons are fermions)

In principle, one can separate the two contributions!



Example

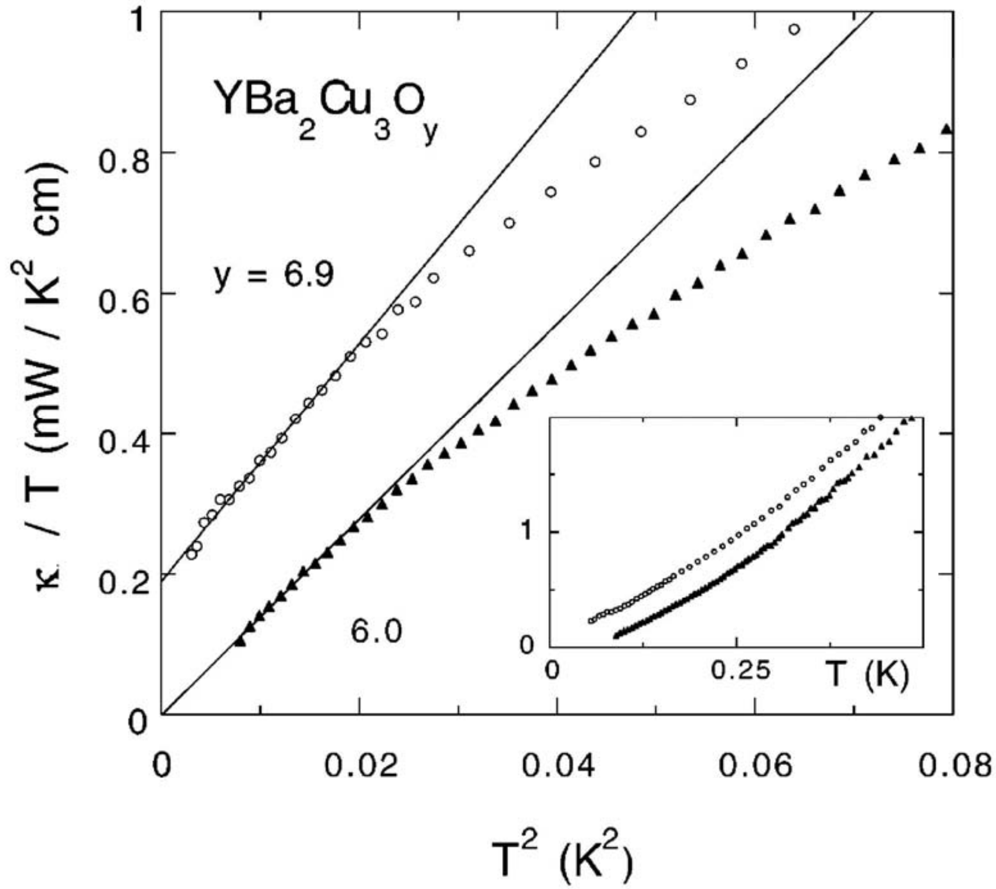


FIG. 1. *a*-axis thermal conductivity of the two $YBa_2Cu_3O_y$ crystals, one superconducting ($y = 6.9$; circles) and one insulating ($y = 6.0$; triangles). Main panel: κ/T vs T^2; lines are fits to $a + bT^2$ for $T < 0.15$ K. Inset: κ/T vs T.

Taillefer *et al.*, 1997

Thermal conductivity of superconductors

- Above T_c a superconductor is a metal

(mobile electrons carry heat!)

- Below T_c, mobile electrons condensate in a macroscopic quantum state: electronic heat carriers vanish!

- A superconductor can be assimilated to a thermal insulator

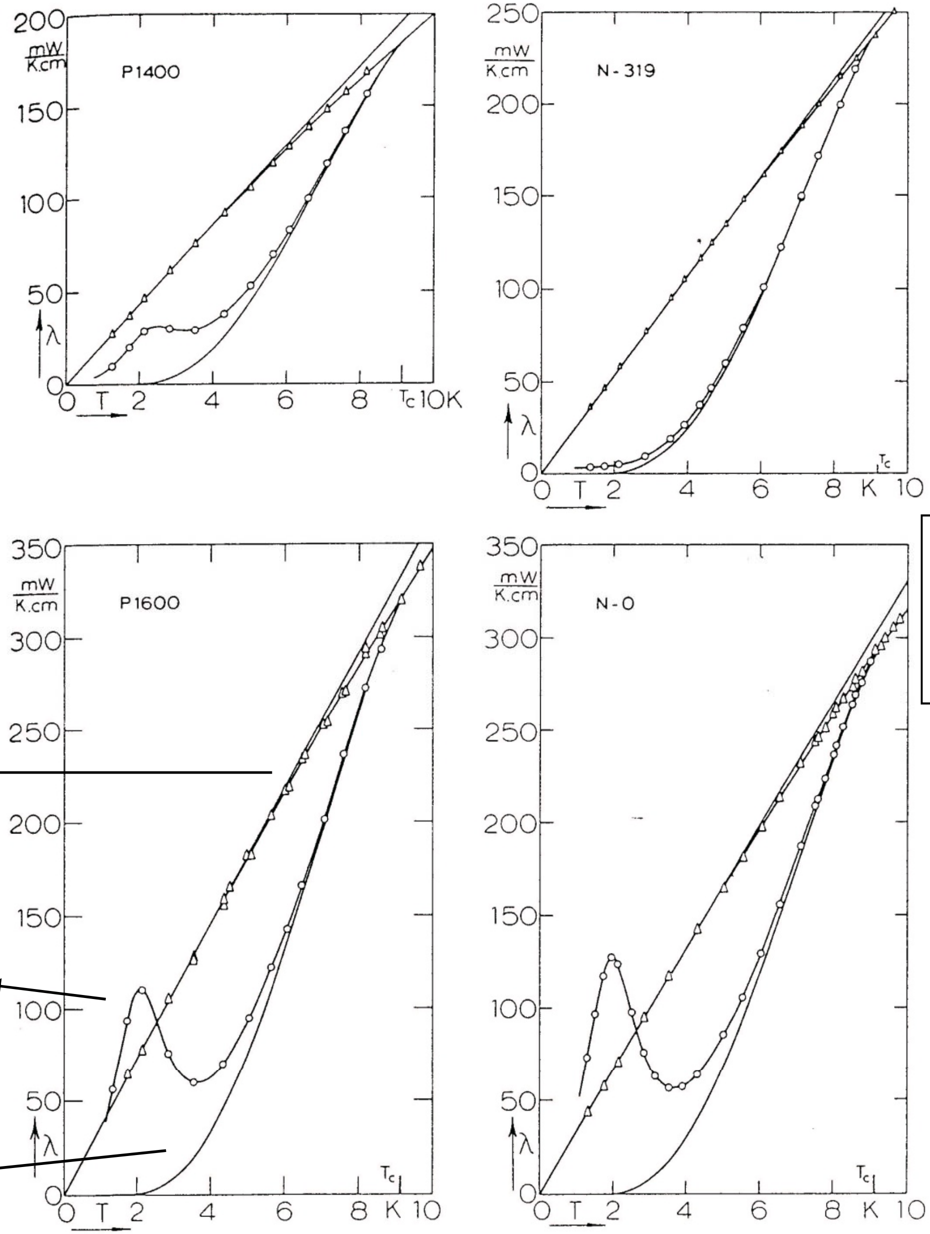
In conventional superconductors

- Electron thermal conductivity decreases exponentially
- Phonon thermal conductivity increases due to a diminished electron scattering

At finite temperature, the temperature dependence of thermal conductivity in a superconductor is complex !

Example: niobium

A phonon peak, whenever the phonon mean-free-path can increase below T_c!



normal

superconducting

e^- component

Kes *et al.*
J. Low Temp. Phys. (1974)

Unconventional superconductors

- The order parameter of the unconventional superconductors is less symmetric than the Fermi surface of the mother metal.

- The gap function can vanish along particular orientations (nodes).

- Nodal quasi-particles can carry heat!

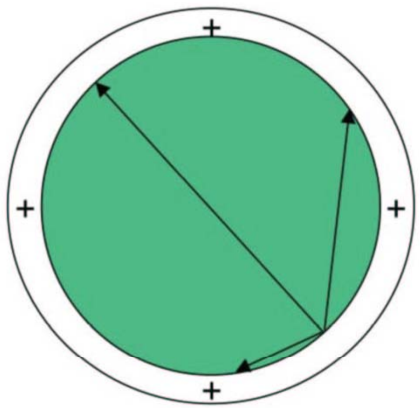
Effect of an unconventional superconducting transition on thermal transport

- The electronic thermal transport does NOT decrease exponentially

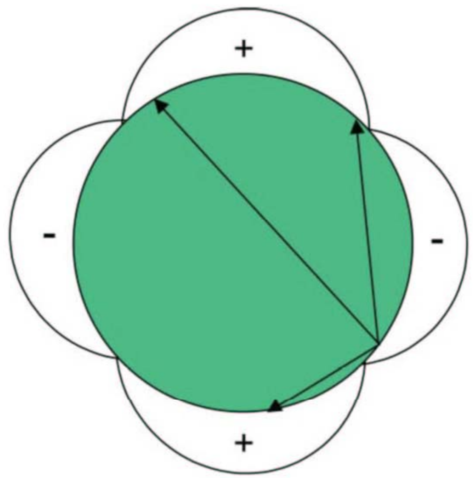
- It can even increase below T_c (due to an increase in the electronic mean-free-path

$$\kappa = 1/3\ C\ v\ l$$

Scattering events are restricted in an unconventional superconductor

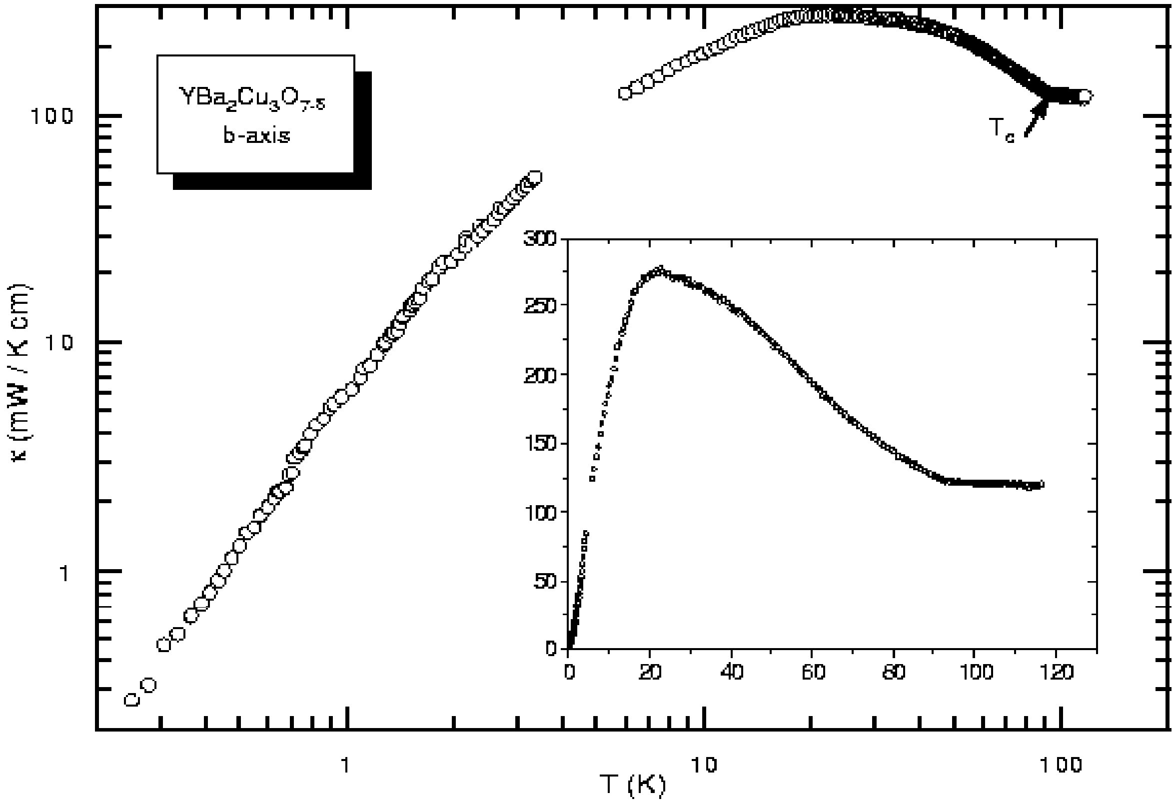


s-wave



d-wave

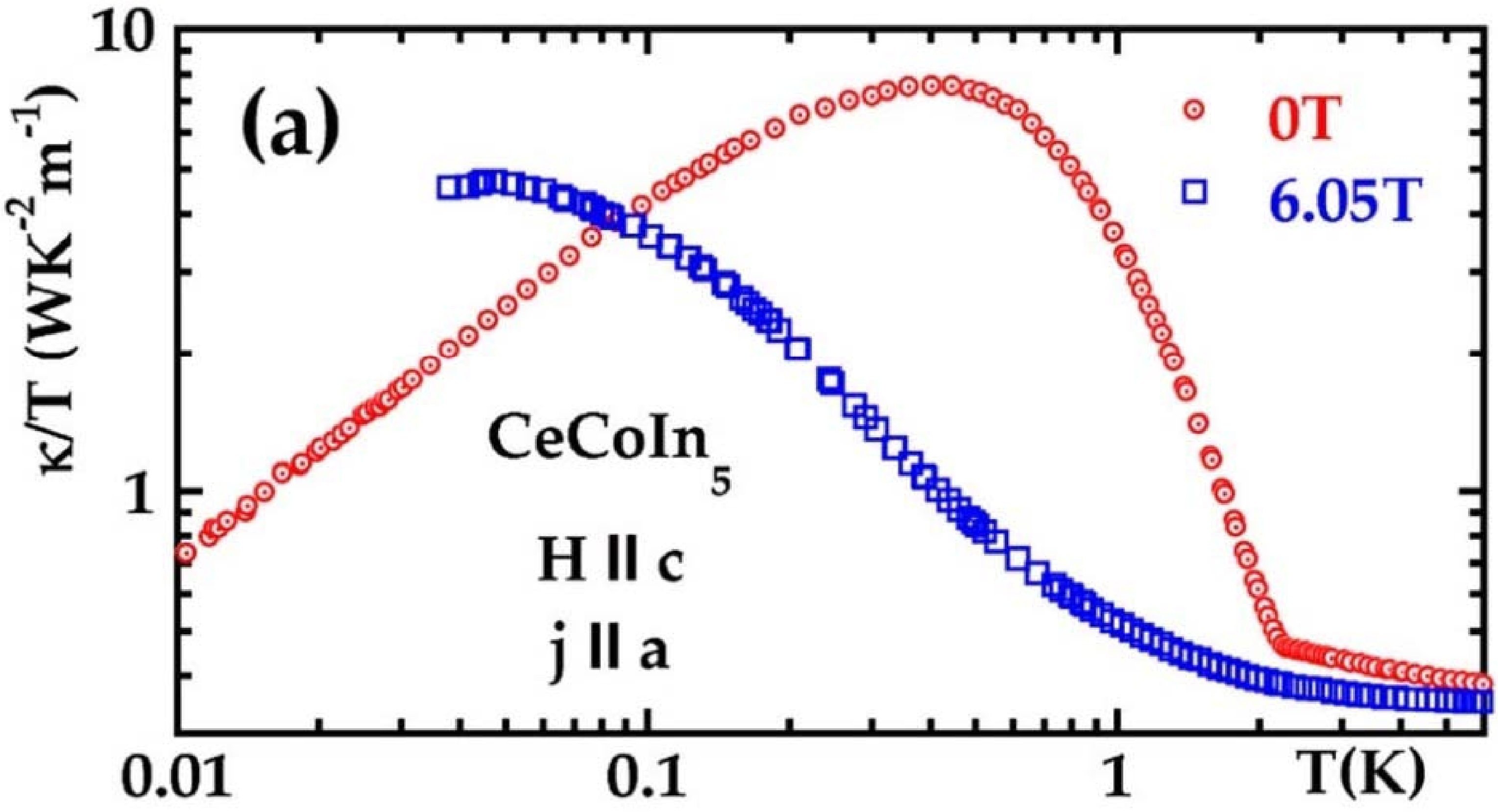
Heat conduction in YBCO



Aubin 1997

The increase in thermal conductivity below T_c is due to electrons!

Enhanced electronic thermal conductivity in other unconventional superconductors



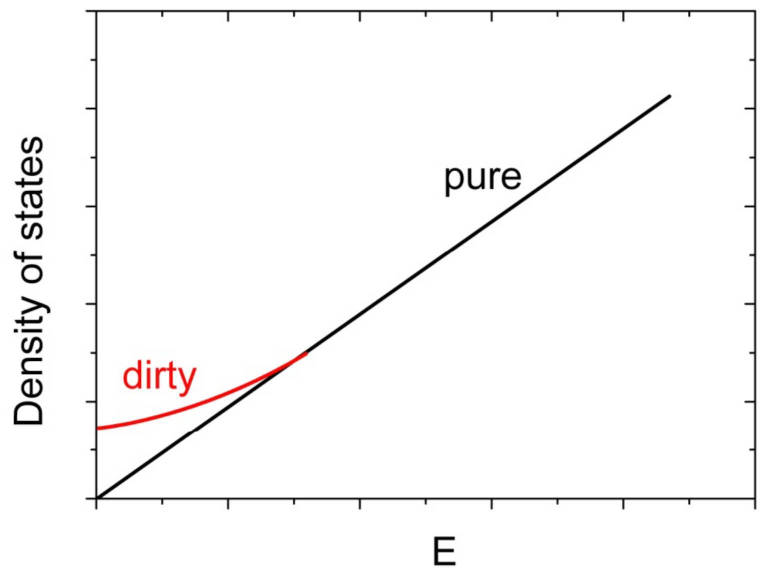
$CeCoIn_5$ (Seyfarth *et al.*, 2008)

Thermal transport in the zero-temperature limit

- A finite linear term in thermal conductivity

- A residual normal fluid of nodal quasi-particles

$\kappa = \kappa_{00}T + \ldots + bT^3$



Example: heavy-fermion superconductor $CePt_3Si$

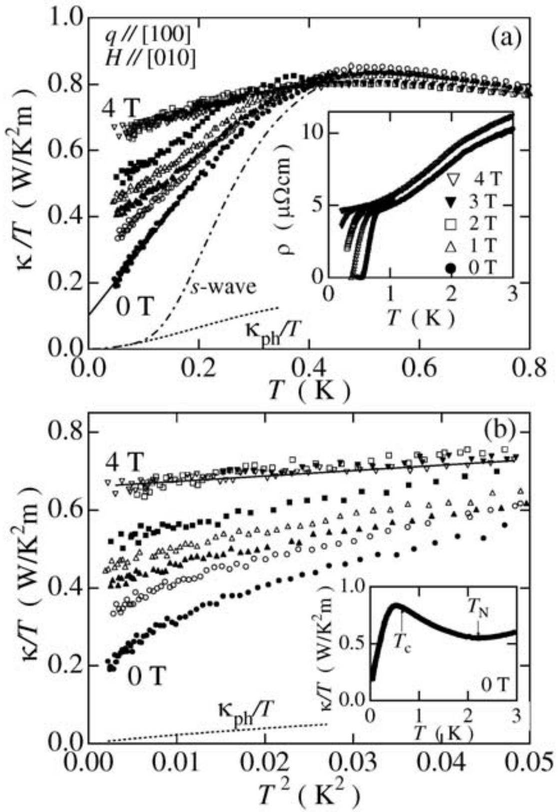


FIG. 1. (a) Main panel: a-axis thermal conductivity of $CePt_3Si$ for the $\boldsymbol{H} \parallel b$ axis, plotted as κ/T vs T (●, 0 T; ○, 0.2 T; ▲, 0.5 T; △, 1 T; ■, 1.5 T; □, 2 T; ▼, 3 T; ▽, 4 T). The solid line is a linear fit in zero field: $\kappa/T = A + BT$. The dashed line is the contribution of phonons κ_{ph}. The dash-dotted line is the thermal conductivity in s-wave superconductors. Inset: T dependence of a-axis resistivity for the $\boldsymbol{H} \parallel b$ axis. (b) The same data at low temperatures plotted as κ/T vs T^2. The solid line is a linear fit in the normal state: $\kappa/T = a + bT^2$. The dashed line represents κ_{ph}. Inset: T dependence of κ/T in zero field.

