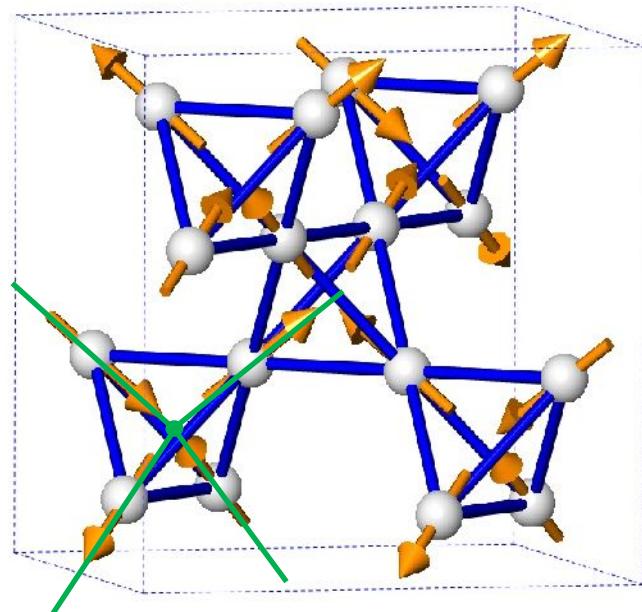
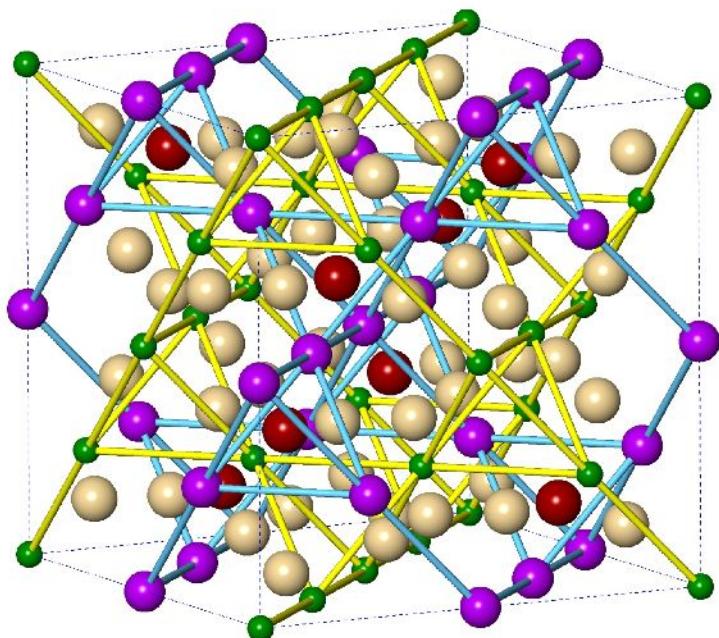


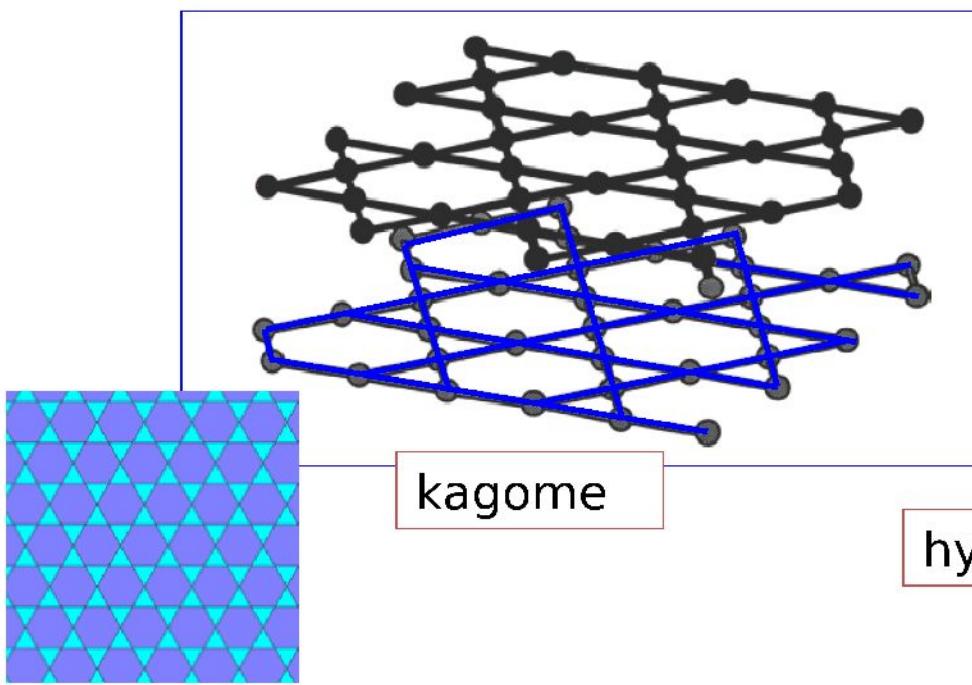
"Oxydes de Pyrochlore Magnétique: Fin de l'Esclavage du Modèle d'Ising"