Izawa *et al.,* PRL '05

A superconductor with no inversion symmetry and yet with nodal quasi-particles!

Universal thermal conductivity

- In a d-wave superconductor, in a first approximation κ_0 is independent of impurity concentration

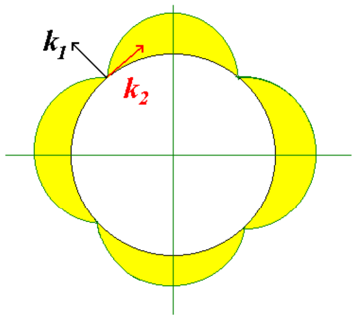
Impurities: → Decrease the mean-free-path

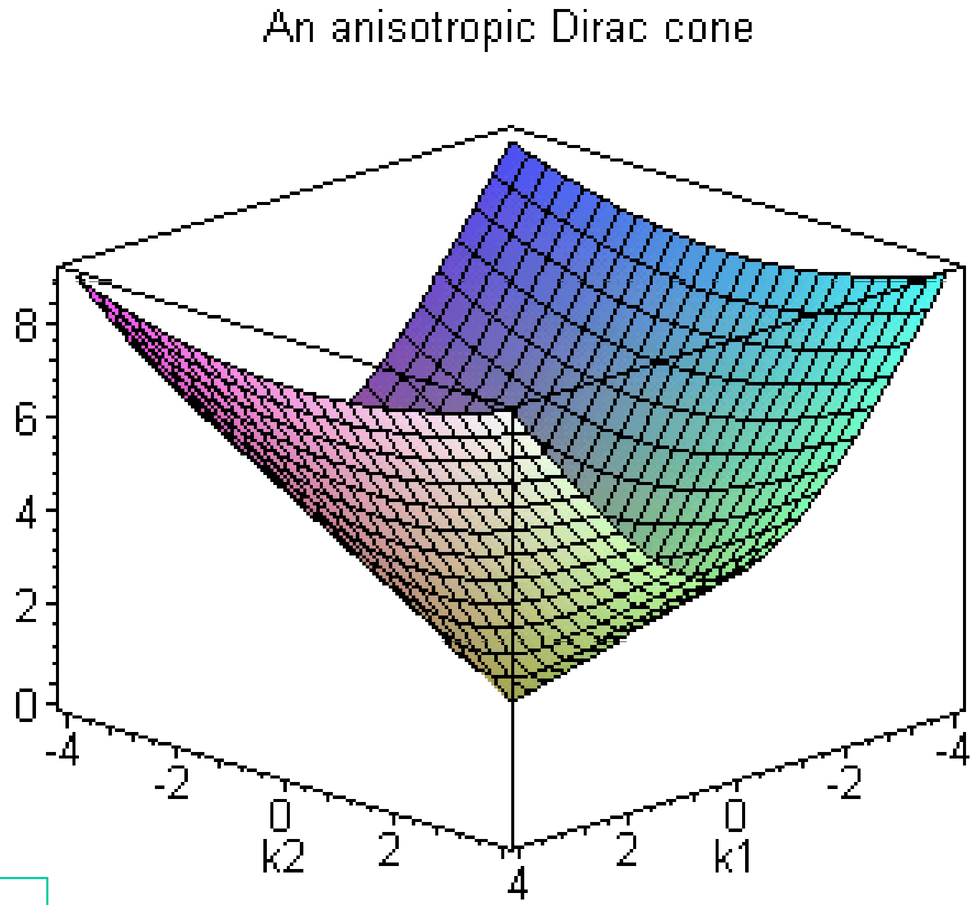
Impurities: → Enhance the specific heat

These two cancel out in a subset of unconventional superconductors.

A TALE OF TWO VELOCITIES!

(Durst , Lee '99)





Excitation spectrum in

the vicinity of a node:

$$E(k) = (\varepsilon_k^2 + \Delta_k^2)^{1/2}$$

$$= (v_F^2 k_1^2 + v_2^2 k_2^2)^{1/2}$$

Fermi velocity : $v_F = d\varepsilon_k/dk_1$

Gap velocity: $v_2 = d\Delta_k/dk_2$

$$\kappa_{00}/T = (nk_B^2/3\hbar)\ (v_F / v_2)$$

Experimental observation of universal thermal conductivity

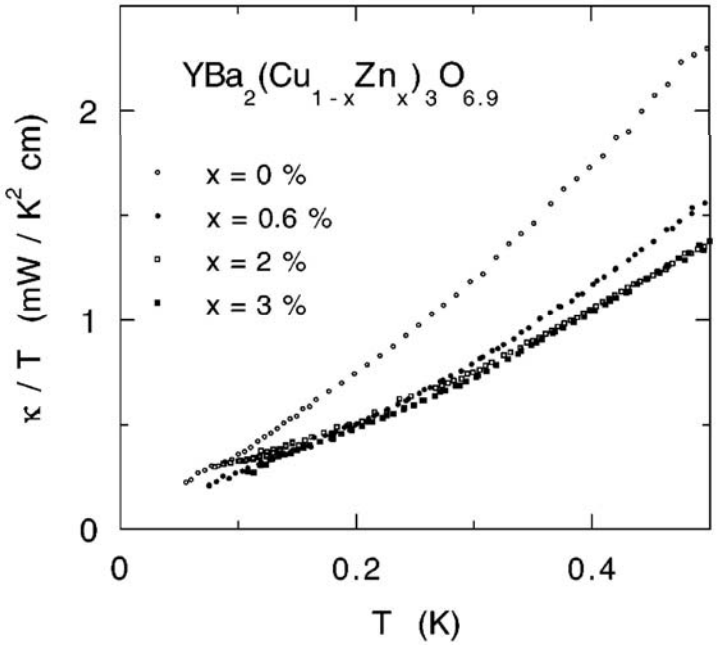


FIG. 2. *a*-axis thermal conductivity of the four Zn-doped crystals, plotted as κ/T vs T.

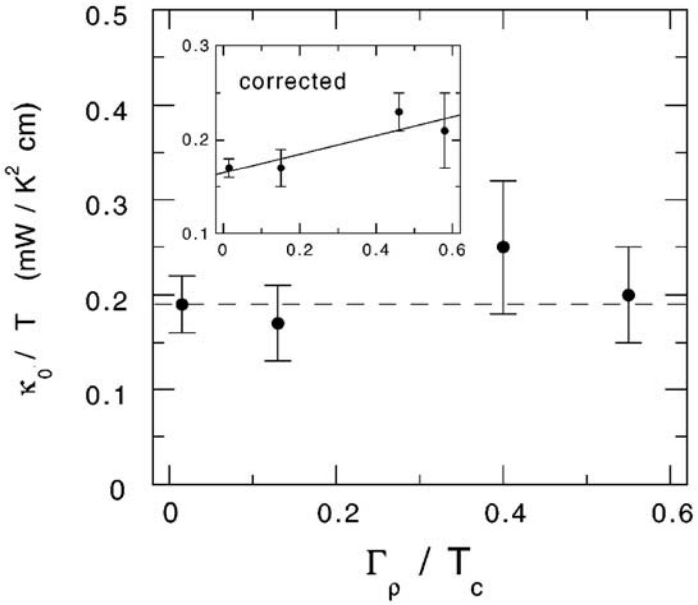


FIG. 3. Residual linear term vs scattering rate for the four crystals of $YBa_2(Cu_{1-x}Zn_x)_3O_{6.9}$; the dashed line indicates a constant at 0.19 $mW\,K^{-2}\,cm^{-1}$. Inset: same, but with corrected values (see text); the solid line is a least-squares fit.

A 30-fold decrease in mean-free-path in Zn-doped YBCO leaves κ_0 unchanged (Taillefer et al., PRL 1997)

Universal conductivity in other unconventional superconductors

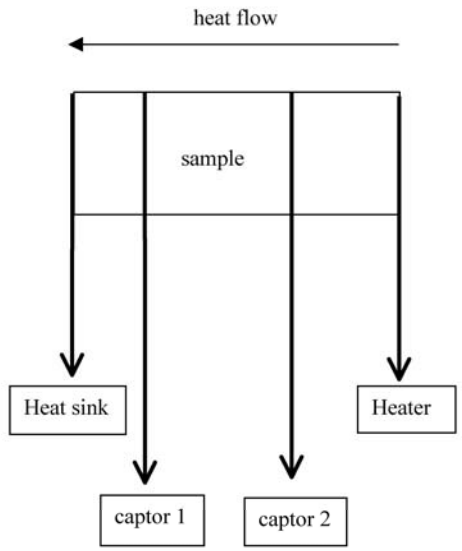
κ_0 independent of impurity concentration checked in:

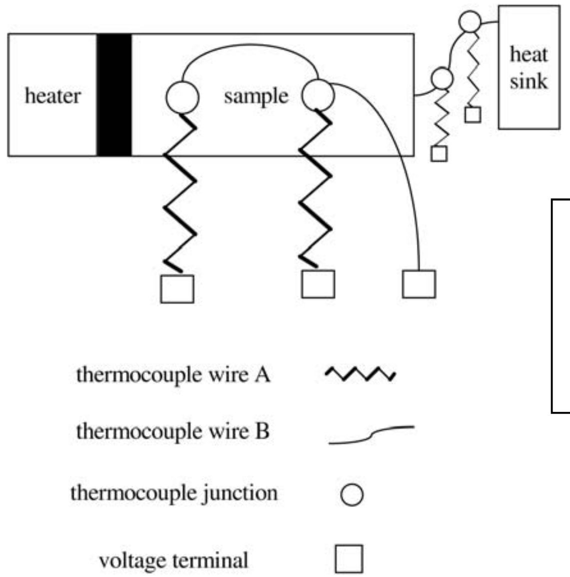
- $Bi_2Sr_2CaCu_2O_8$ (Nakamae *et al.* , 2001)
- Sr_2RuO_4 (Suzuki *et al.* , 2004)

κ_0 of the right order of magnitude found in:

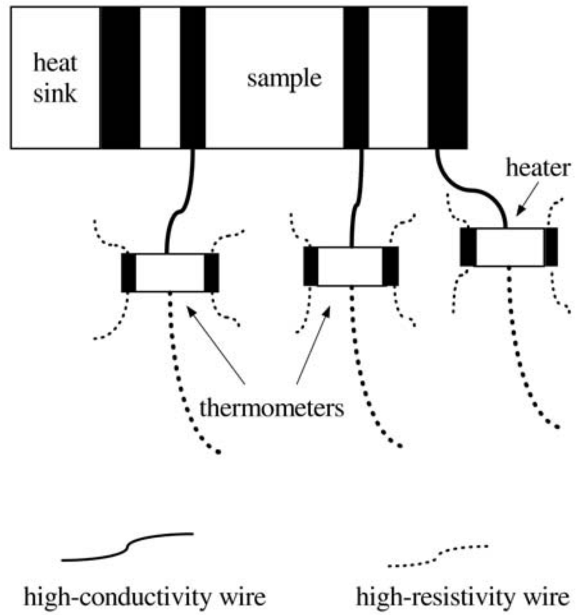
- κ-$(BEDT\text{-}TTF)_2Cu(NCS)_2$ (Belin *et al.,* 1998)
- $CePt_3Si$ (Izawa *et al.,* 2005)
- URu_2Si_2 (Kasahara *et al.,* 2007)
- $CeCoIn_5$ (Seyfarth *et al.,* 2008)

Mesauring thermal conductivity





Temperature captors:
Resistive thermometers
or thermocouples!



Part II- transport equations

Heat and charge current in a solid

$$\vec{J}_e = \sigma \vec{E}$$

$$\vec{J}_Q = \quad - \kappa \vec{\nabla} T$$

$$\vec{J}_e = \sigma \vec{E} - \alpha \vec{\nabla} T$$

$$\vec{J}_Q = \beta \vec{E} - \kappa \vec{\nabla} T$$

$$\beta = \alpha T$$

$\beta = \alpha T$ *Kelvin relation, (1860) Onsager relation (1930)*

Four vectors

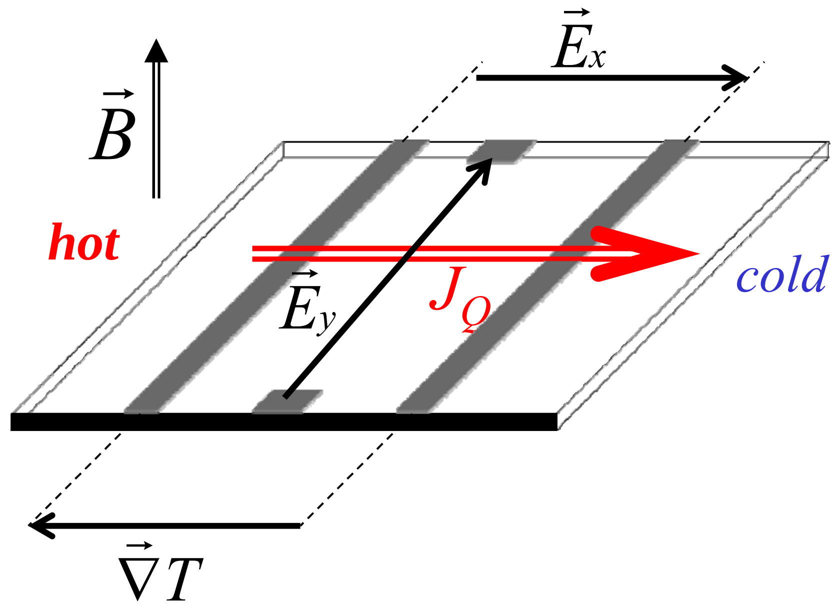
J_e : charge current density
J_Q : heat current density
E : electric field
DT : thermal gradient

Three tensors

σ electric conductivity
κ thermal conductivity
α thermoelectric conductivity

Definition of thermoelectric coefficients

- In presence of a thermal gradient, electrons produce an electric field.
- Seebeck and Nernst effect refer to the longitudinal and the transverse components of this field.



$$S = \frac{-E_x}{\nabla_x T} \qquad N = S_{xy} = \frac{-E_y}{\nabla_x T} \qquad \left[\nu = \frac{-E_y}{B_z \nabla_x T}\right]$$

Link to the Peltier tensor: α

Seebeck coefficient

$$S = \frac{\alpha_{xx}}{\sigma_{xx}}$$

Nernst coefficient

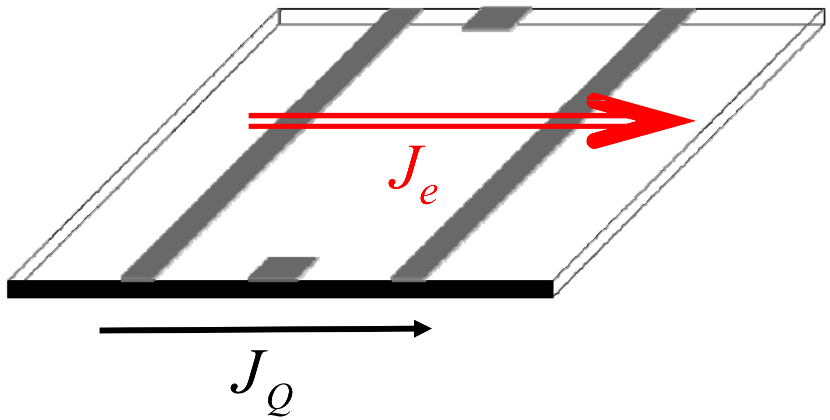
$$\upsilon = N / B = \frac{\alpha_{xy}\sigma_{xx} - \alpha_{xx}\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2}$$

Experimentally, what is measured is S and ν .

Seebeck and Peltier coefficients

Peltier effect: A thermal gradient created by an electric current

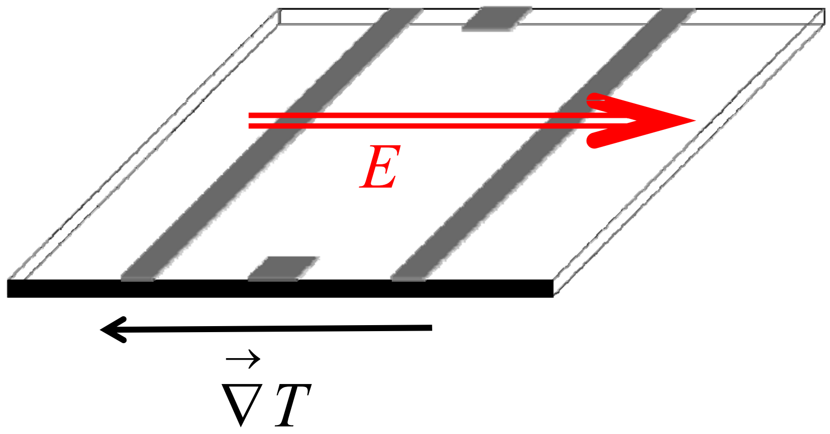
$$\Pi = \frac{J_Q}{J_e}$$



Seebeck and Peltier coefficients

Seebeck effect: An electric field created by a thermal current

$$S = \frac{-E_x}{\nabla_x T}$$

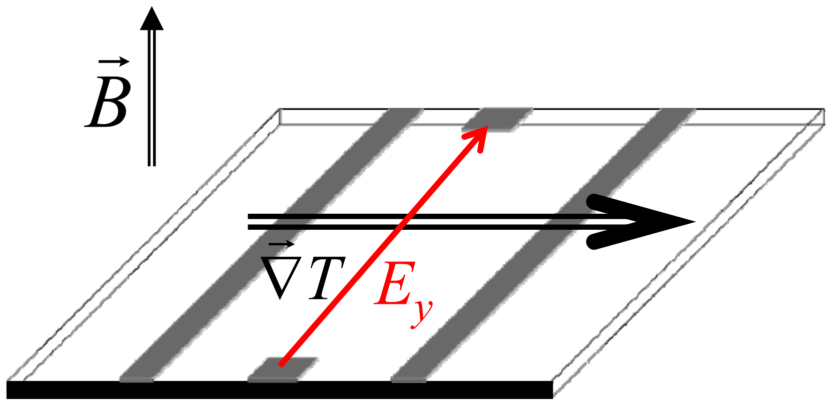


The Kelvin relation : $\Pi = ST$

Nernst and Ettingshausen coefficients

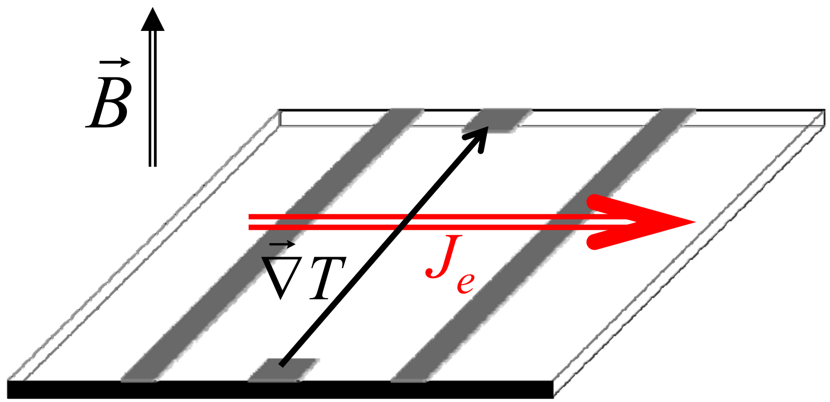
Nernst coefficient

$$S_{xy} = \frac{-E_y}{\nabla_x T}$$



Nernst and Ettingshausen coefficients

Ettingshausen coefficient
$\varepsilon = \nabla_y T / J_e$

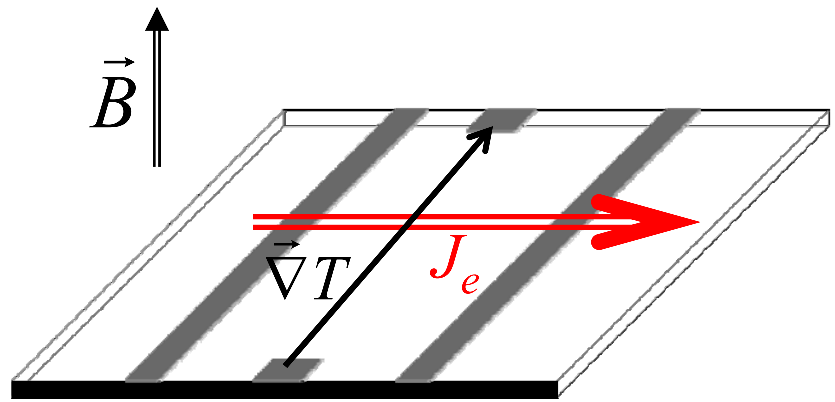


Nernst and Ettingshausen coefficients

$S_{xy} = \varepsilon\, \kappa / T$

Bridgman relationship,

Thermal conductivity



Ettingshausen coefficient

$[\varepsilon = \nabla_y T / J_e]$

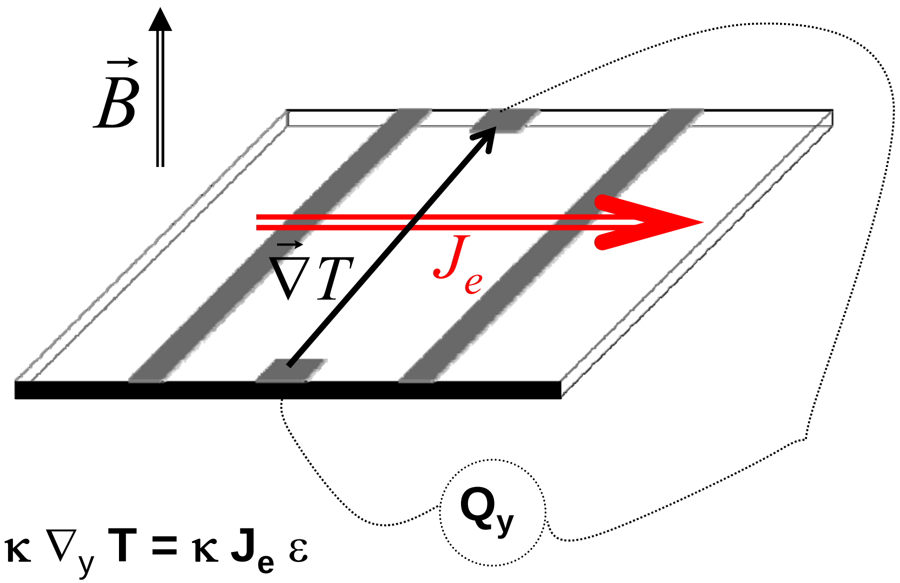
Nernst and Ettingshausen coefficients

$S_{xy} = \varepsilon\, \kappa \,/\, T$

Thermal conductivity

Ettingshausen coefficient
$[\varepsilon = \nabla_y T / J_e]$

Bridgman relationship,
Sommerfeld and Frank, Rev. Mod. Phys., 1931



$\mathbf{Q_y = \kappa\, \nabla_y\, T = \kappa\, J_e\, \varepsilon}$

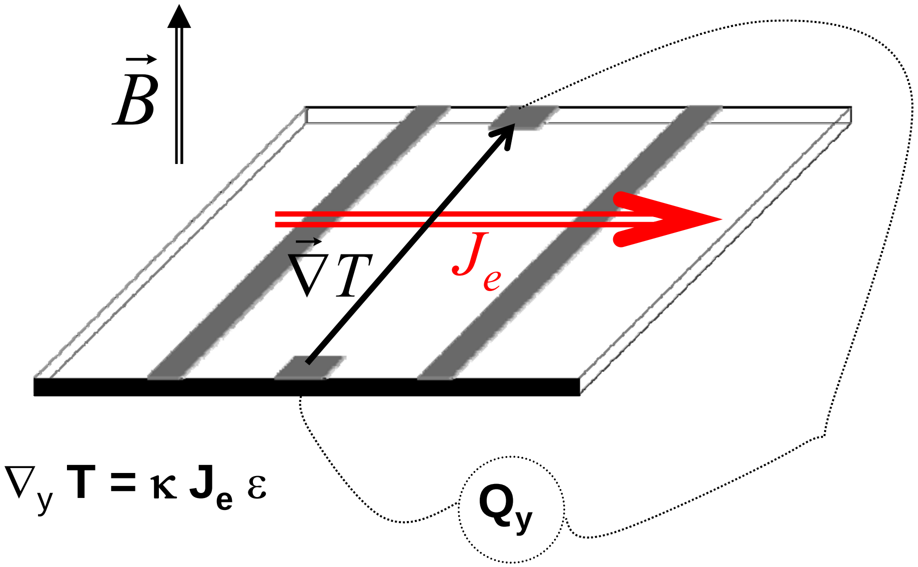
Nernst and Ettingshausen coefficients

$S_{xy} = \varepsilon\, \kappa\, /\, T$

Thermal conductivity

Ettingshausen coefficient
$[\varepsilon = \nabla_y T / J_e]$

Bridgman relationship,
Sommerfeld and Frank, Rev. Mod. Phys., 1931



$Q_y = \kappa\, \nabla_y T = \kappa\, J_e\, \varepsilon$

Energy cost : $(Q_y / T)\, \nabla_y T = \kappa\, J_e\, \varepsilon\, \nabla_y T / T$

Can be provided by an Electric field E_x
$J_e E_x = k\, J_e\, \varepsilon\, \nabla y\, T / T \longrightarrow E_x / \nabla_y T = \kappa\, \varepsilon / T$

The Semi-classic picture

$$\vec{J}_e = \sigma \vec{E} - \alpha \vec{\nabla} T$$

$$\vec{J}_Q = \alpha T \vec{E} - \kappa \vec{\nabla} T$$

$$\overline{\alpha} = \frac{\pi^2}{3} \frac{k^2{}_B}{e} T \frac{\partial \overline{\sigma}}{\partial \varepsilon}\Big|_{\varepsilon_F}$$

$$\overline{\kappa} = \frac{\pi^2}{3} \frac{k^2{}_B}{e^2} T \overline{\sigma}$$

The Wiedemann-Franz law!

The Wiedemann-Franz law

- Strictly valid in absence of inelastic scattering
- A robust signature of a Fermi liquid
  (expected to break down in case of spin-charge separation)
- Even in Fermi liquids a finite downward deviation is expected in presence of inelastic scattering

Carriers of charge are carriers of heat

$$\frac{\kappa}{T\sigma} = \frac{\pi^2}{3}\frac{k^2{}_B}{e^2}$$

The ratio of two conductivities is linked to the ratio of the two quanta of charge and entropy!

Can be derived using the kinetic equation!

$$C = \frac{\pi^2}{3} k_B^2 T N(\varepsilon_F)$$

$$\sigma = \frac{1}{3} e^2 N(\varepsilon_F) v_F \ell_e$$

$$\kappa = \frac{1}{3} C_{el} v_F \ell = \frac{\pi^2}{3}\frac{k^2{}_B}{e^2} T\sigma$$

The effect of an electric field and a thermal gradient on a Fermi surface

T. Ziman
Electrons and phonons

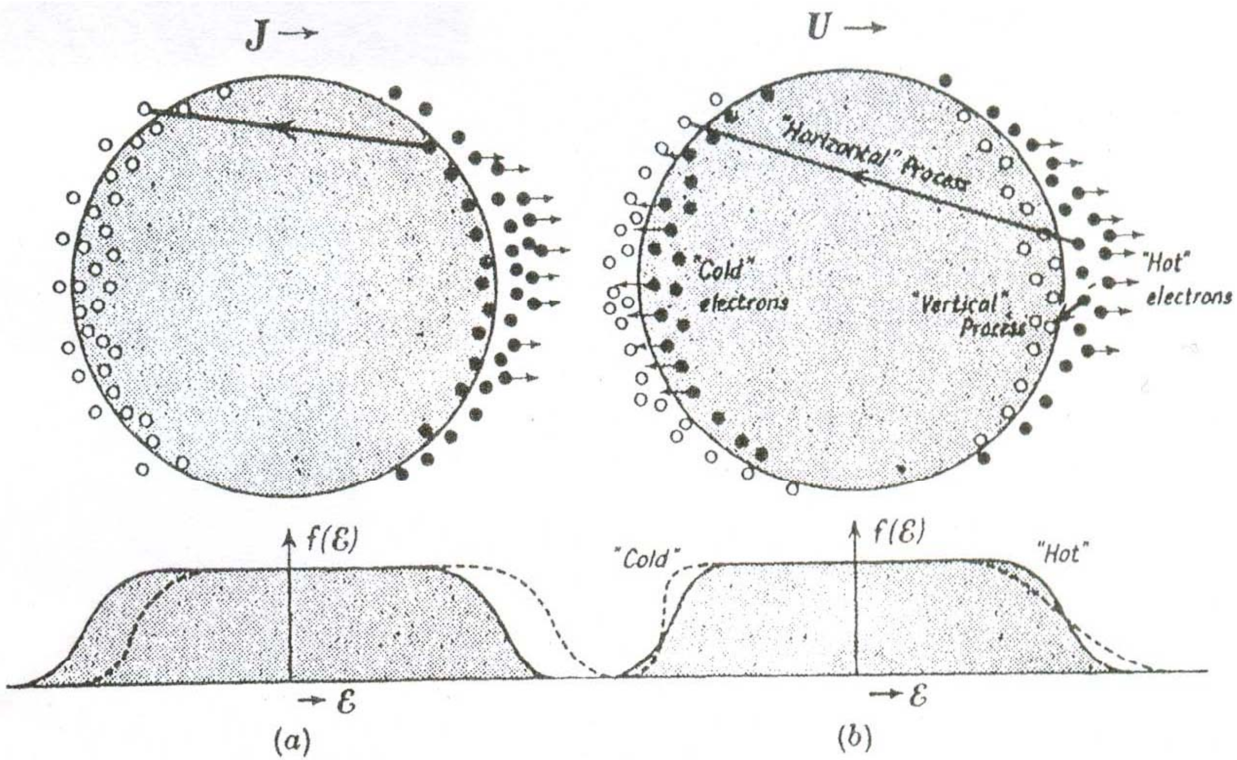
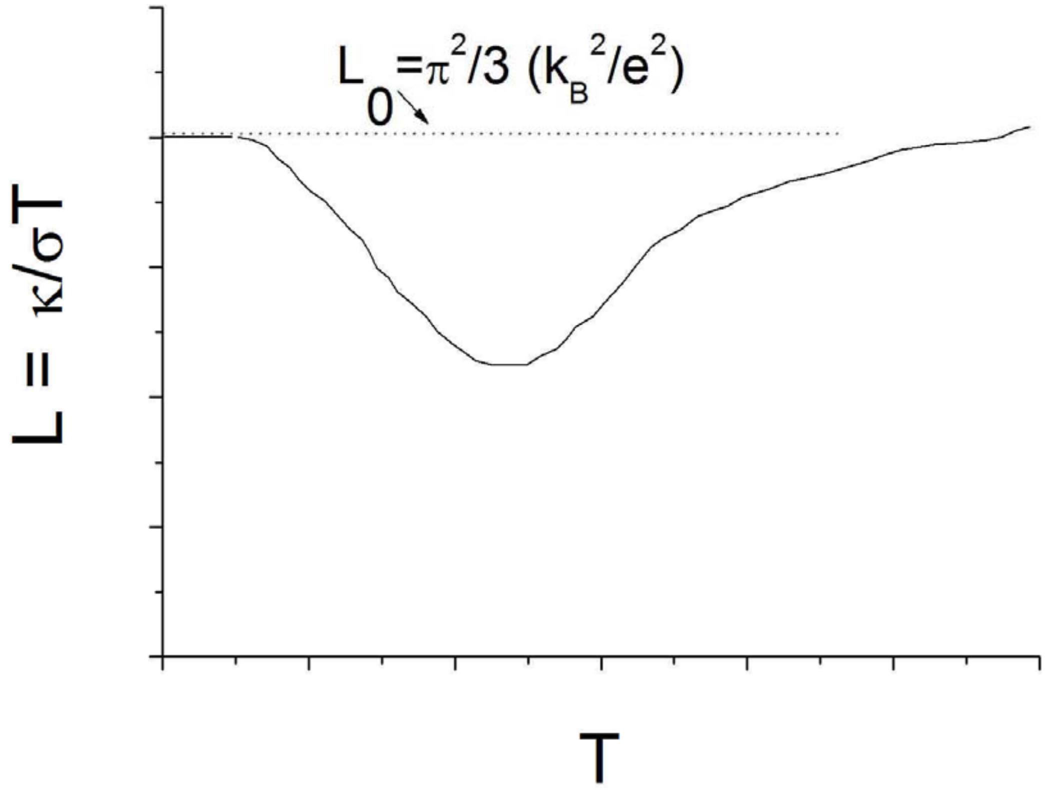


FIG. 122. The Fermi surface and distribution functions: (*a*) in electrical conduction; (*b*) in thermal conduction.

● extra electron ——— equilibrium distribution.
○ electron deficiency --------- steady state transport distribution.

The electric field displaces the Fermi surface, but a thermal gradient makes it fuzzier in the hotter end!

The Wiedemann-Franz law is recovered only at T= 0 and at high temperatures!



At high temperatures vertical scattering beomes marginal because of the thermal broadening of the Fermi surface

III. Correlated electrons

Landau theory of Fermi liquids

- Why band theory is successful in spite of its neglect of electronic interactions?
- Interaction electrons can be mapped to non-interacting quasi-particles with the same spin and charge
- Normalized mass of quasi-particles contains all information on the magnitude of the interactions

Heavy Fermi liquids

- Intermetalics containing a 4f (Ce, Yb…) or 5f (U, Pu,…) element.
- High temperature: A Fermi sea and localised spins
- Low temperature: Heavy quasi-particles as a result of hybridization of f electrons and conduction electtrons

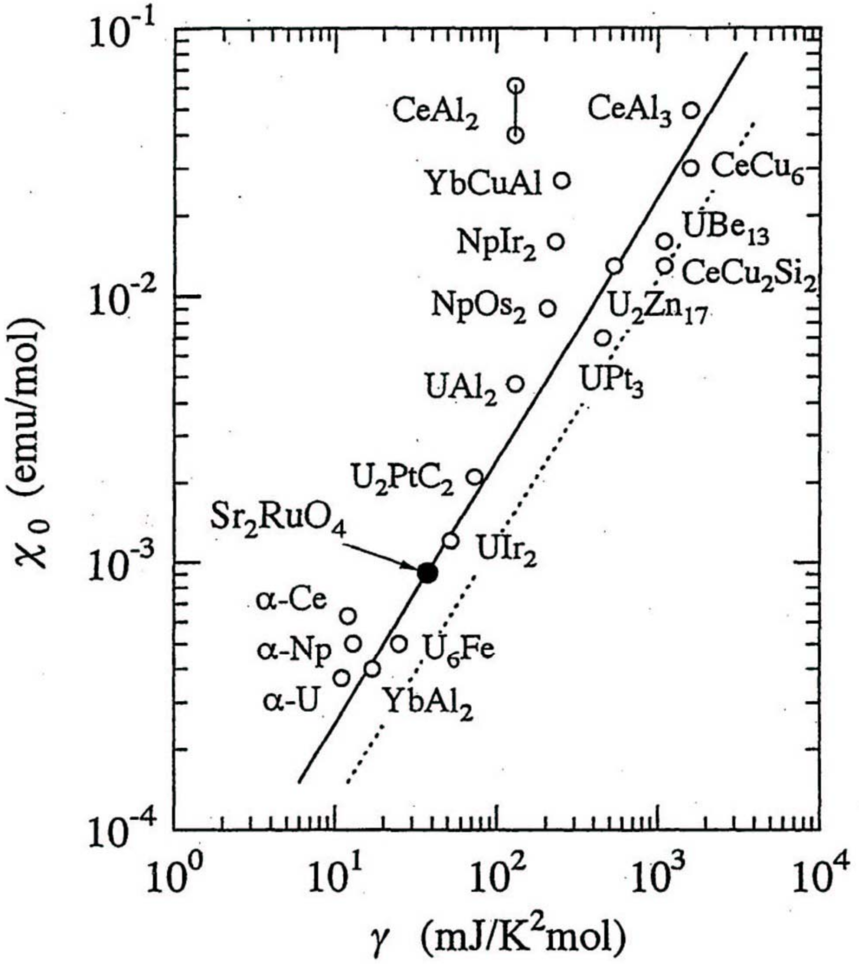
Heavy Fermi liquids

- Enhanced specific heat $$\gamma = \frac{\pi^2}{3} k_B^2 N(\varepsilon_F)$$
- Enhanced Pauli Susceptibility $$\chi = \mu_B^2 N(\varepsilon_F)$$
- Enhanced T^2-resistivity $$\rho = \rho_0 + A\,T^2$$ $$A \propto N(\varepsilon_F)^2$$