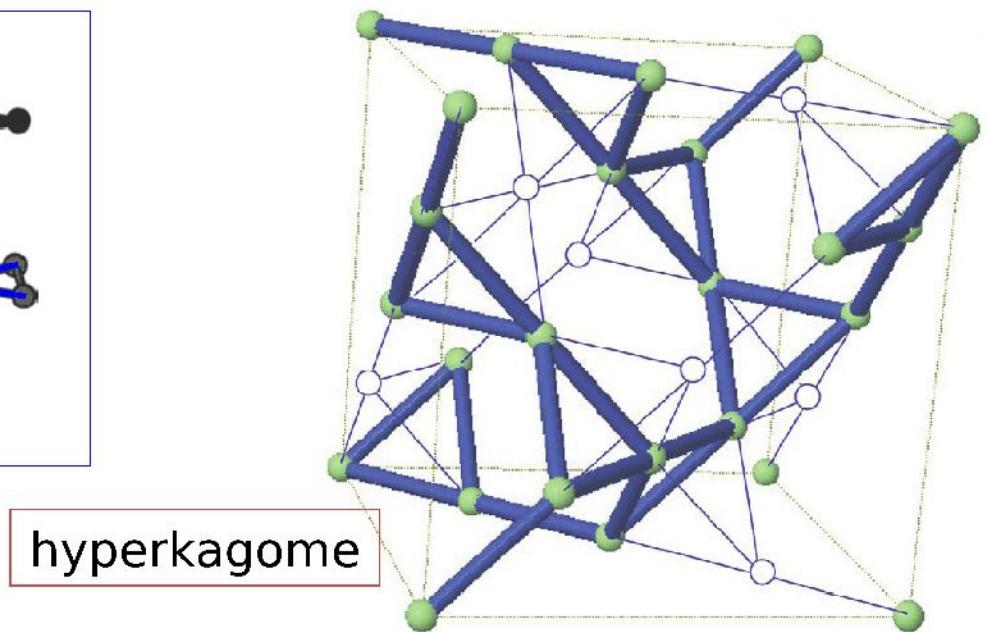
Michel Gingras

*Department of Physics & Astronomy, University of Waterloo, Waterloo, Ontario, Canada
&*

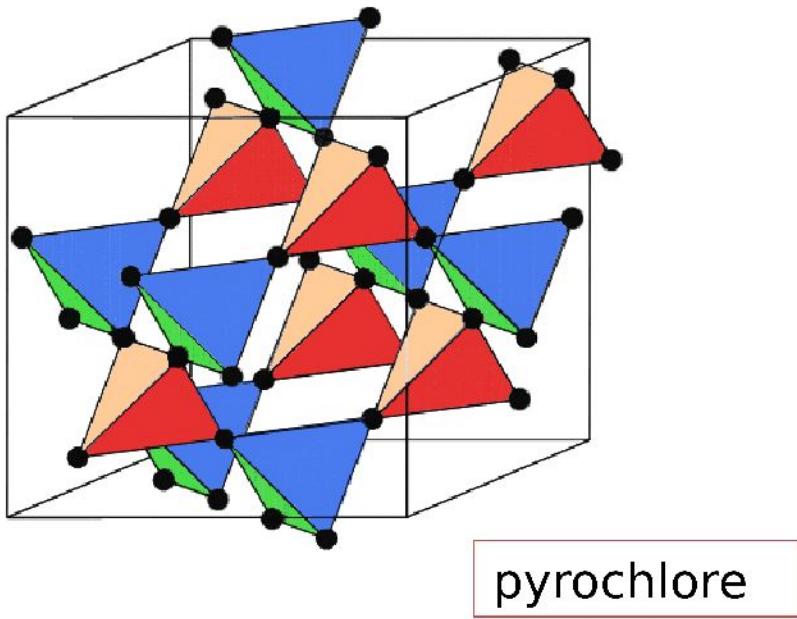
Canadian Institute for Advanced Research/Quantum Materials Program



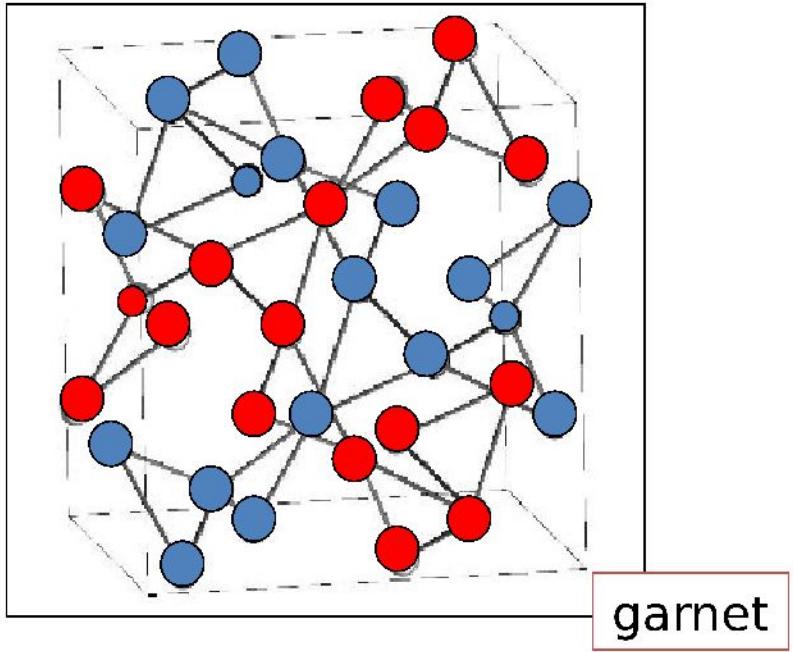
kagome



hyperkagome



pyrochlore



garnet

What is the issue at stake? (I)

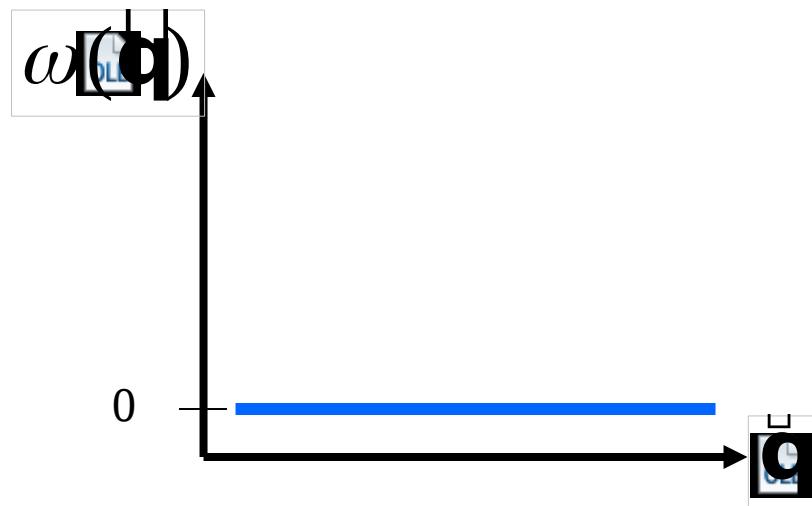
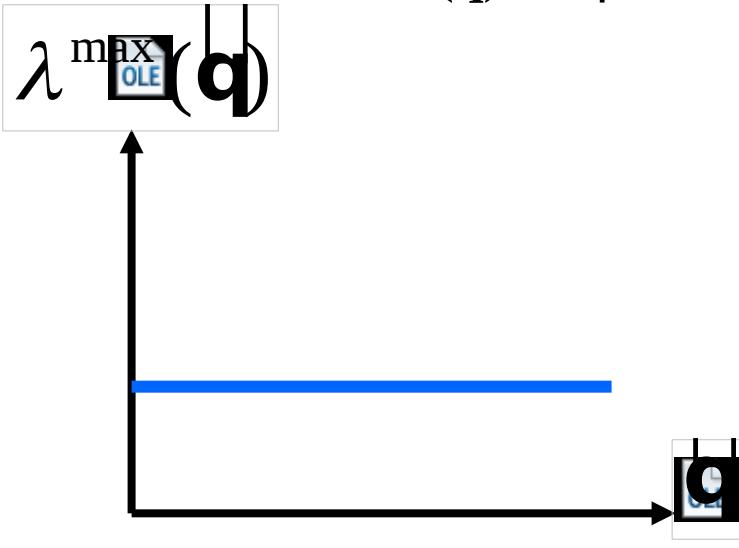
1) Mean-field: staggered susceptibility  $\chi(\mathbf{q})$

$$\chi(\mathbf{q}) \sim \sum_{\alpha} \frac{1}{1 - \lambda_{\alpha}(\mathbf{q}) / 3T}$$

- $\chi(\mathbf{q}_{\text{ord}})$ is a maximum.
- $\lambda_{\alpha}(\mathbf{q})$ maximum at \mathbf{q}_{ord} , $\lambda_{\max}(\mathbf{q}_{\text{ord}})$.
- $T_{\text{cMF}} = \lambda_{\max}(\mathbf{q}_{\text{ord}})/3$.

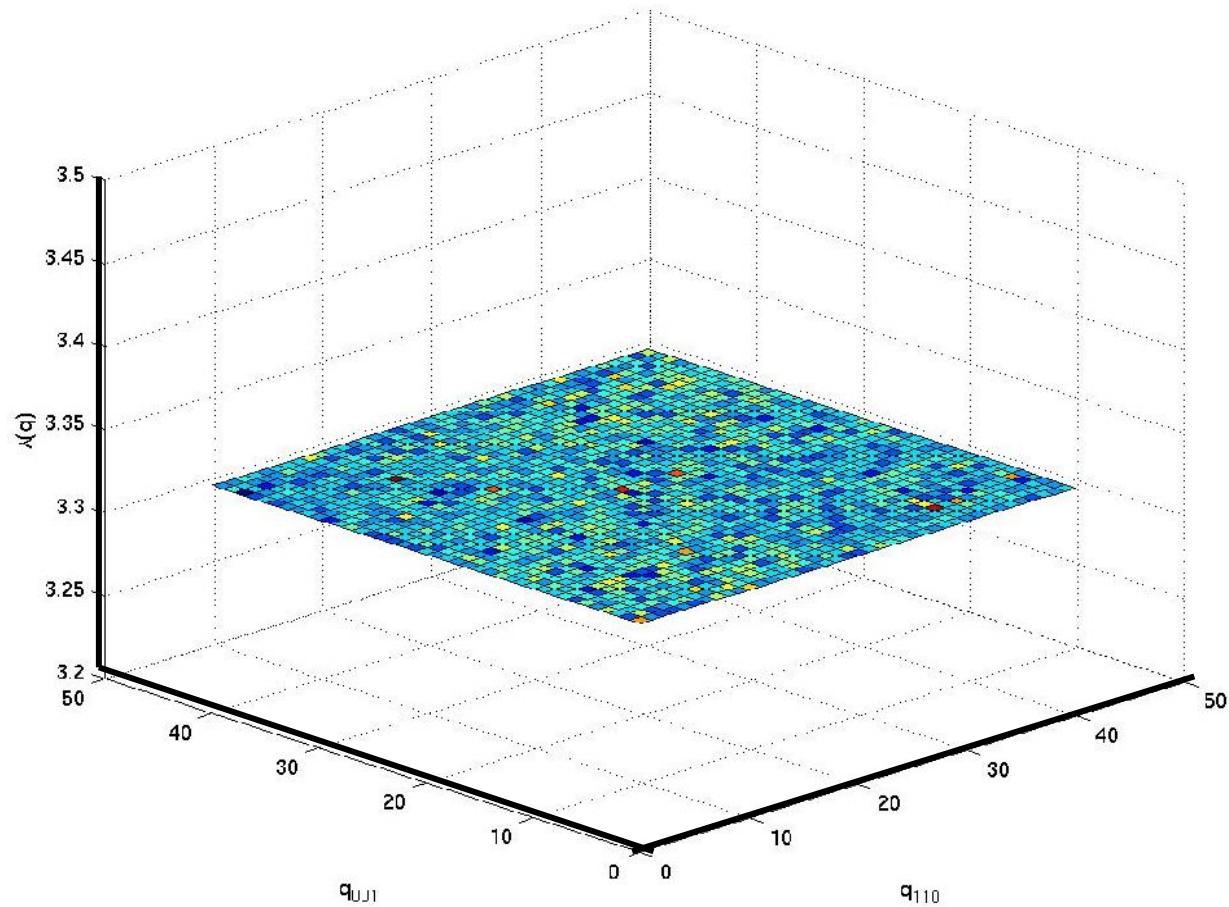
 $\lambda(\mathbf{q}) \sim$ Fourier transform of the spin-spin interactions $J(\mathbf{q})$