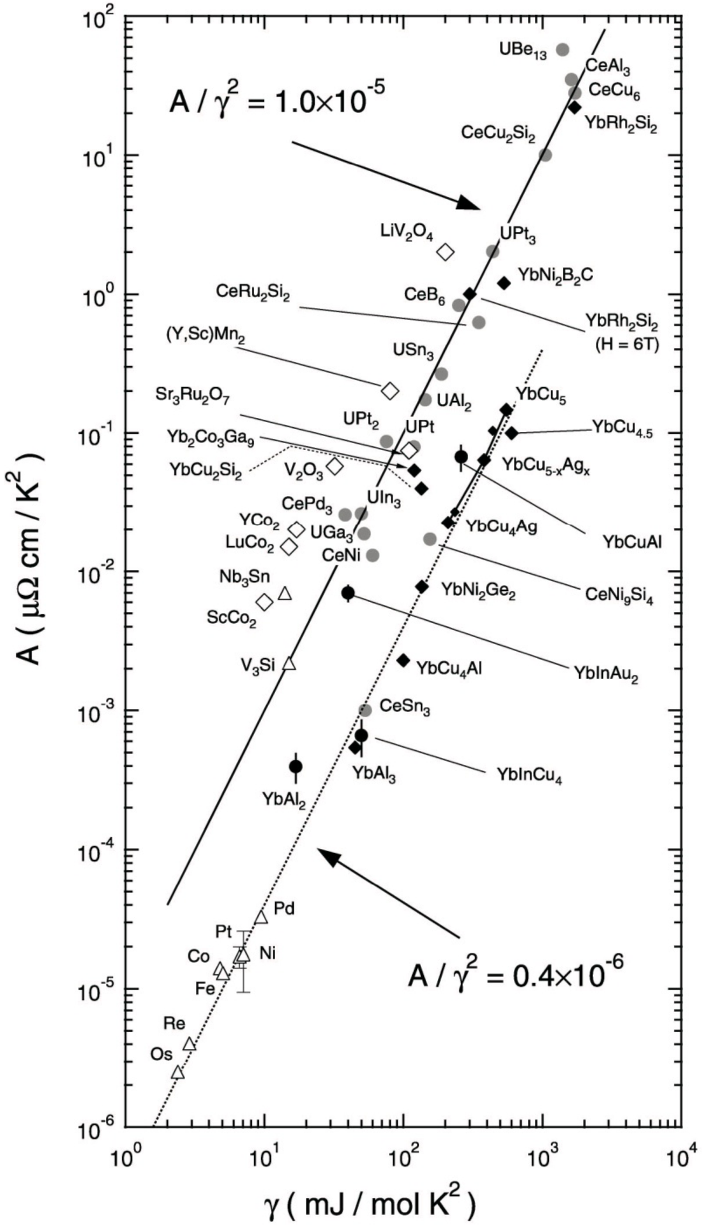
Fermi liquid ratios

The Wilson ratio

$$R_W = \frac{\pi^2}{3}\frac{k^2{}_B}{\mu_B^2}\frac{\chi}{\gamma}$$



Fermi liquid ratios

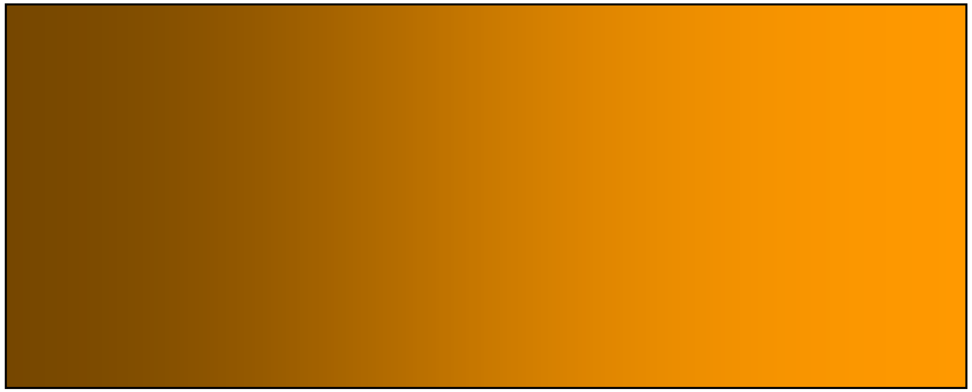


The Kadowaki-Woods ratio

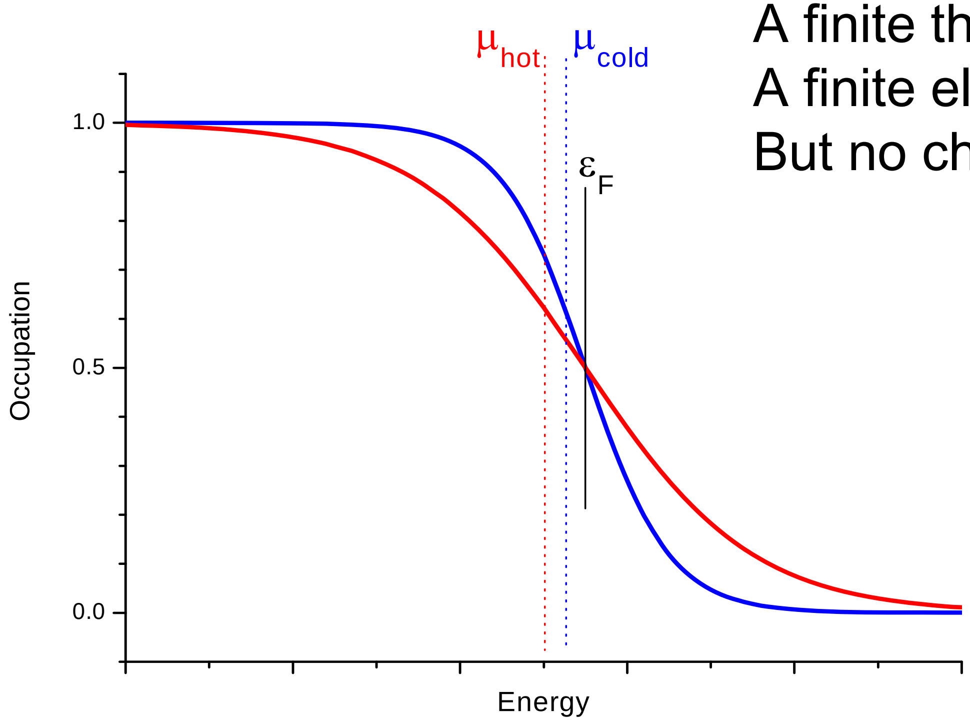
$$KW = \frac{A}{\gamma^2}$$

Thermoelectric response

hot



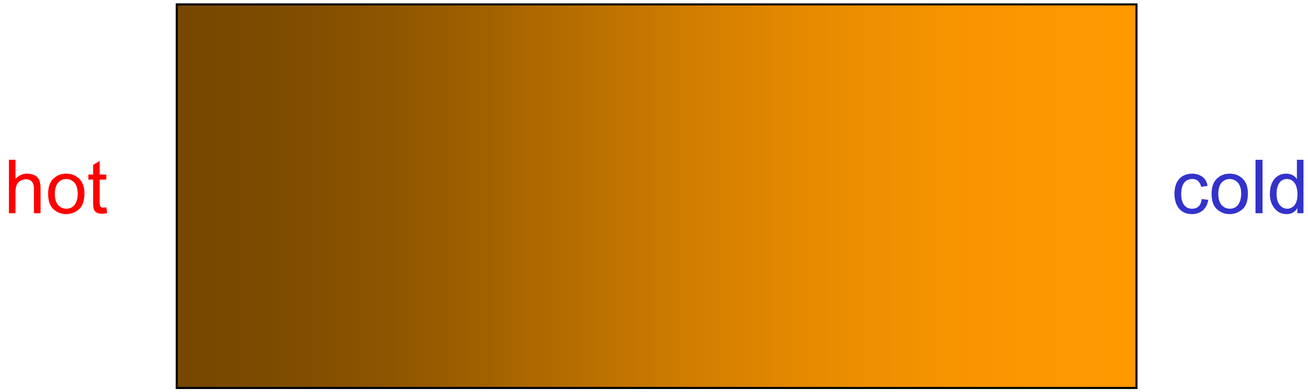
cold



A finite thermal gradient!
A finite electric field!
But no charge current!

Response to a gradient in chemical potential!

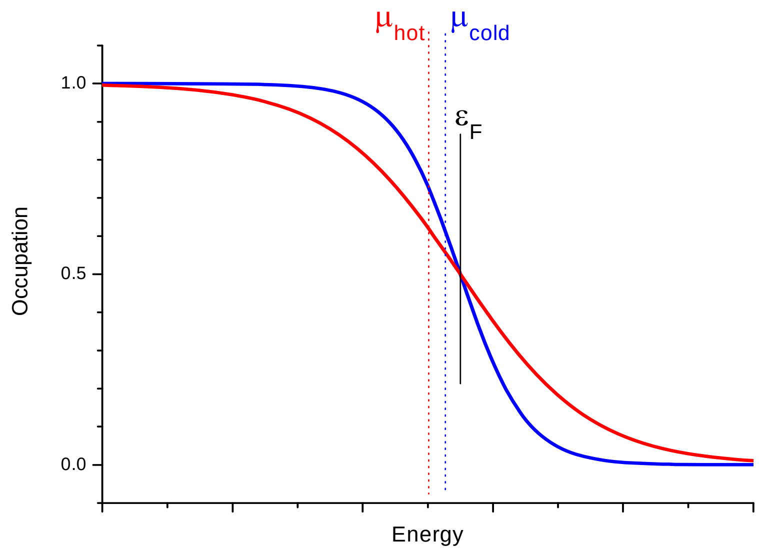
Thermoelectric response



Carriers with a charge, q, and an entropy, S_{ex} will suffer two forces:

- Electric force: $F = E\,q$
- Thermal force: $F = S_{ex}\,\nabla T$

$$S = E/\nabla T = S_{ex}/q$$



Thermopower measures entropy per [charged] carrier.

$S = (k_B\,T/\,E_F)/\,e$

Seebeck coefficient of the free electron gas

In the Boltzmann picture thermopower is linked to electric conductivity:

$$S = -\frac{\pi^2}{3}\frac{k_B^2 T}{e}\left(\frac{\partial \ln \sigma(\epsilon)}{\partial \epsilon}\right)_{\varepsilon F}$$

This yields:

$$S = -\frac{\pi^2}{3}\frac{k_B^2 T}{e}\left[\left(\frac{\partial \ln \tau(\epsilon)}{\partial \epsilon}\right)_{\varepsilon F} + \frac{\int d\mathbf{k}\delta(\epsilon_F - \epsilon(\mathbf{k}))\mathbf{M}^{-1}(\mathbf{k})}{\int d\mathbf{k}\delta(\epsilon_F - \epsilon(\mathbf{k}))v(\mathbf{k})v(\mathbf{k})}\right]$$

transport Thermodynamic

For a free electron gas, with $\tau = \tau_0{}^{\xi}$, this becomes:

$$S = -\frac{\pi^2}{3}\frac{k_B^2}{e}\frac{T}{\epsilon_F}\left(\frac{3}{2} + \xi\right)$$

Thermopower and specific heat

In a free electron gas (with ξ=0):

$$S = -\frac{\pi^2}{3}\frac{k_B^2 T}{e}\frac{N(\epsilon_F)}{n} \qquad\qquad C_{el} = \frac{\pi^2}{3}k_B^2 T N(\epsilon_F)$$

Thermopower is a measure of specific heat per carrier

The dimensionless ratio: $$q = N_{Av} e \frac{S}{T\gamma}$$

is equal to –1 (+1) for free electrons (holes)

[if one assumes a constant mean-free-path , then ξ=1/2 and q=2/3]

Thermoelectricity in real metals

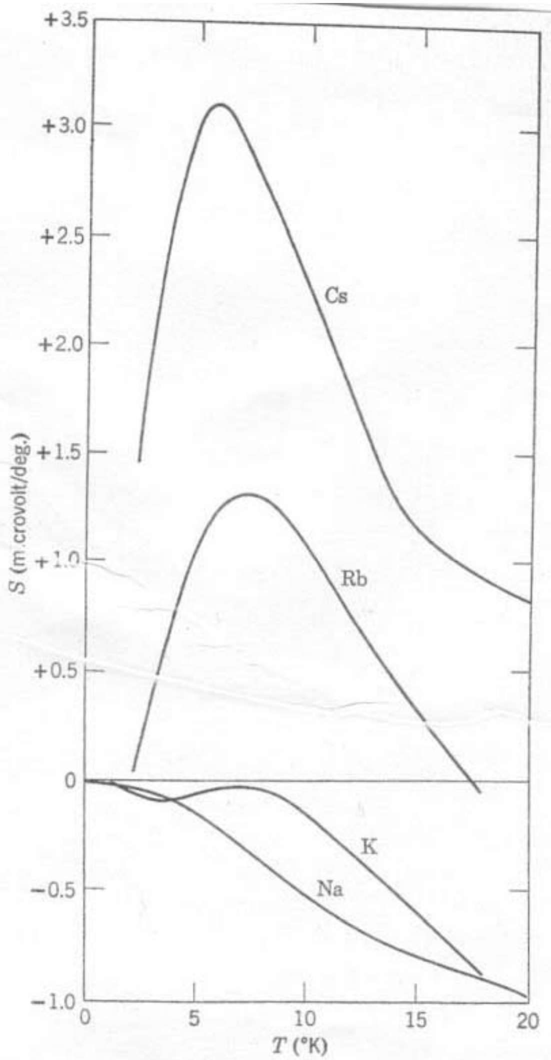


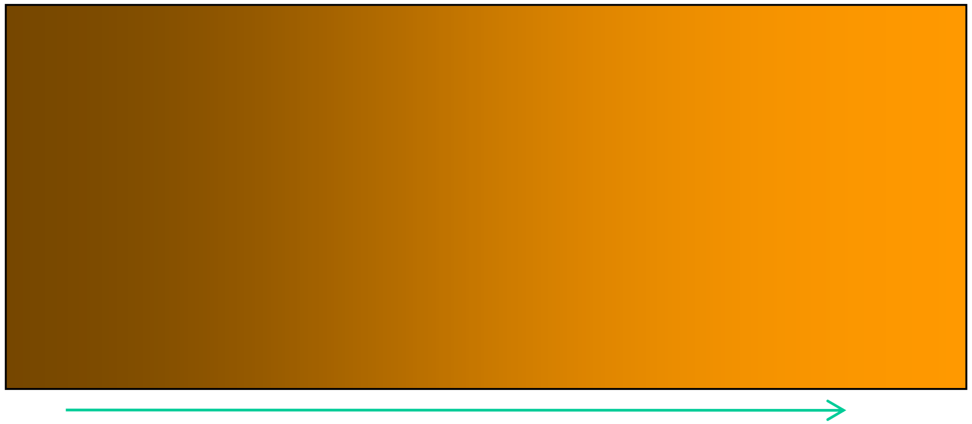
Fig. 28. Absolute thermoelectric power of alkali metals at very low temperatures. The data are taken from MacDonald, Pearson, and Templeton (1958) and, in general, the purest specimen has been selected in each case here. The positive hump in the thermoelectric power due to phonon-drag is particularly striking at the low temperatures in rubidium and caesium and already is making its presence felt in potassium between 5 to 10° K.

- Even in simplest metals, the free-electron-gas picture does not work at finite temperature !

Structure in the thermopower of AlKali metals (MacDonald 1961) !

Phonon Dragg

hot



cold

Flow of heat-carrying phonons

•If electrons and phonons exchange energy then electrons would be dragged by phonons

Lattice specific heat

Phonon dragg thermopower

$$S_g = \frac{C_L}{Ne}\alpha$$

Carrier density

e⁻ - ph coupling ($0<\alpha<1$)

Order of magnitude of phonon dragg

$(T/\Theta_D)^3$

$$S_g / S_{diff} = \frac{C_L}{C_e \beta}\alpha$$

Frequency of ph-e^- scattering events

T/T_F

e^- per atom

- Expected to vanish in the zero-temperature limit
- Becomes negligible when $C_e >> C_L$

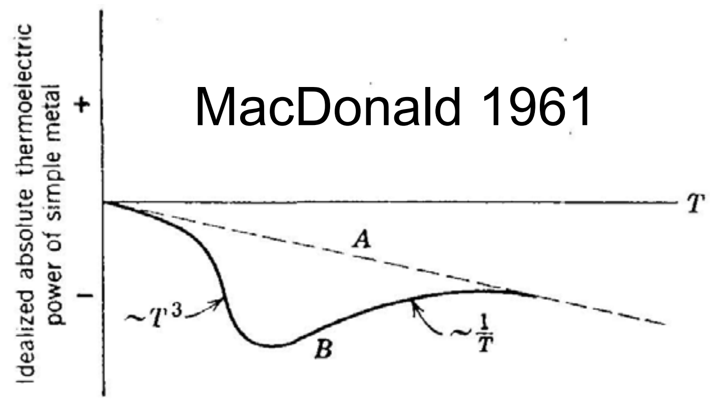


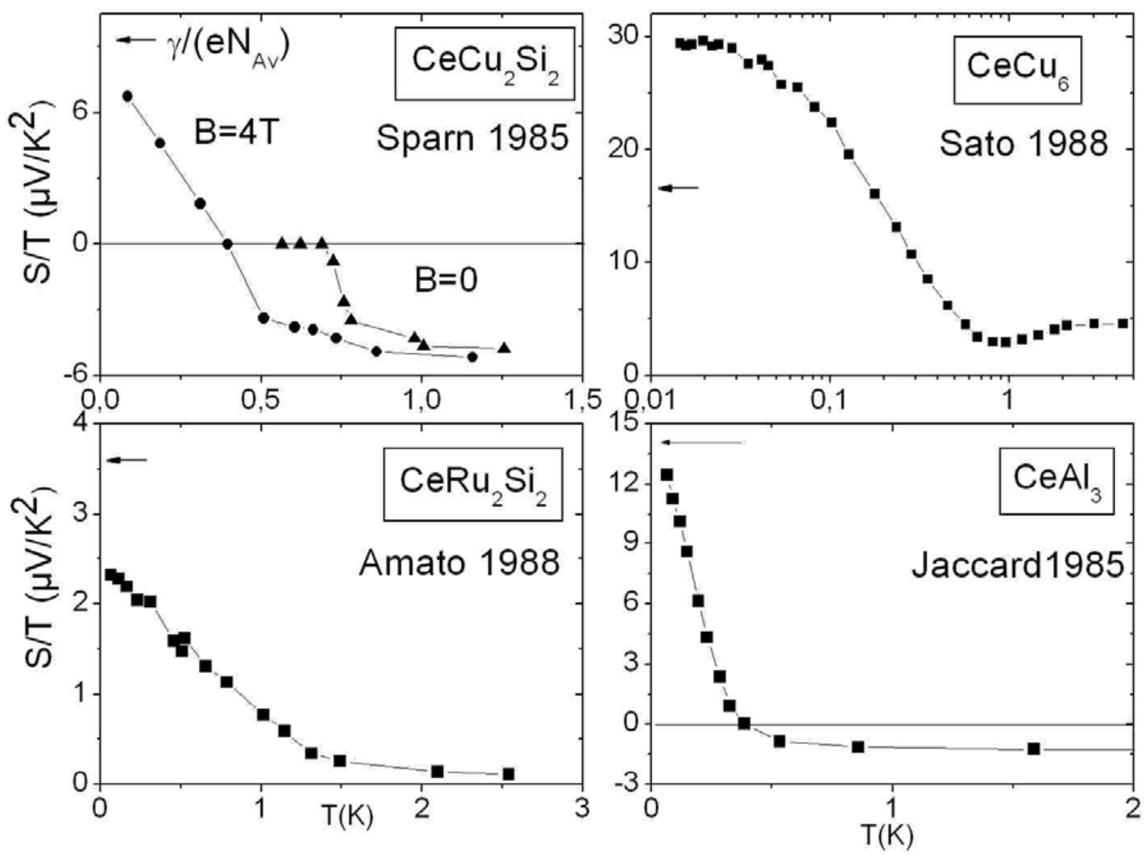
Fig. 8. Sketch of idealized absolute thermoelectric power of a simple quasi-free electron metal.

A: Electron diffusion component of thermoelectric power approximately proportional to *T*.

B: Phonon-drag component with magnitude increasing as T^3 at very low temperatures ($T \ll \theta$), and decaying as $1/T$ at "high" temperatures ($T \gtrsim \theta$).

Heavy electrons in the T=0 limit

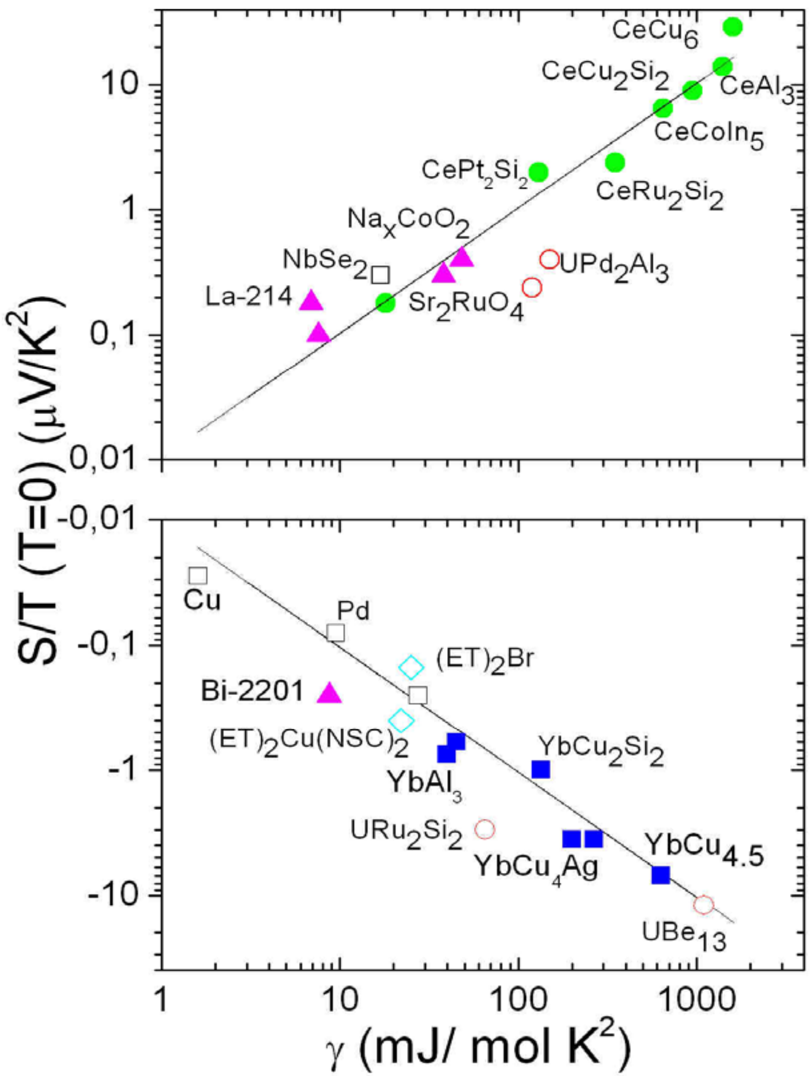
Two-decades-old data replotted!