2) Excitations $\omega(\mathbf{q})$ ("spin waves") around classical ground states



Example: Nearest-neighbor Ising AF on pyrochlore lattice (or, equivalently, n.n. spin ice)

1nn cut-off



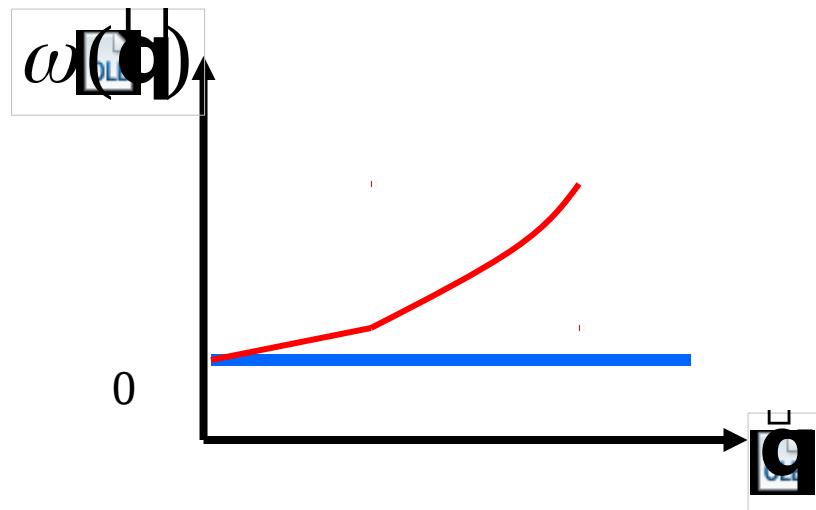
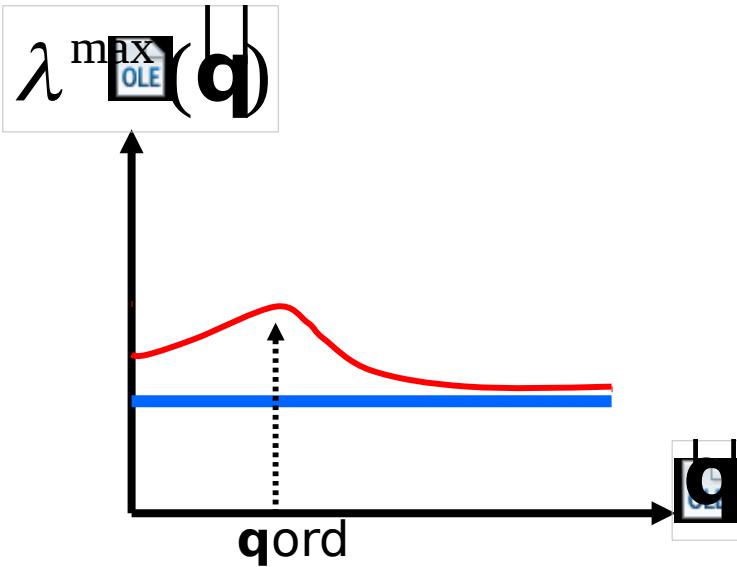
What is the issue at stake?

(II)

$$H = H_0 + H'$$

- H' {
- $J_{ij}(|r_{ij}|)$ beyond nearest neighbor
 - single-ion anisotropy
 - spin-phonon coupling
 - random disorder
 - long-range dipole-dipole interaction

□ In highly frustrated magnets, $1/S$ fights against H'



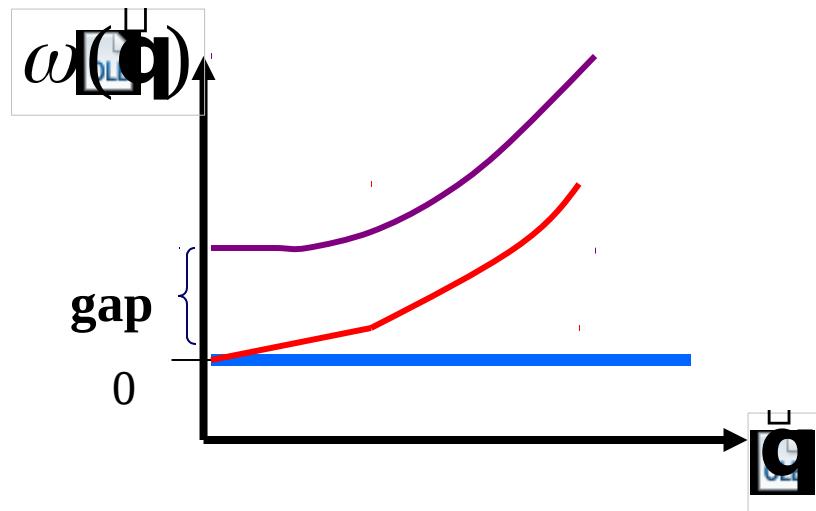
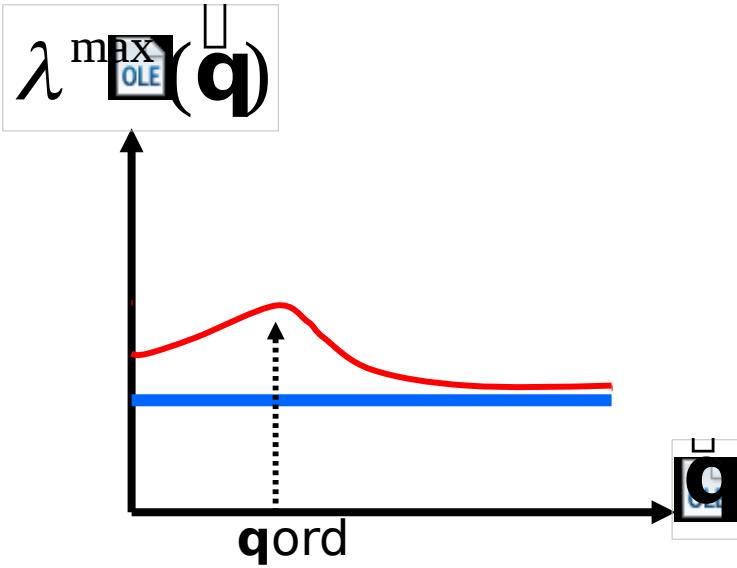
What is the issue at stake?

(II)

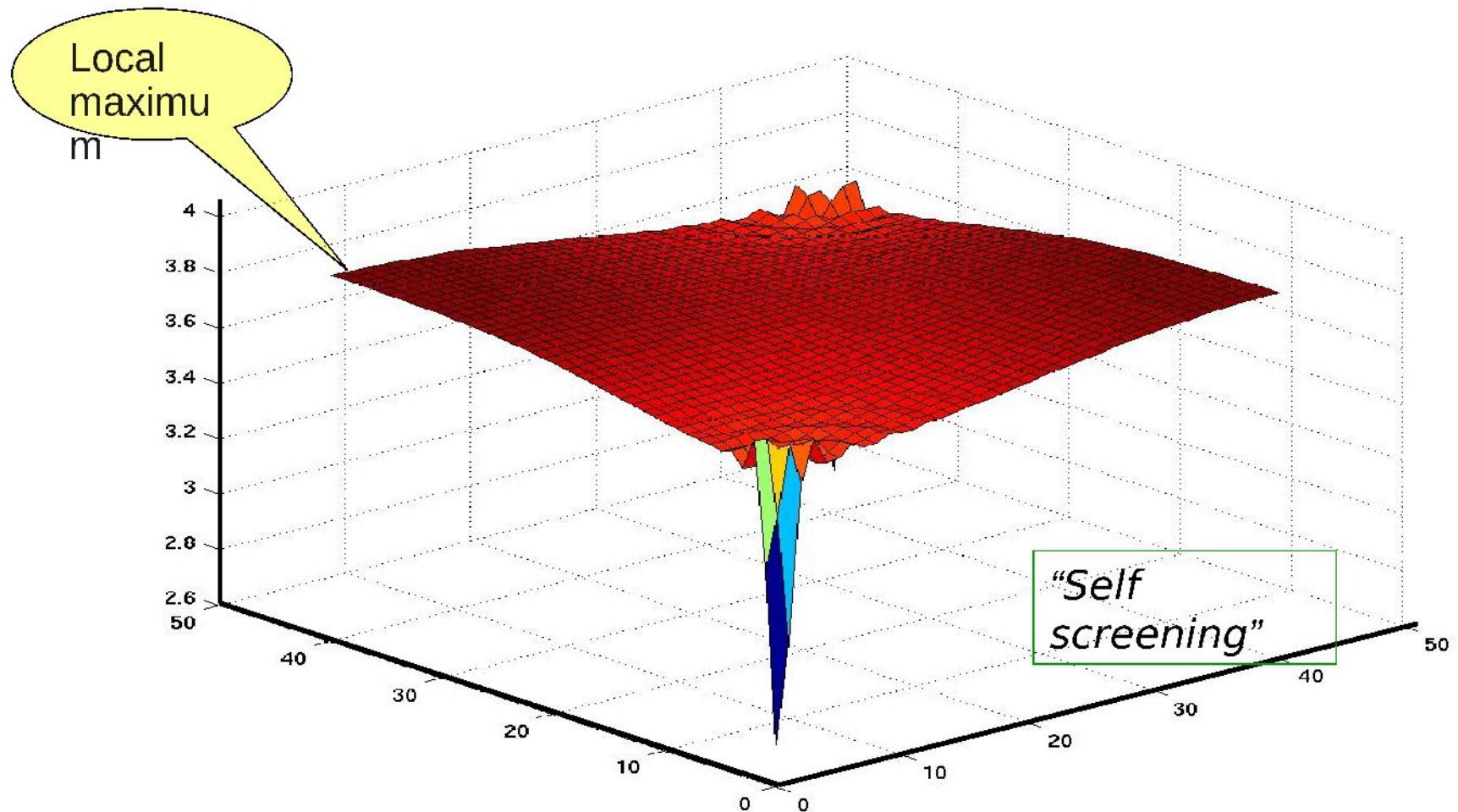
$$H = H_0 + H'$$

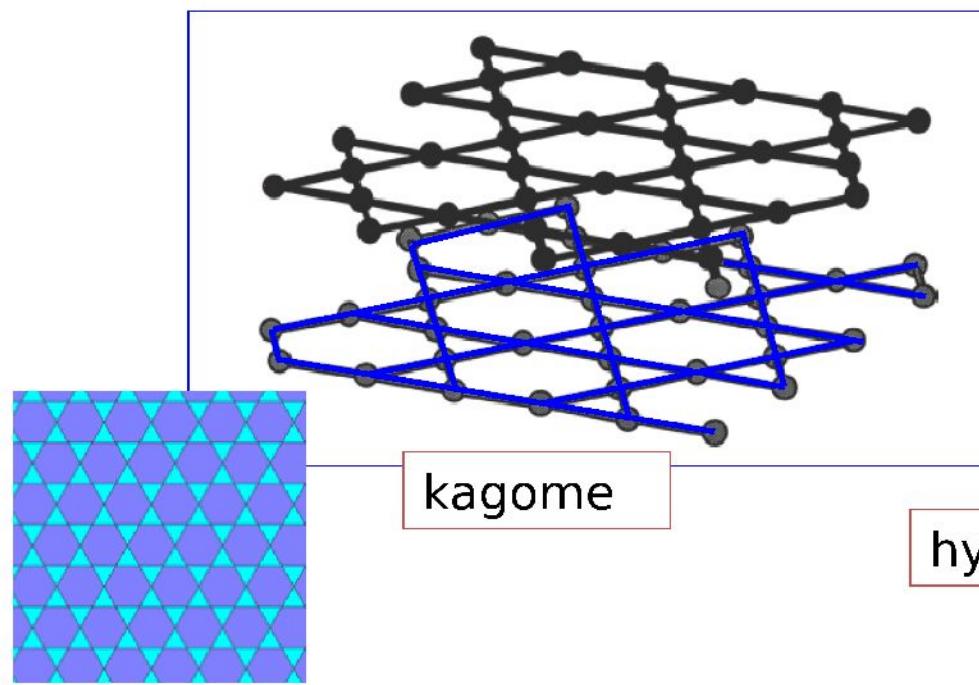
- H' {
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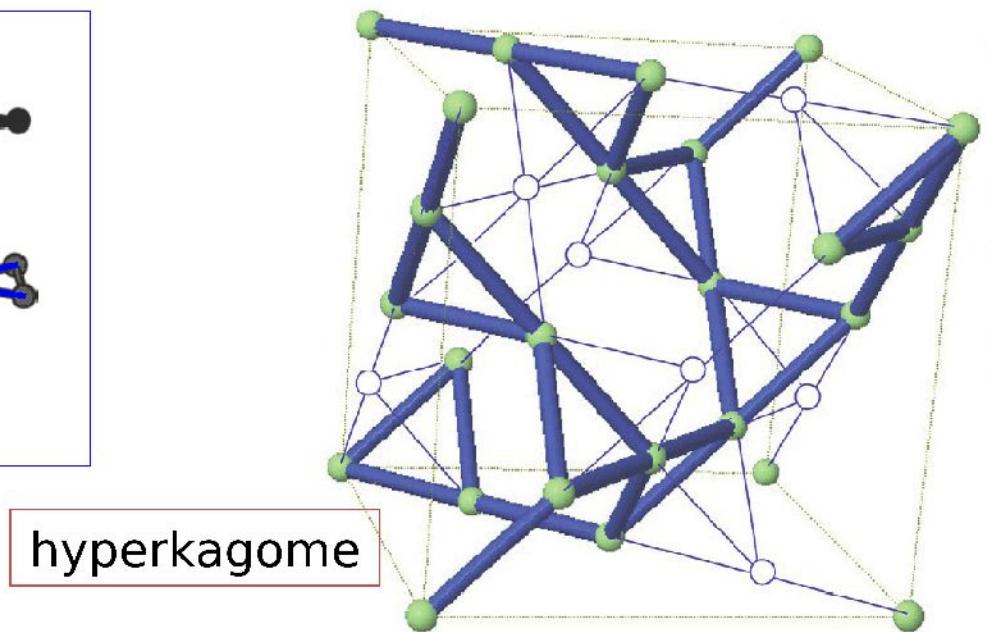