Extrapolated to T=0, data yields a q close to unity!

Another plot linking two distinct signatures of electron correlation

KB, D. Jaccard, J. Flouquet 2004



Compound	S/T (μ V / K^2)	Remarks	γ (mJ /mol K^2)	q
$CeCu_2Si_2(B=4T)$	9[30]	polycrystal	950[52]	0.9
$CeCu_6$	29[33]	along [010]	1600[53]	1.7
$CeAl_3$	14[32]	polycrystal	1400[32]	1..0
$CeRu_2Si_2$	2.4[31]	in-plane	350[54]	0.7
$CeCoIn_5$ (B=6T)	6[55]	in-plane	650[56]	0..9
$CePt_2Si_2$	2[57]	along [110]	130[58]	1.5
$CeSn_3$	0.18[59]	polycrystal	18[60]	1.0
CeNiSn	50[61]	polycrystal	45[62]	107
$YbCu_{4.5}$	-7[63]	polycrystal	635[64]	-1.1
YbCuAl	-3.6[65]	polycrystal	267[66]	-1.3
$YbCu_4Ag$	-3.6[67]	polycrystal	200[68]	-1.7
$YbCu_2Si_2$	-1[20, 69]	polycrystal	135[70]	-0.7
$YbAl_3$	-0.6[20]	polycrystal	45[71]	-1.3
$YbInAu_2$	-0.75[69]	polycrystal	40[72]	-1.8
UPt_3	unknown	none observed[35]	430[35]	–
UBe_{13}(B=7.5T)	-12[36]	polycrystal	1100[73]	-1.1
UNi_2Al_3	0.24[37]	polycrystal	120[74]	0.2
UPd_2Al_3	0.4[37]	S $\perp c$	150[75]	0.3
URu_2Si_2	-3[38]	S $\perp c$	65[76]	-4.5
κ-(BEDT-TTF)$_2$Cu[N(CN)$_2$]Br	-0.4[41]	in-plane	22[77]	-1.7
κ-(BEDT-TTF)$_2$Cu(NSC)$_2$	-0.15[42]	in-plane	25[78]	-0.6
(TMTSF)$_2$ClO$_4$	unknown	No report found	11[79]	–
Sr_2RuO_4	0.3[44]	in-plane	38[4]	0..8
$SrRuO_3$	unknown	No report found	30[80]	–
$Sr_3Ru_2O_7$	unknown	No report found	38[81]	–
$SrRhO_3$	0.03[82]	polycrystal	7.6[83]	1..3
Na_xCoO_2	0.4[45]	in-plane	48[46]	0..8
$La_{1.7}Sr_{0.3}CuO_4$	0.18[47]	ceramic	6.9[6]	2.5
$Bi_2Sr_2CuO_{6+\delta}$	-0.25[48]	ceramic	8.7[84]	-2.8
$NbSe_2$	0.3[51]	in-plane	17[85]	1.7
Pd	-0.08[16]	polycrystal	9.5[86]	-0.8
Cu	-0.028[50]	along [231]	1.6[11]	-1.7
constantan (%43Ni-%57Cu)	-0.25[14]	wire	27.4[87]	-0.9

Table 1. Reported magnitudes of linear thermopower and specific heat for a number of metals. The significance of the coefficient $q = \frac{S}{T}\frac{N_{Av}e}{\gamma}$ is discussed in the text.

Data since 2004

	γ	S/T	q
UPt_3	430	2.5	0.6
$NpPd_5Al_2$	200	-1.3	-0.65
$CeNi_2Al_3$	30	0.27	0.9
$PrFe_4As_{12}$	340	-1	0.3
$YbRh_2Si_2$	750	-6.7	-0.8
FeTe	34	-0.3	-0.9
MgB_2	3	0.04	-1.3
$SrRu0_3$	30	0.24	0.8
$SrRh_2O_4$	10	0.22	2.2

In semi-metals q is large!

- URu_2Si_2 (q=-11)
- $PrFe_4P_{12}$ (q=-58)
- $PrRu_4P_{12}$ (q=-43)
- CeNiSn (q=107)
- $Bi_{0.96}Sb_{0.04}$ ($q=10^4$)

In these systems the FS occupies a small fraction (~1/2q) of the BZ.

Theory on correlation between S and γ

Miyake & Kohno, JPSJ (2005)

$$\lim_{\varepsilon\to\mu}\frac{\partial \ln \tau_{imp}(\epsilon)}{\partial \epsilon} \propto \left(\frac{\partial \ln N^{*}(\epsilon)}{\partial \epsilon}\right)_{\varepsilon=\mu} \sim \frac{1}{\tilde{\epsilon}_F}$$

In both unitary and Born limits, q ~ $\pm$1

Zlatic, Monnier, Freericks & Becker, PRB 2007

(Single-impurity Anderson model)

Paul & Kotliar, PRB 2001
Near a QCP both expected to diverge logarithmically

Haule and Kotliar, CorrelatedThermoelectricity workshop (2008)
DMFT

Measuring the Fermi energy of an electronic system

- Resistivity: $$A = \frac{h}{e^2} a \frac{1}{\varepsilon_F^2}$$
- Specfific heat $$\gamma = \frac{\pi^2}{3} k_B^2 n \frac{1}{\varepsilon_F}$$
- Thermopower $$\frac{S}{T} = \frac{\pi^2}{3} \frac{k_B}{e} \frac{1}{\varepsilon_F}$$

Electronic correlations…

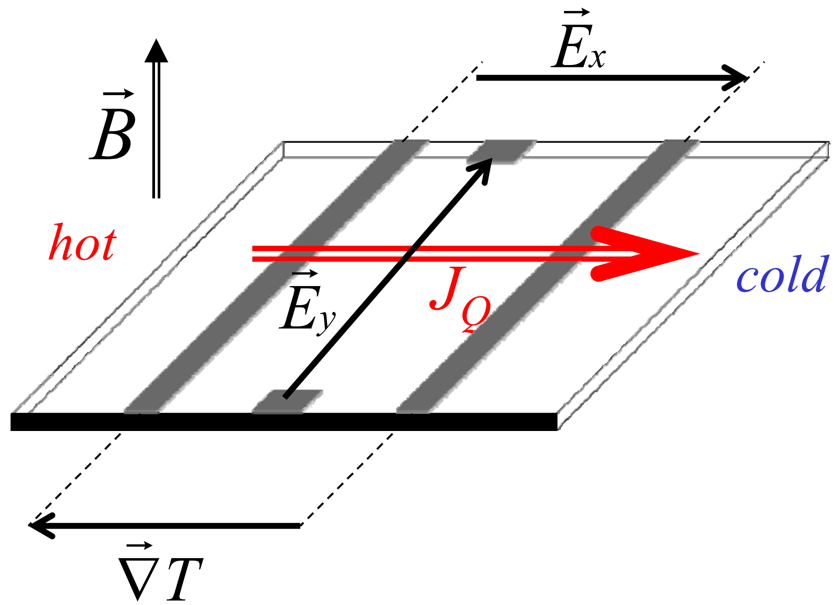
- Do not enhance or diminish the absolute value of σ or κ components in the T=0 limit

- But, they do enhance the magnitude of α, the thermoelectric response

IV. Nernst effect

Thermoelectric coefficients

- In presence of a thermal gradient, electrons produce an electric field.
- Seebeck and Nernst effect refer to the longitudinal and the transverse components of this field.

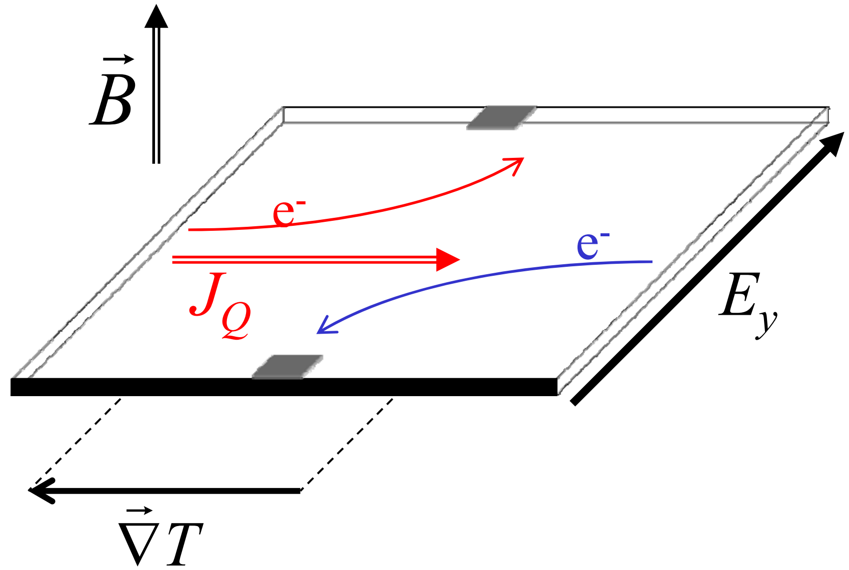


$$S = \frac{-E_x}{\nabla_x T} \qquad N = e_y = S_{xy} = \frac{-E_y}{\nabla_x T} \qquad [\nu = \frac{-E_y}{B_z \nabla_x T}]$$

Nernst effect in a single-band metal

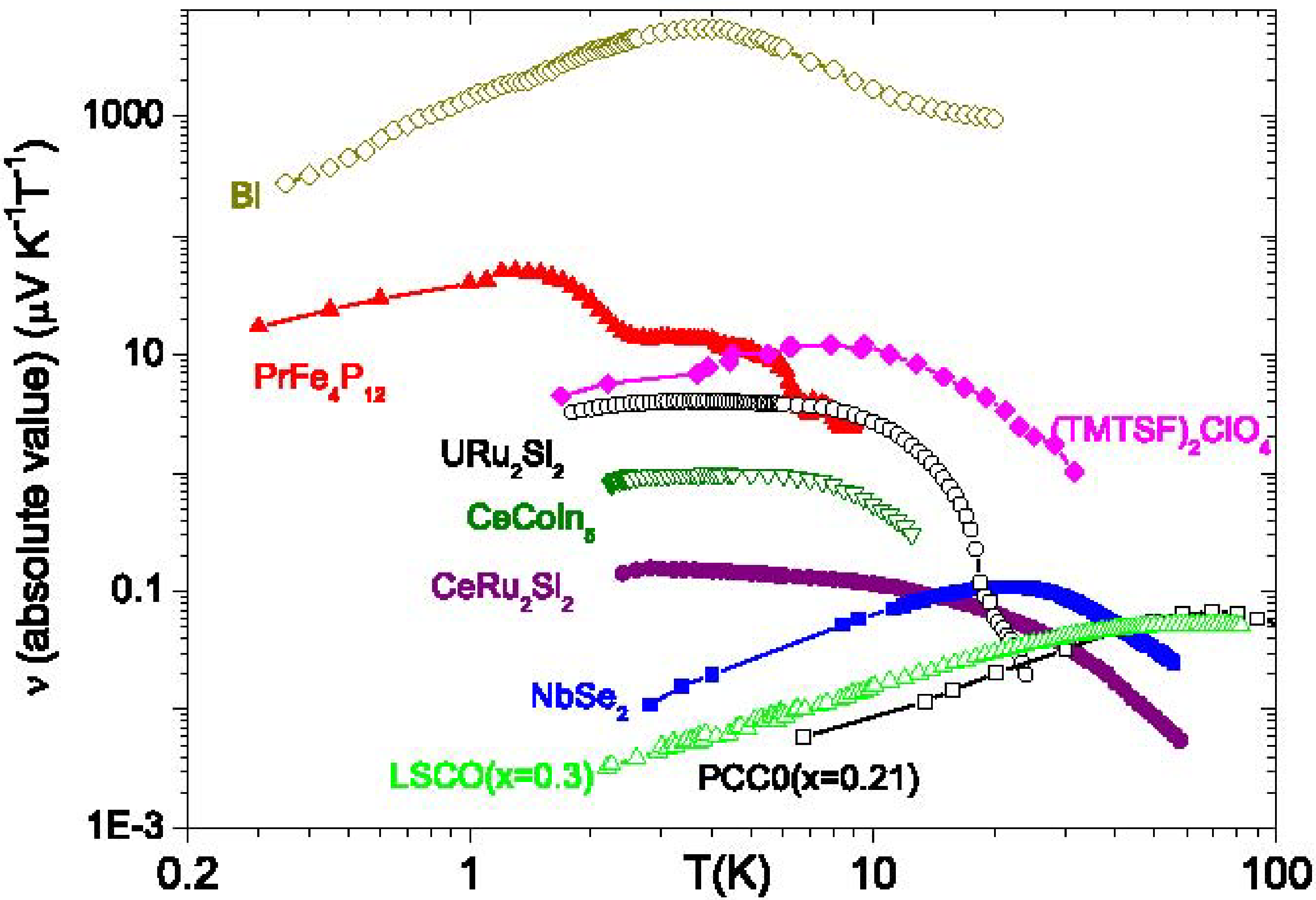
Absence of charge current leads to a counterflow of hot and cold electrons:

$$\boxed{J_Q \neq 0 \; ; J_e = 0 \; ; E_y = 0}$$

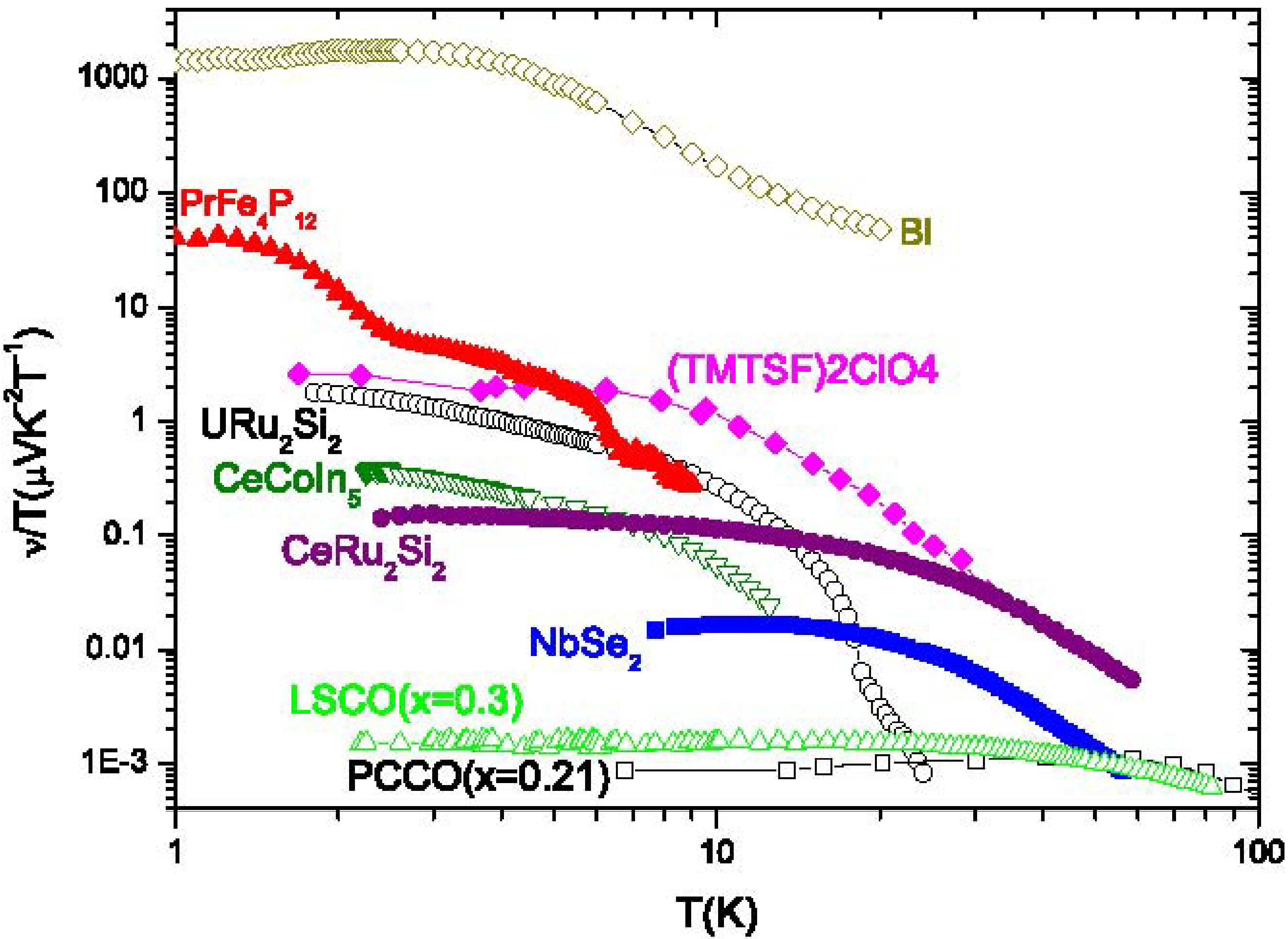


In an ideally simple metal, the Nernst effect vanishes!
(« Sondheimer cancellation », 1948)

Nernst coefficient in remarkable metals!



A large diffusive component in the zero-temperature limit!



Close-up on Sondheimer cancellation

$$\vec{J}_e = \sigma \vec{E} - \alpha \vec{\nabla} T$$
$$\vec{J}_Q = \alpha T \vec{E} - \kappa \vec{\nabla} T$$

J_e=0

$$N = \frac{E_y}{\nabla_x T} = \frac{\alpha_{xy}\sigma_{xx} - \alpha_{xx}\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2}$$

Boltzmann picture: $\overline{\alpha} = \frac{\pi^2}{3}\frac{k_B^2 T}{e}\frac{\partial\overline{\sigma}}{\partial\epsilon}\Big|_{\varepsilon_F} \longrightarrow N = \frac{\pi^2}{3}\frac{k_B^2 T}{e}\frac{\partial\Theta_H}{\partial\epsilon}\Big|_{\varepsilon_F}$

If the Hall angle, Θ_H, does not depend on the position of the Fermi level, then the Nernst signal vanishes!

Roughly, the Nernst coefficient tracks $\omega_c\tau / E_F$…

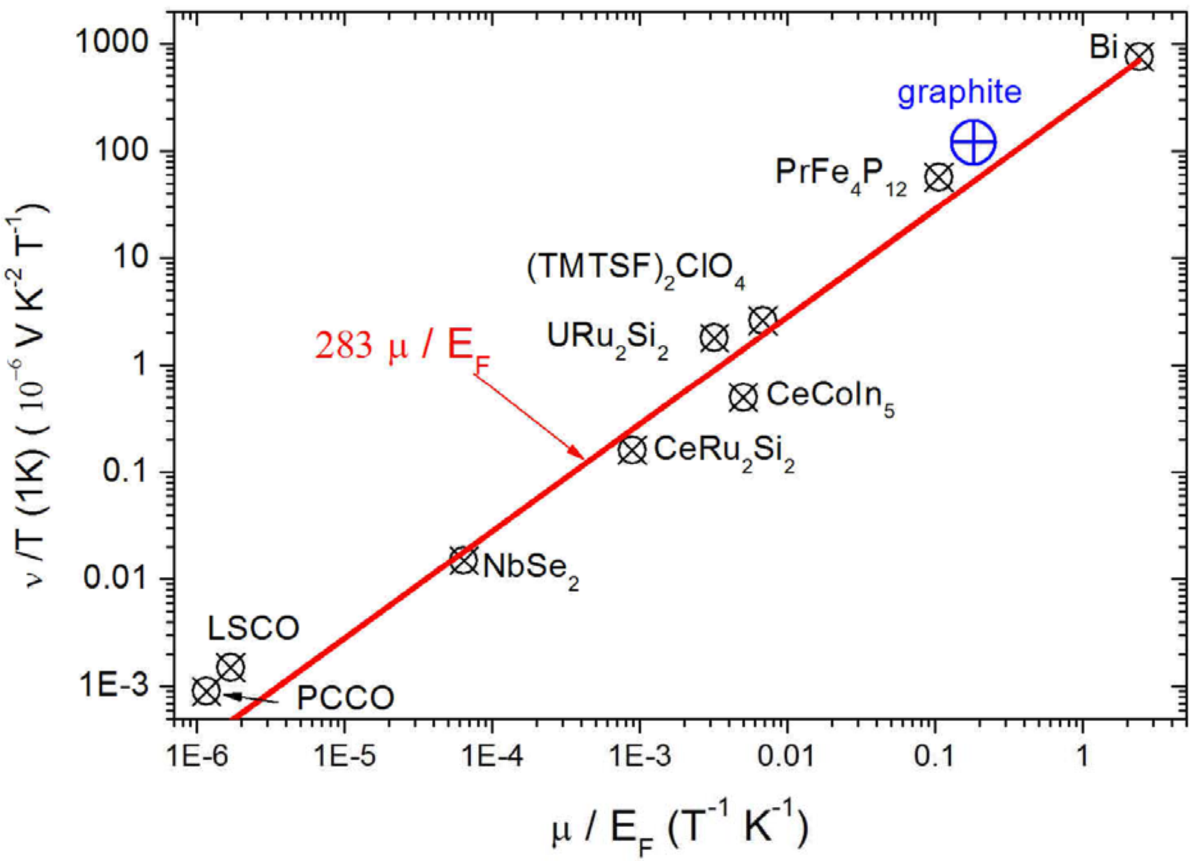
$$N = \frac{\pi^2}{3}\frac{k_B^2 T}{e}\frac{\partial\Theta_H}{\partial\epsilon}\Big|_{\epsilon_F}$$

$$N \sim \pi^2/3\; k_B^2 T/e\; \omega_c\tau / E_F$$

Table 1. The magnitude of the Nernst coefficient divided by temperature at low temperature, together with estimations of the electronic mobility and the Fermi energy in various metals. The fourth column yields the expected magnitude of ν/T according to equation (10).