Example: Long-range dipolar spin ice

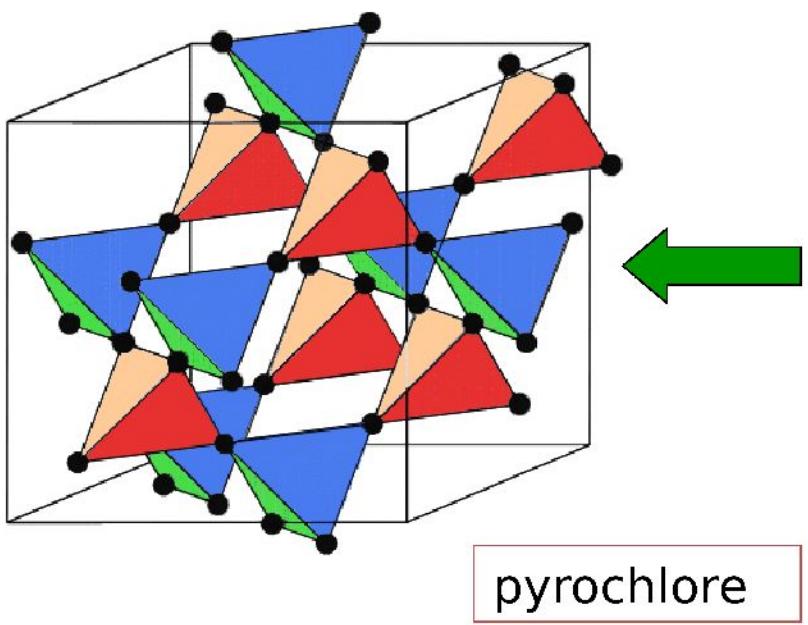




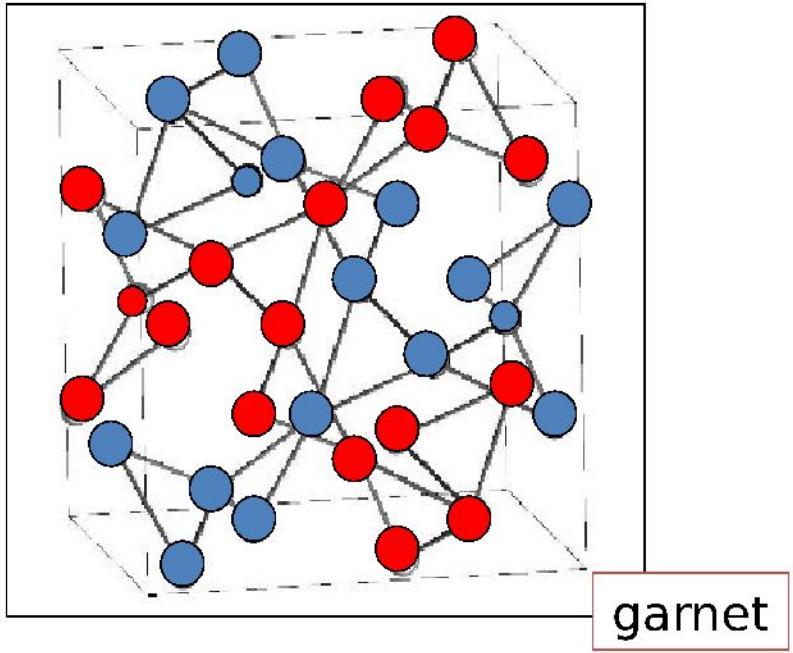
kagome



hyperkagome



pyrochlore



garnet

Outline

1. Frustrated rare-earth pyrochlore oxides

- Hamiltonian, crystal field, effective Hamiltonian

2. Examples of phenomena

- Ising – $(\text{Ho},\text{Dy})_2(\text{Ti},\text{Sn},\text{Ge})_2\text{O}_7$: spin ice
- Ising – $\text{Tb}_2\text{Ti}_2\text{O}_7$: spin ice/quadrup. fluct.
- Heisenberg" – $\text{Gd}_2\text{Ti}_2\text{O}_7$: multiple transitions
- XY AF – $\text{Er}_2\text{Ti}_2\text{O}_7$: order-by-disorder
- XY FM – $\text{Yb}_2\text{Ti}_2\text{O}_7$: quantum spin ice (?)