System	ν/T (μV K^{-2} T^{-1})	μ (T^{-1})	ϵ_F (K^{-1})	$\frac{\pi^2}{3}\frac{k_B}{e}\frac{\mu}{\epsilon_F}$ (μV K^{-2} T^{-1})
Bi	750	420	130	914
$CeRu_2Si_2$	0.16	0.2	180	0.25
$CeCoIn_5$	0.5	0.3	60	1.4
URu_2Si_2	1.8	0.08	25	0.9
$PrFe_4P_{12}$	57	0.85	8	30
$(TMTSF)_2ClO_4$	2.6	0.75	110	1.9
$La_{1.7}Sr_{0.3}CuO_4$	0.0015	0.01	5900	4.8×10^{-4}
$Pr_{1.79}Ce_{0.21}CuO_4$	9×10^{-4}	0.005	4300	3.3×10^{-4}
$NbSe_2$	0.015	0.09	1400	0.018

Recipe for a large diffusive Nernst response:



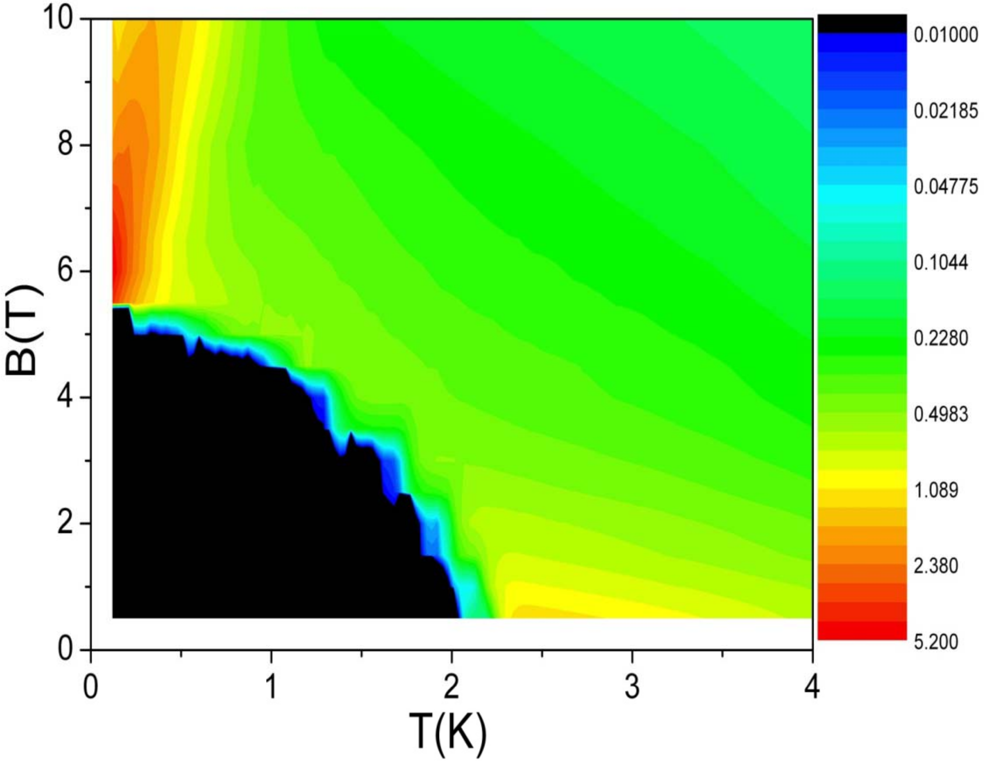
- High mobility
- Small Fermi energy
- Ambipolarity

In graphite at 20 K and 1T
S_{xy} ~3mVK^{-1}, ρ ~3mΩcm
S_{xy}^2/ρ = 3000 μW K^{-2}cm^{-1}

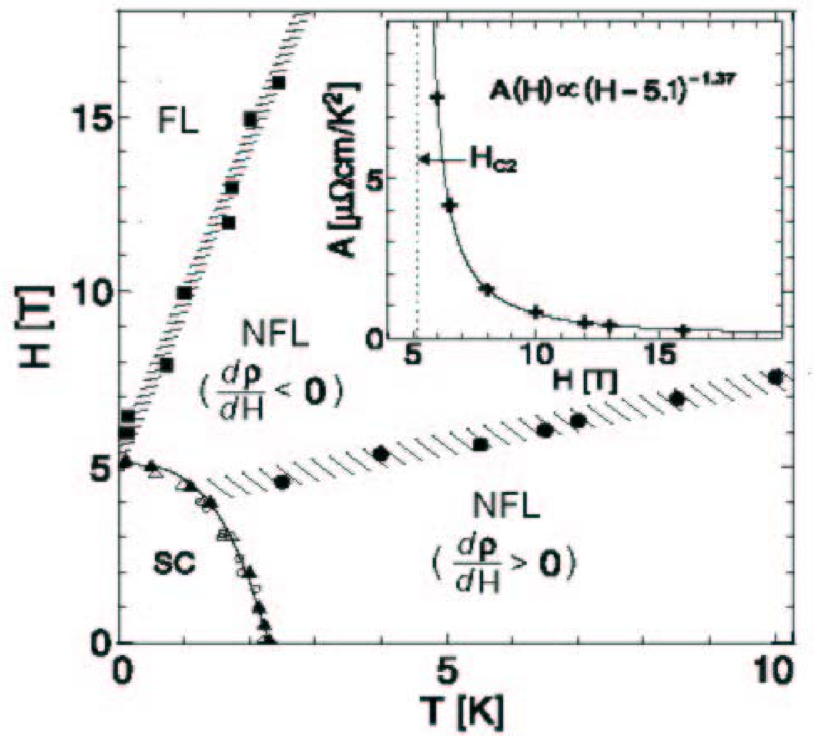
Nernst effect as a probe of quantum criticality

The case of $CeCoIn_5$

Izawa et al. 2007



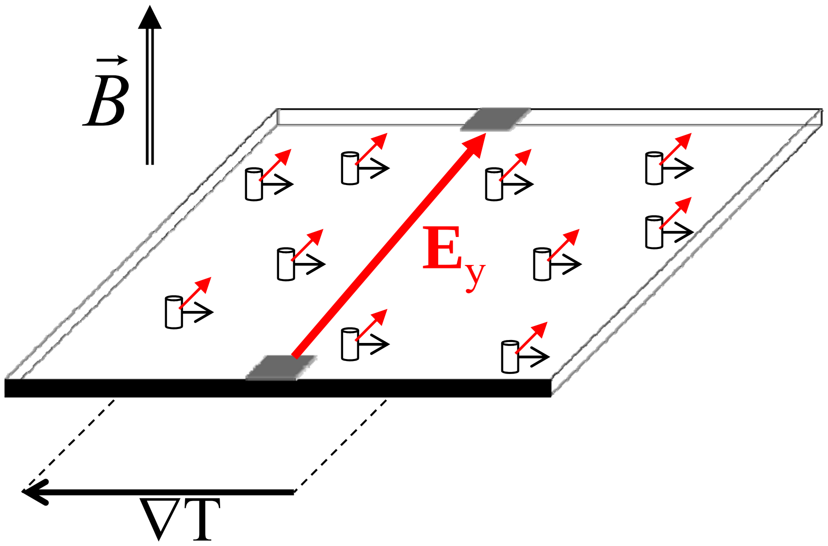
Paglione *et al. ,2003,* Bianchi *et al. 2003*



logarithmic color plot of ν/T

Nernst effect directly reveals the quantum critical point!

Nernst effect in the vortex state



A superconducting vortex is:

- A quantum of magnetic flux
- An entropy reservoir
- A topological defect

- Thermal force on the vortex : $F=-S_\phi \nabla T$ (S_ϕ : vortex entropy)
- The vortex moves
- The movement leads to a transverse voltage: $E_y=v_x B_z$

Nernst effect in optimally-doped YBCO

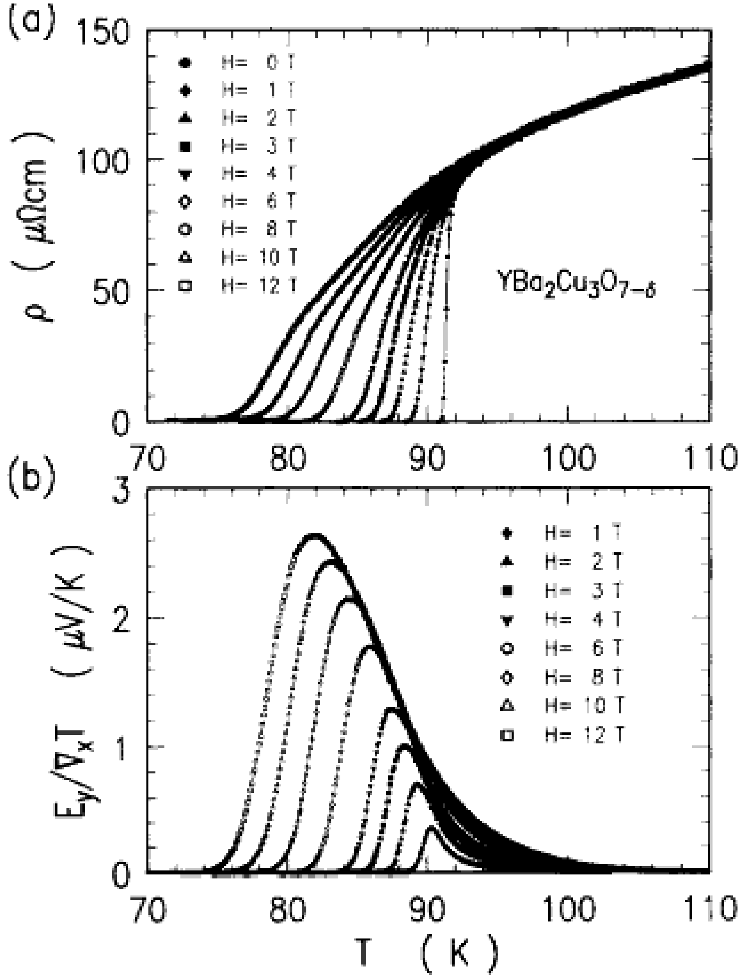


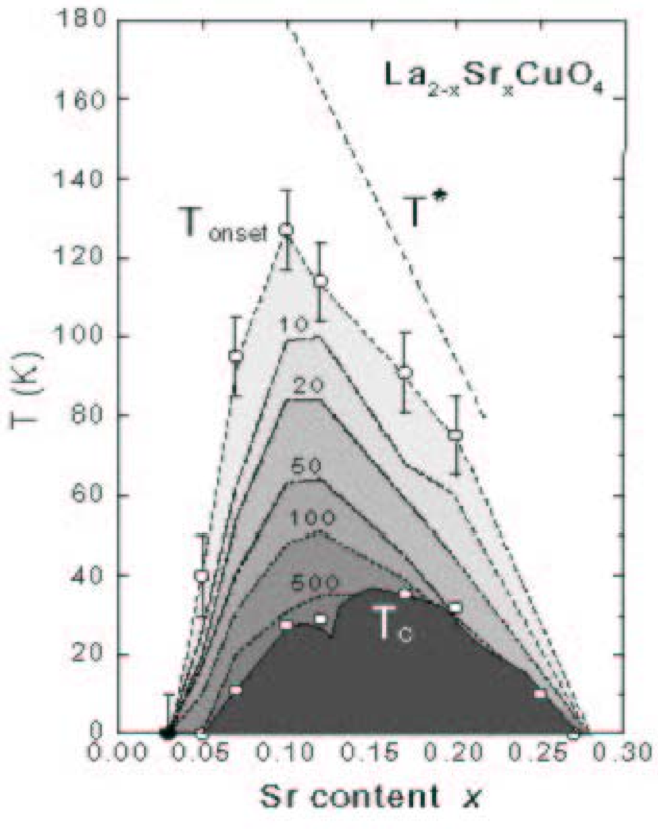
FIG. 3. Resistivity ρ (a) and normalized Nernst electric field $E_y/\nabla_x T$ (b) versus temperature for an epitaxial, c-axis-oriented $YBa_2Cu_3O_{7-\delta}$ film at different magnetic fields applied parallel to the c axis of the film.

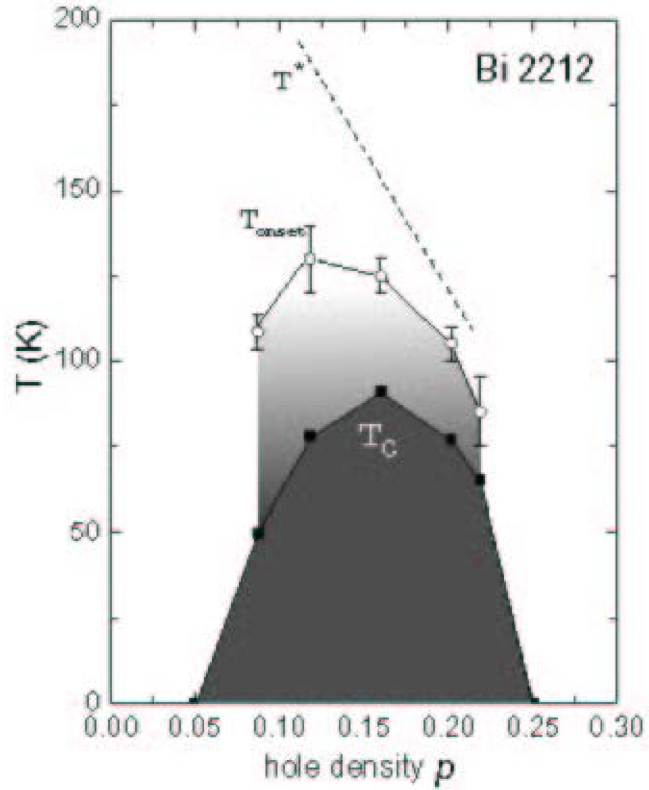
The Nernst coefficient is finite only in the vortex liquid state!

(Ri, Huebner et al. 1994)

Vortex-like excitations in the normal state of the underdoped cuprates?

Ong et collaborators 2006



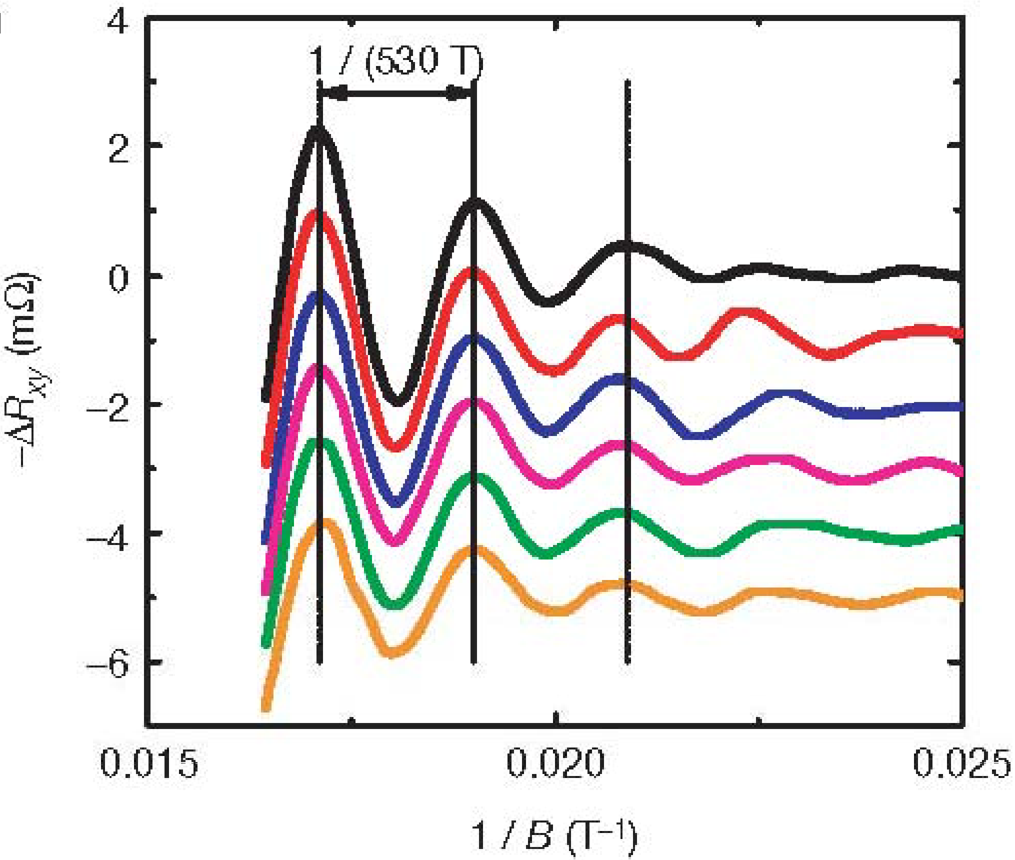


A finite Nernst signal in a wide temperature range above T_c

Quantum oscillations and the Fermi surface in an underdoped high-T_c superconductor

Nicolas Doiron-Leyraud[1], Cyril Proust[2], David LeBoeuf[1], Julien Levallois[2], Jean-Baptiste Bonnemaison[1], Ruixing Liang[3,4], D. A. Bonn[3,4], W. N. Hardy[3,4] & Louis Taillefer[1,4]

a



A small Fermi surface
Highly mobile electrons

Electron pockets in the Fermi surface of hole-doped high-T_c superconductors

David LeBoeuf[1], Nicolas Doiron-Leyraud[1], Julien Levallois[2], R. Daou[1], J.-B. Bonnemaison[1], N. E. Hussey[3], L. Balicas[4], B. J. Ramshaw[5], Ruixing Liang[5,6], D. A. Bonn[5,6], W. N. Hardy[5,6], S. Adachi[7], Cyril Proust[2] & Louis Taillefer[1,6]

PRL **104**, 057005 (2010) PHYSICAL REVIEW LETTERS week ending 5 FEBRUARY 2010

Nernst and Seebeck Coefficients of the Cuprate Superconductor $YBa_2Cu_3O_{6.67}$: A Study of Fermi Surface Reconstruction

J. Chang,[1] R. Daou,[1,*] Cyril Proust,[2,3] David LeBoeuf,[1] Nicolas Doiron-Leyraud,[1] Francis Laliberté,[1] B. Pingault,[1] B. J. Ramshaw,[4] Ruixing Liang,[4,3] D. A. Bonn,[4,3] W. N. Hardy,[4,3] H. Takagi,[5] A. B. Antunes,[6] I. Sheikin,[6] K. Behnia,[7] and Louis Taillefer[1,3,†]

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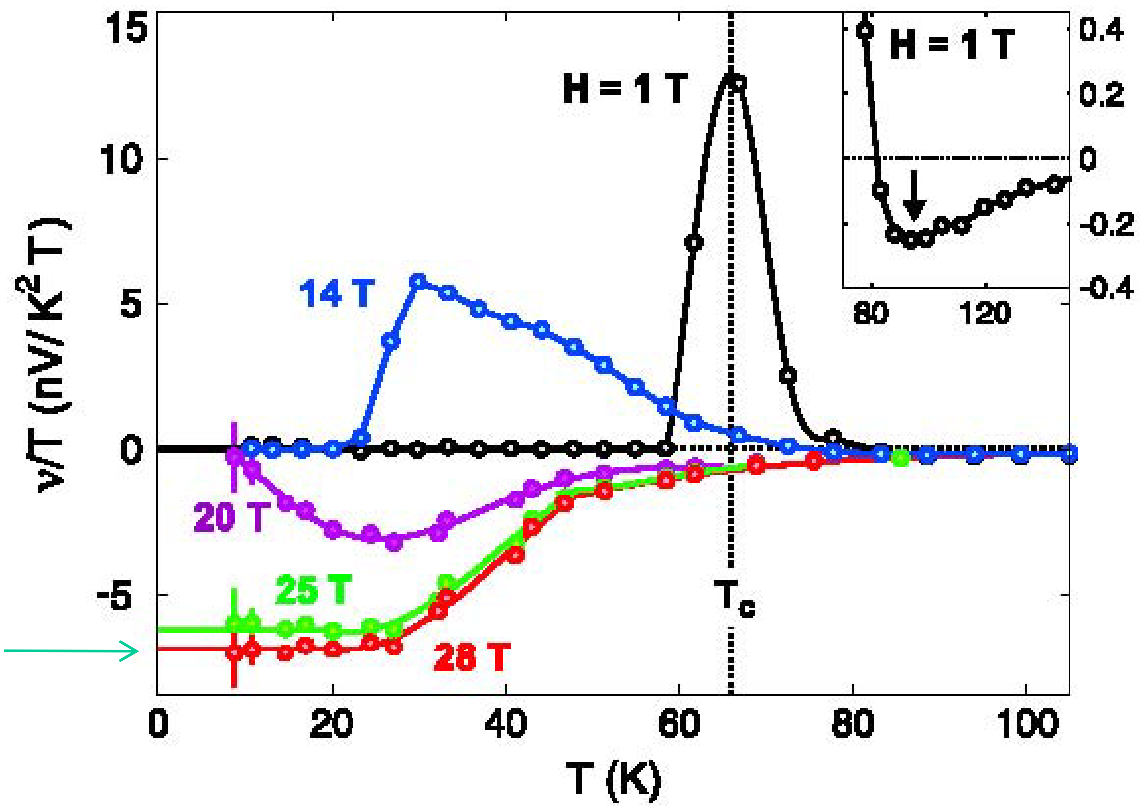
(Received 28 July 2009; published 5 February 2010)

The small electron pocket generates a sizeable negative Nernst signal!



Within a factor of 2 of what is expected for the normal state!

Nernst effect due to Gaussian fluctuations of the amplitude of the superconducting order parameter (Usshishkin, Sondhi & Huse, 2002)

In 2D:

$$\alpha_{xy}^{SC} = \frac{1}{6\pi} \frac{k_B e}{\hbar} \frac{\xi^2}{\ell_B^2}$$

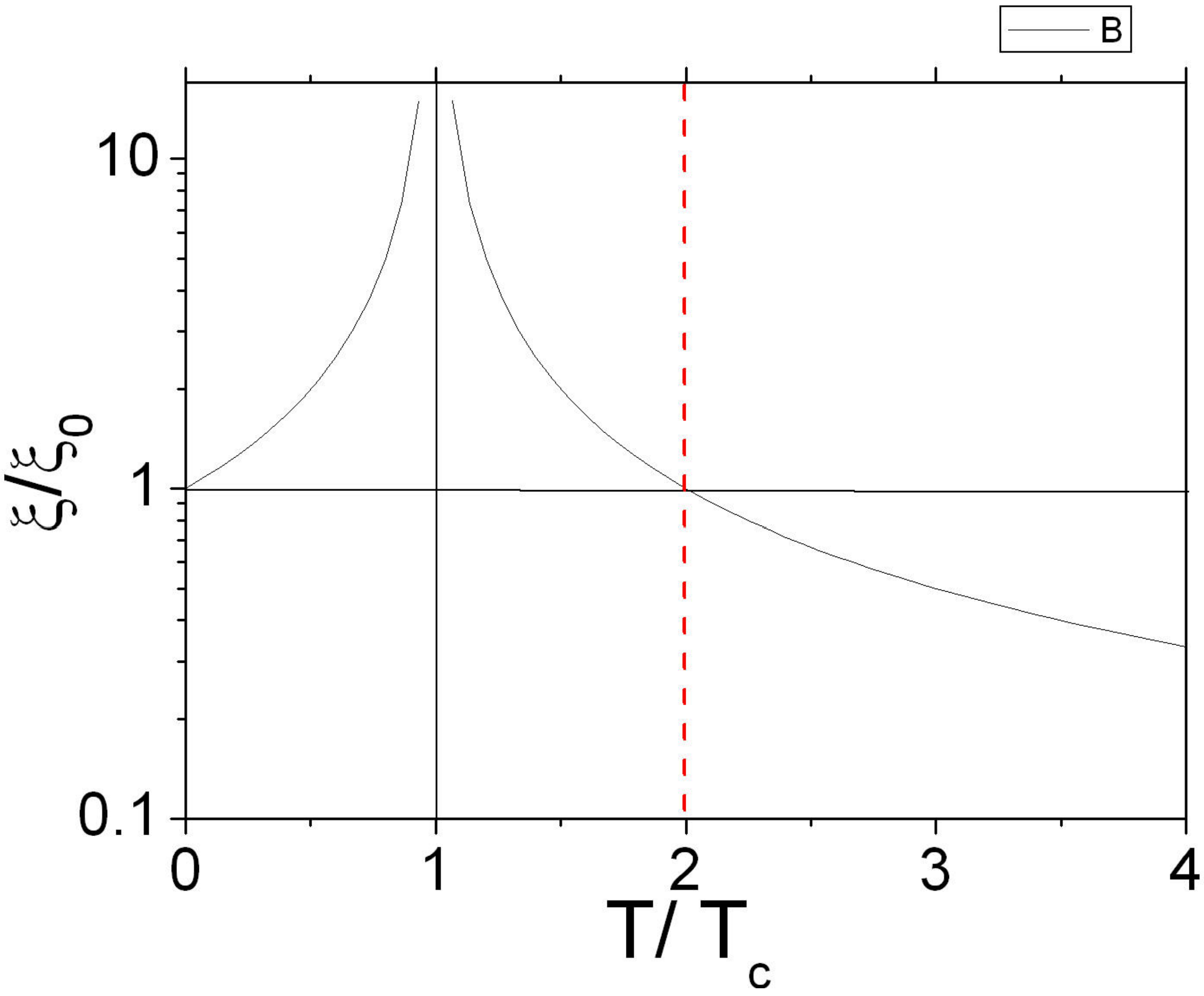
$$\ell_B = \sqrt{\frac{\hbar}{eB}}$$

Magnetic length

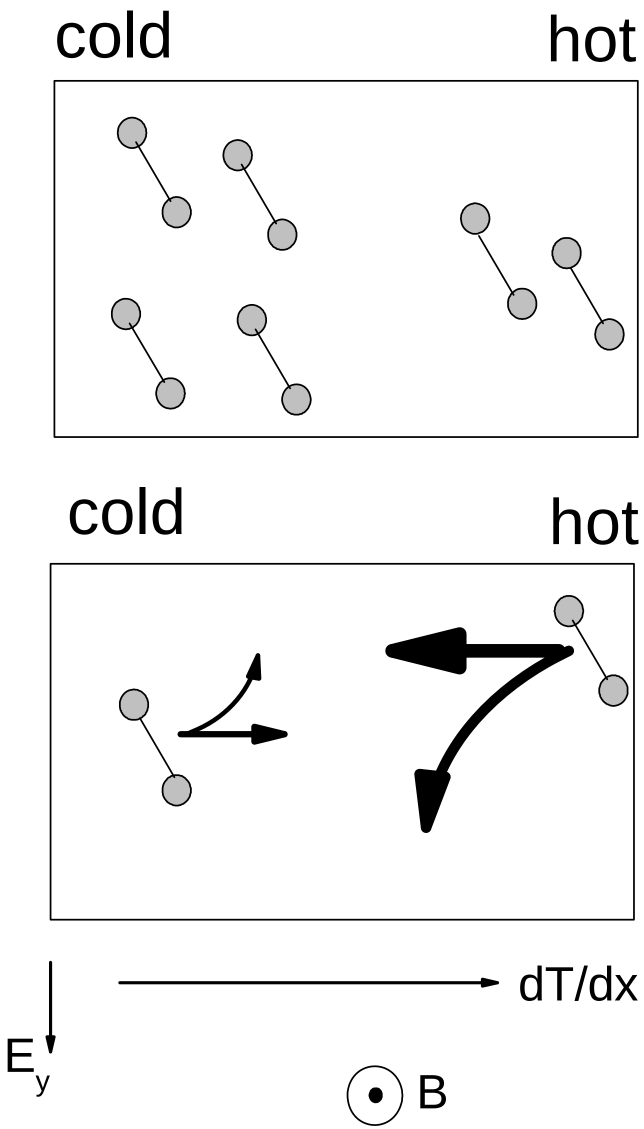
Quantum of thermo-electric conductance (21 nA/K)

In two dimensions, the coherence length is the unique parameter!

The coherence length above T_c



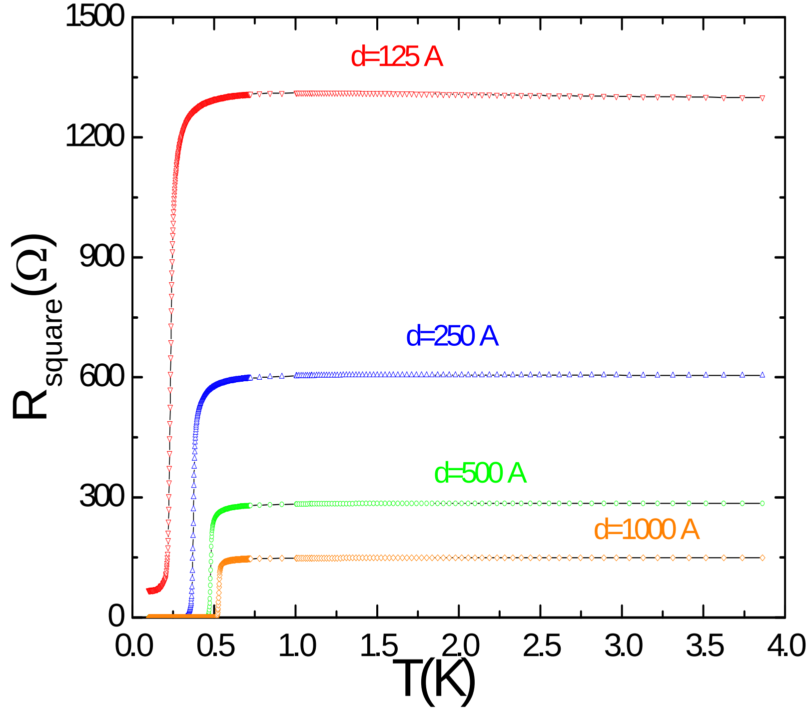
How do the fluctuating Cooper pairs generate a Nernst signal?

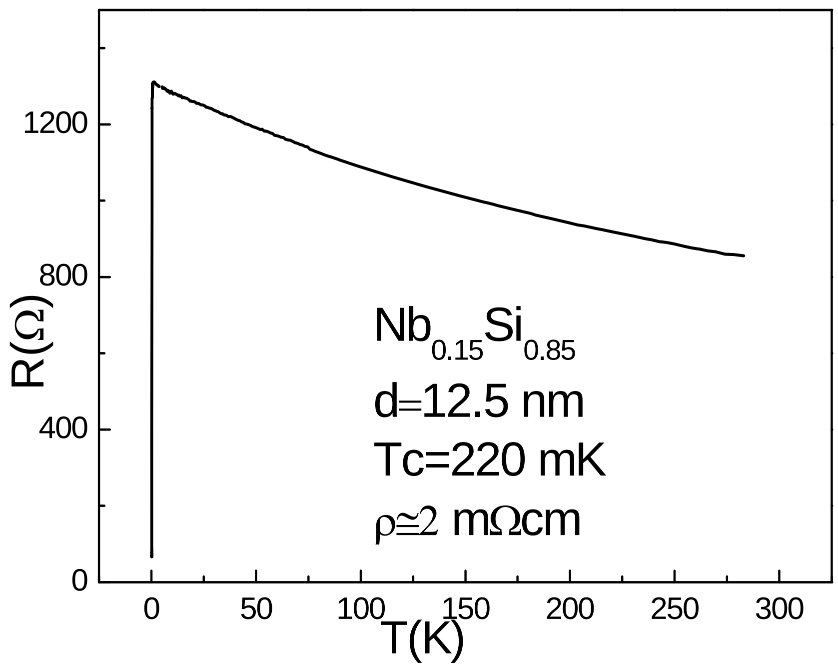


- Above T_c, the lifetime of the Cooper pairs decrease with increasing temperature

- Therefore, those pairs which travel from the hot side along the cold side live longer!

Superconductivity in $Nb_{0.15}Si_{0.85}$ thin films



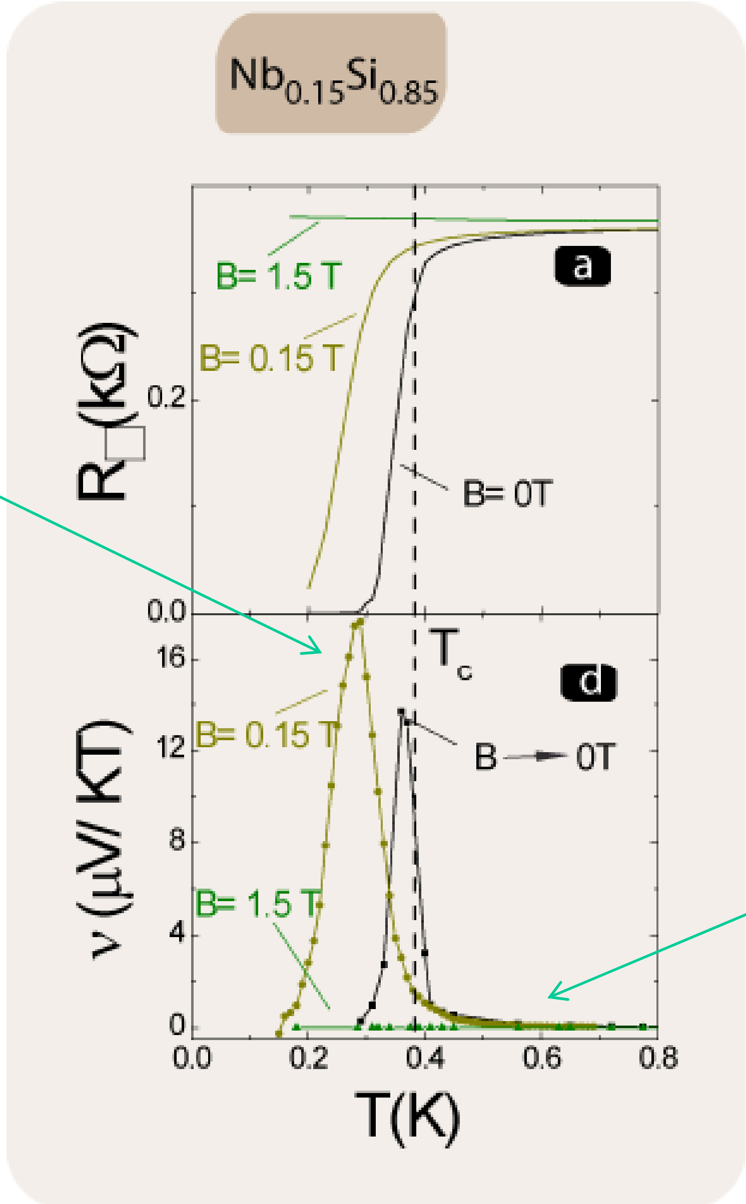


The normal state is a simple dirty metal: l_e~a~ $1/k_F$!

Nernst effect across the resistive transition

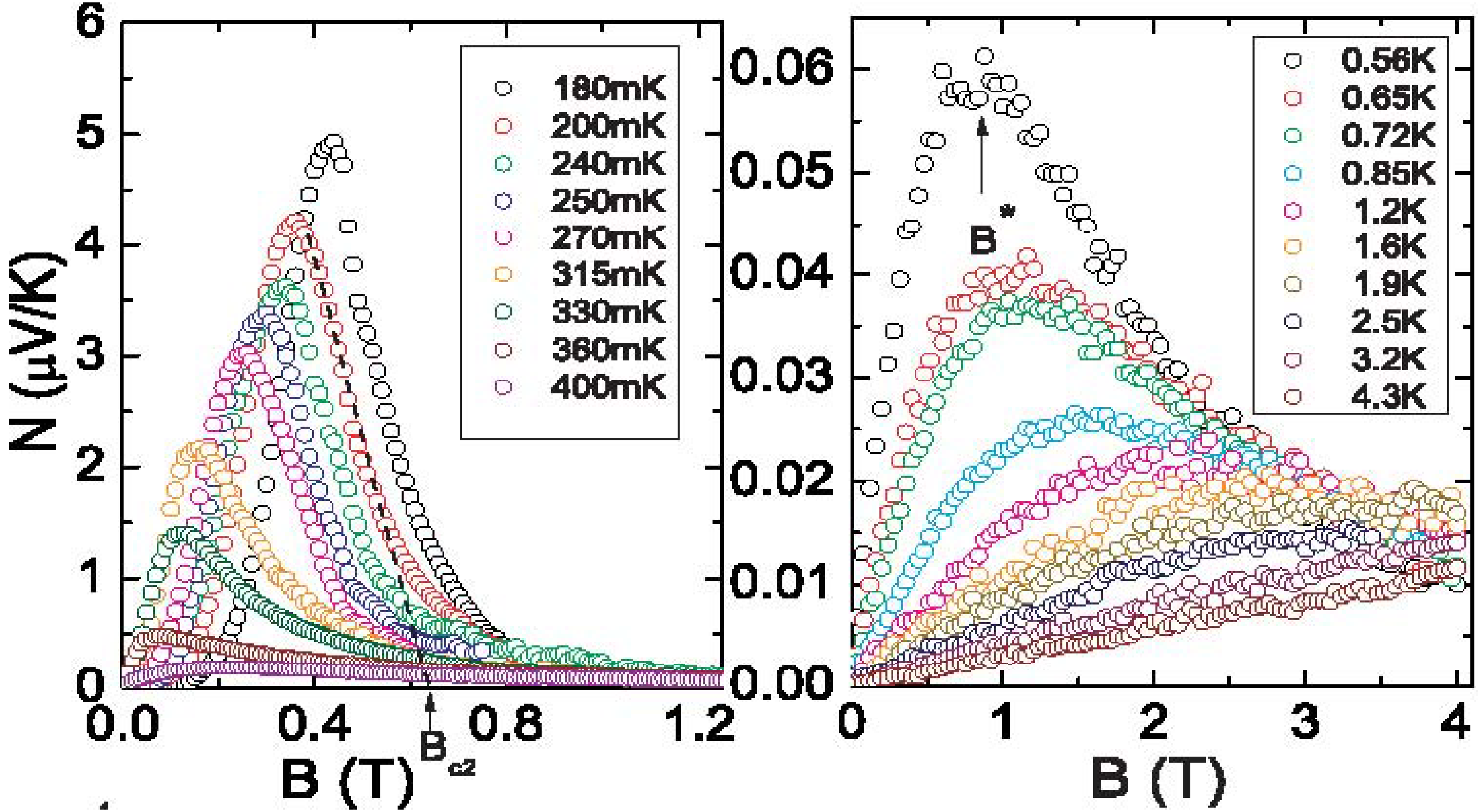
- A large vortex signal below T_c

Pourret 2006

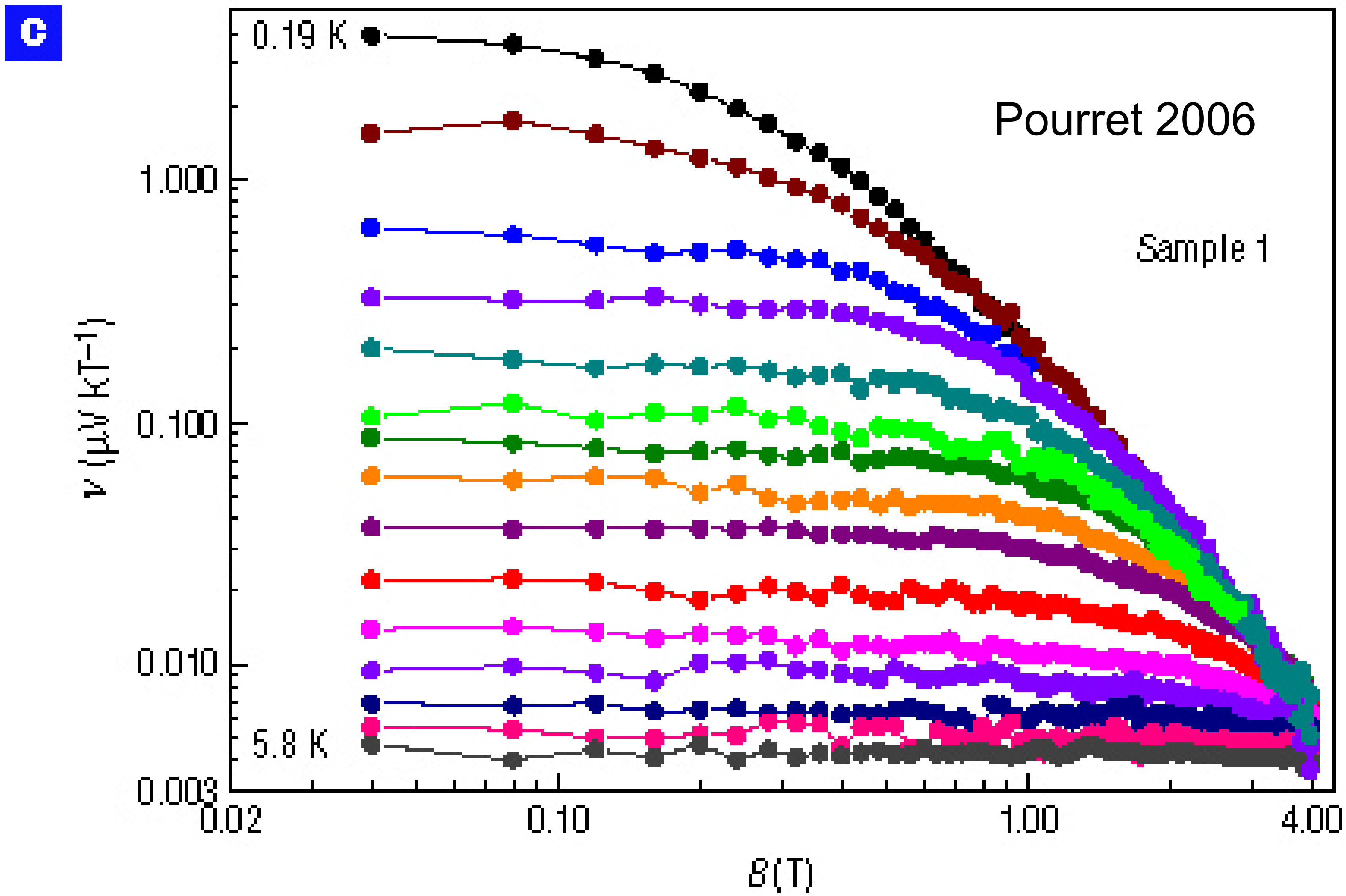


•A long tail above T_c

A signal distinct from the vortex signal

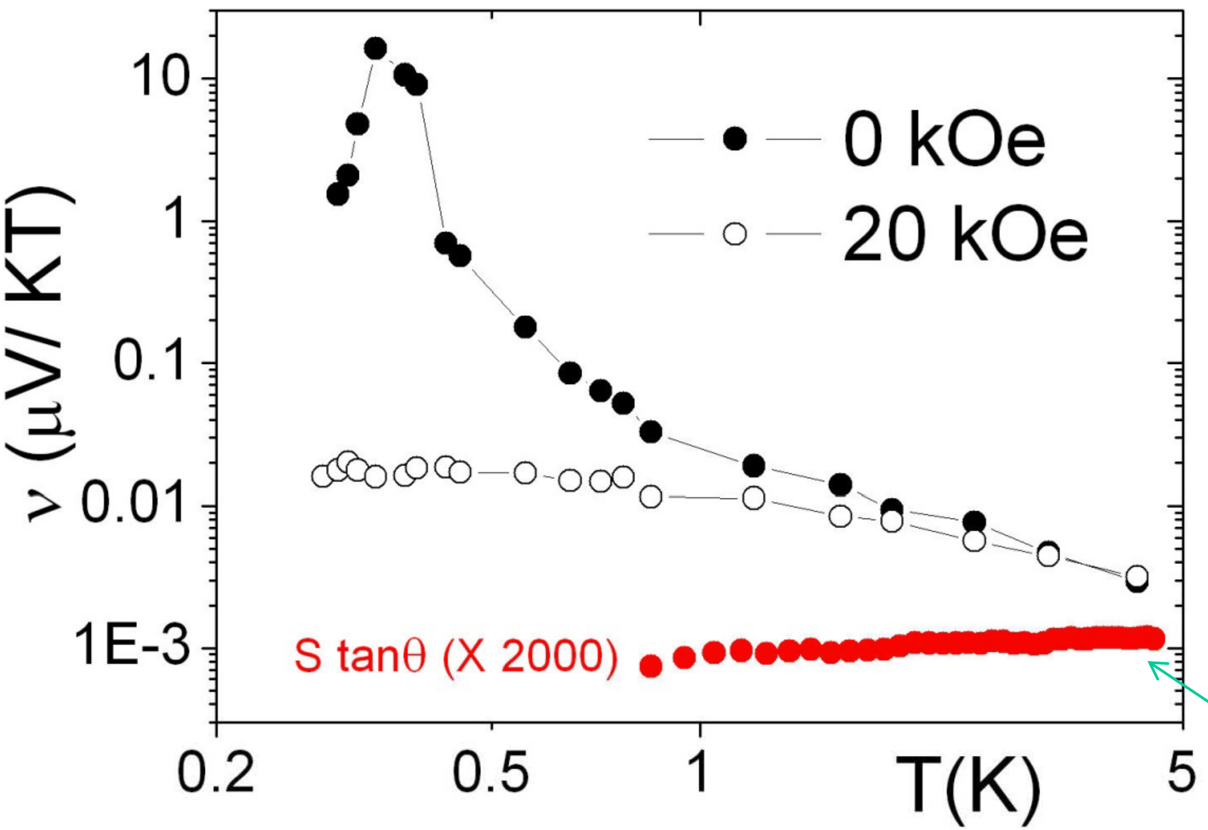


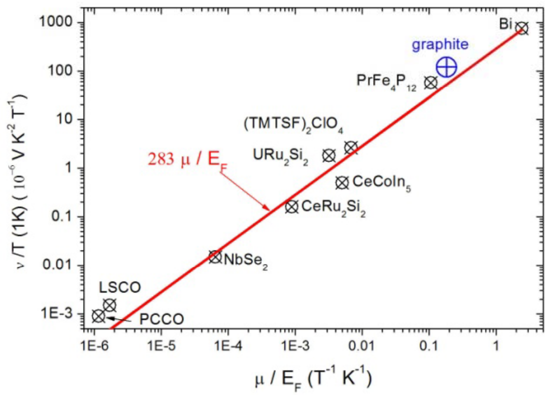
Deep into the normal state!



Can this signal come from normal electrons?

The Nernst signal of the normal electrons is negligible!





$\mu \sim 10^{-5}$ T

$\varepsilon_F \sim 10^4$ K

Even at 6K :
The expected normal state contribution is three orders of magnitude smaller than ν/T !

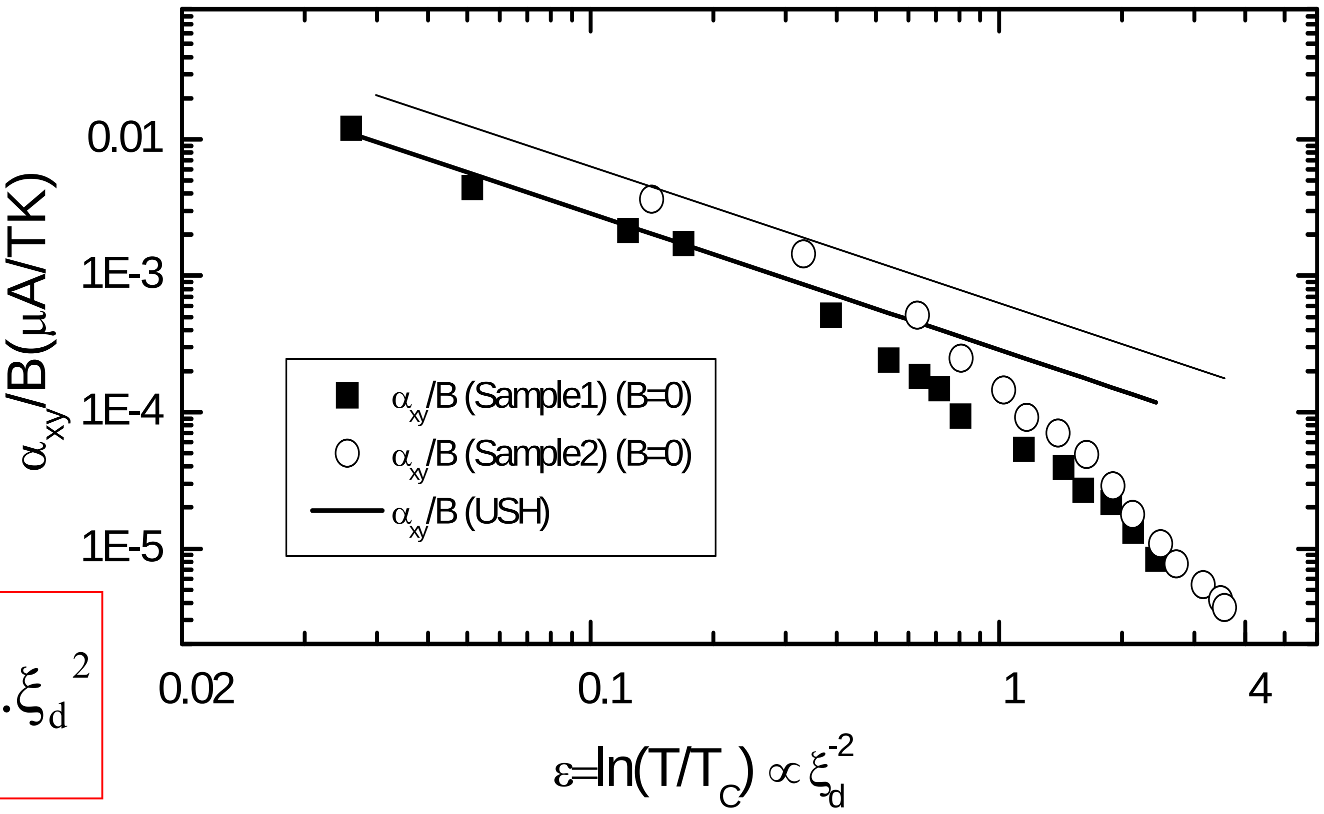
Comparison with theory

Experiment:

$$\frac{\nu}{R_{square}} = \frac{\alpha_{XY}^{SC}}{B}$$

Theory:

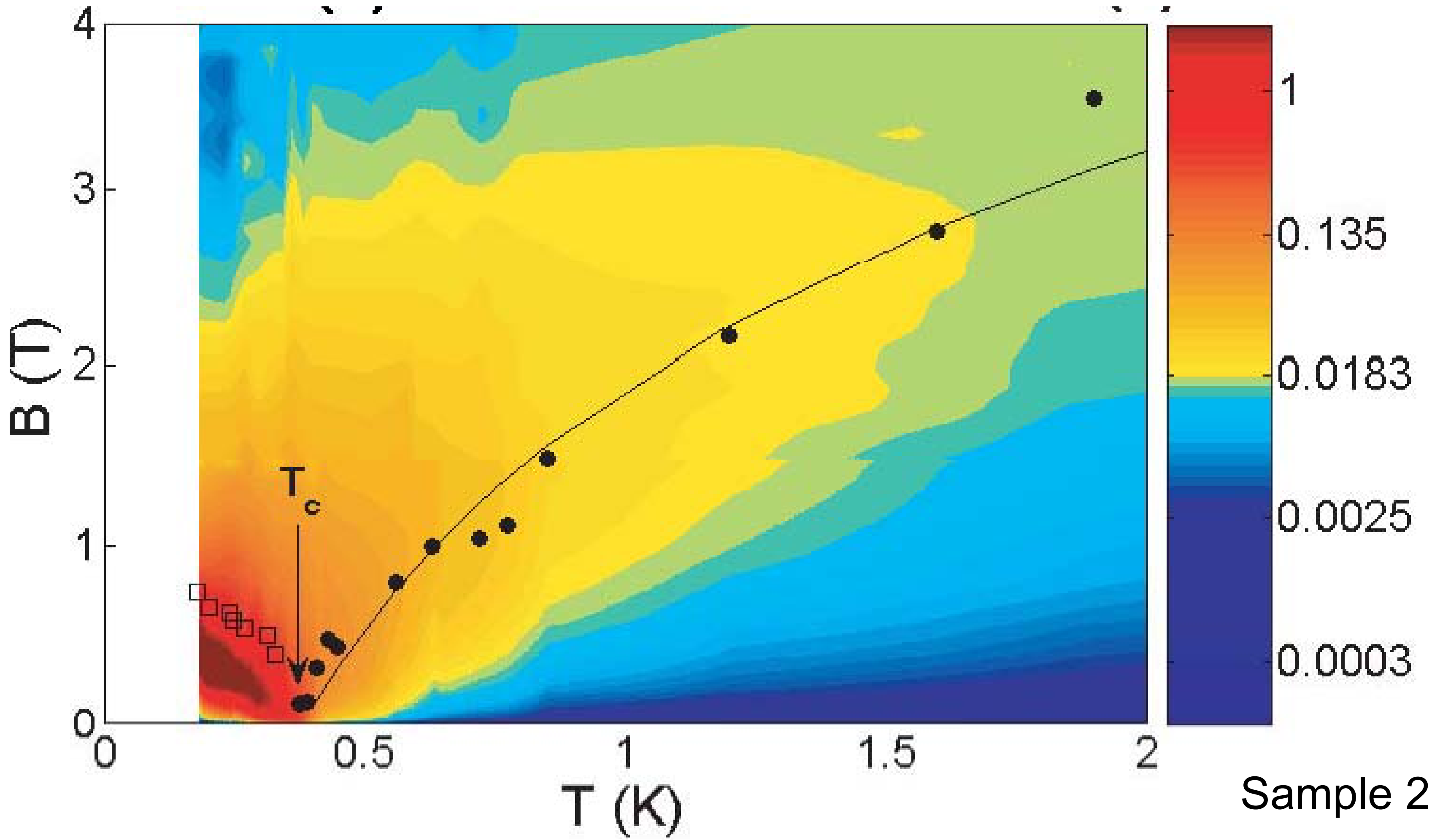
$$\boxed{\frac{\alpha_{xy}}{B}_{USH} = \left(\frac{k_B e^2}{6\pi\hbar^2}\right) . \xi_d^{\,2}}$$



Satisfactory agreement close to T_c

Pourret 2008

The ghost critical field



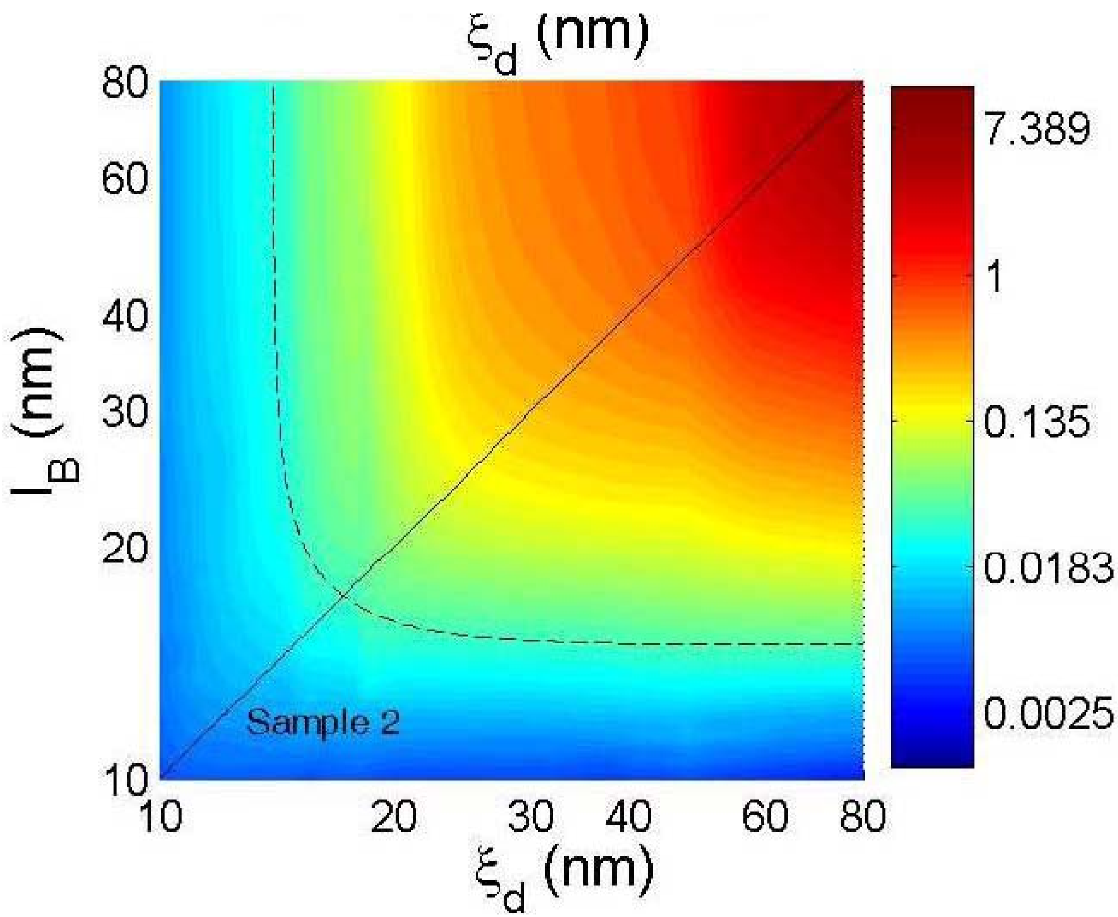
Sample 2

Contour plot of $N = -E_y/(dT/dx)$

A unique correlation length

Contour plot of the Nernst coefficient ν=N/B

Pourret 2008



$$\xi_d = \frac{1}{\sqrt{\varepsilon}} 0.36 \sqrt{\frac{3}{2} \frac{\hbar v_F \ell}{k_B T_c}}$$

$$\epsilon = ln(\frac{T}{T_c})$$

$$l_B = \sqrt{\frac{\hbar}{2eB}}$$