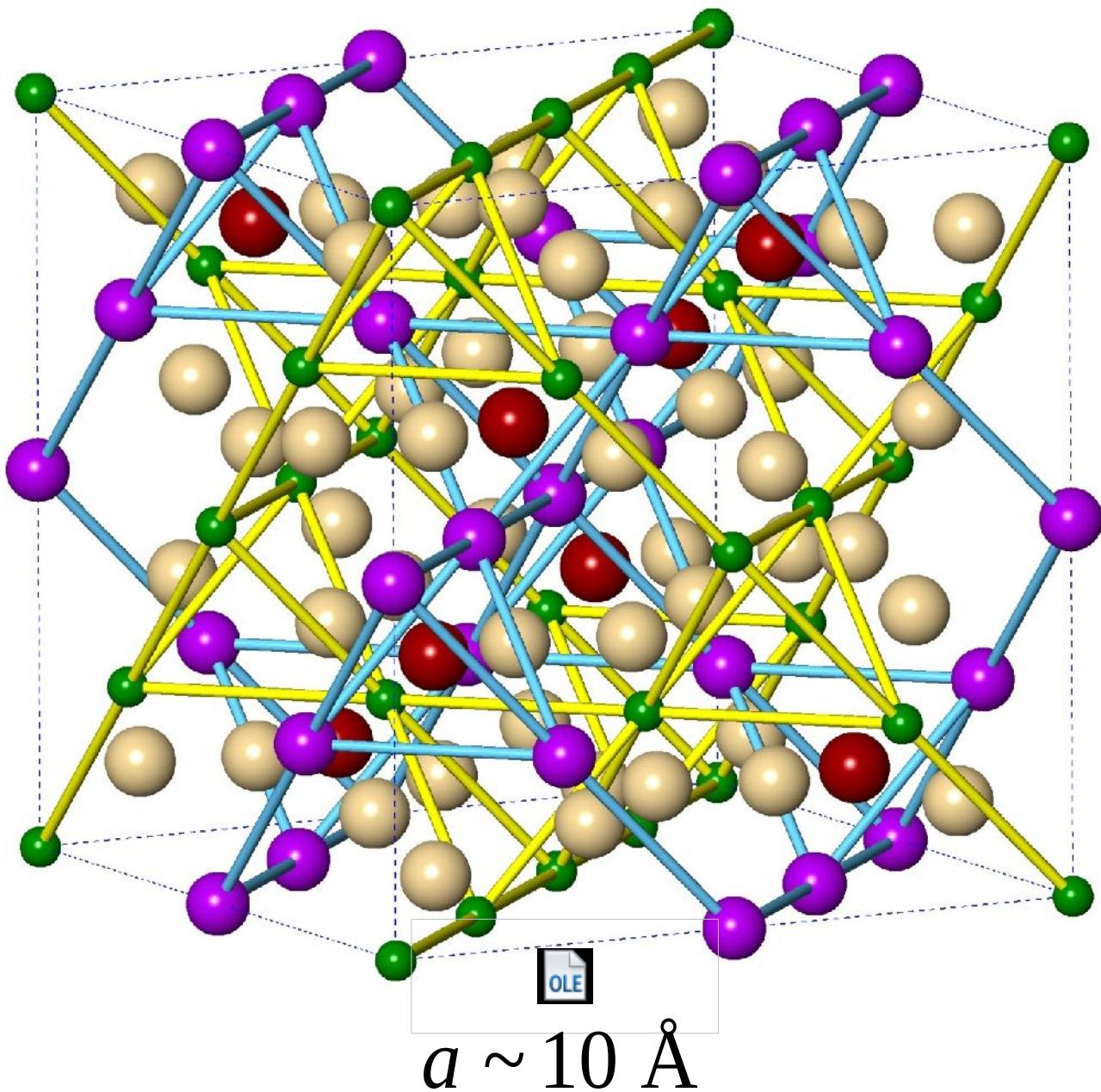
3. Conclusion

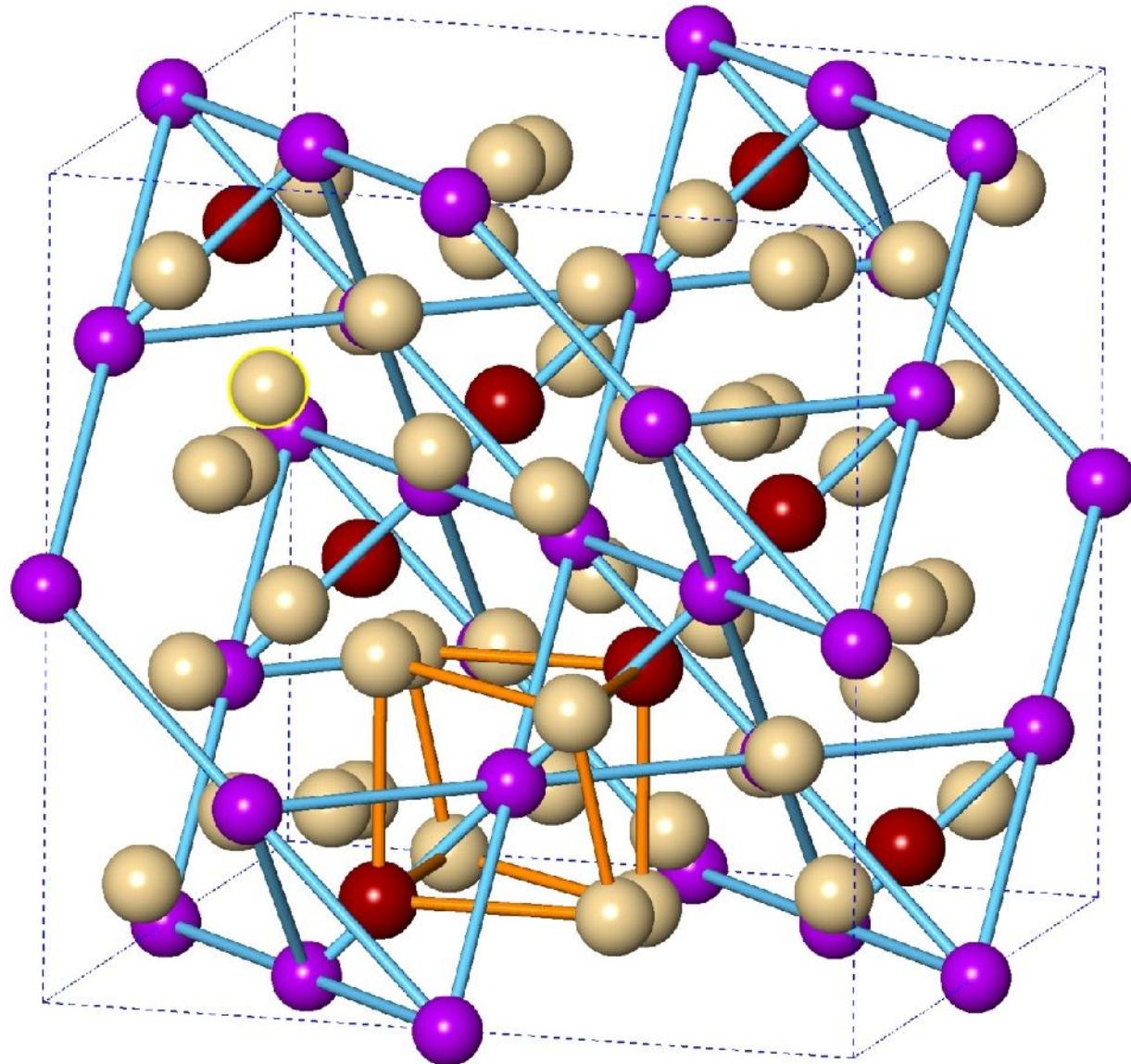
Smorgasbord of Phenomena



Magnetic pyrochlore oxides

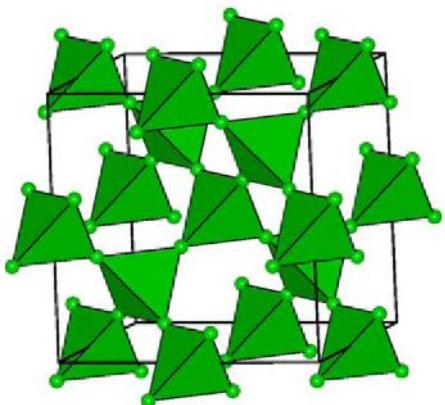
Jason S. Gardner, Michel J. P. Gingras, and John E. Greedan
Rev. Mod. Phys. **82**, 53 (2010).



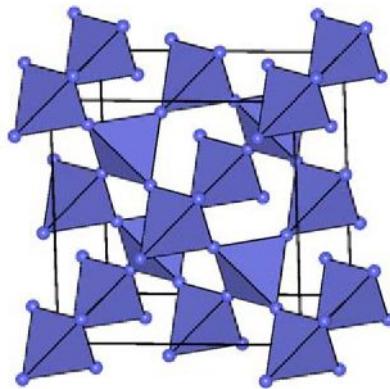


x
y
z

A- Site



B- Site

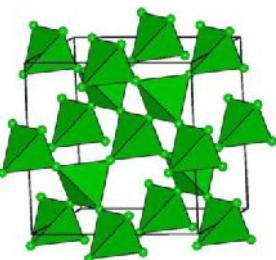


A₂B₂O₇

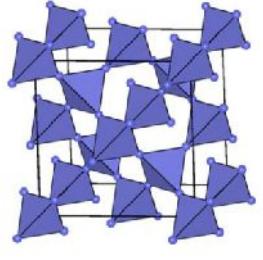
Possible A-site elements
and B site elements

		18/VIII																											
		2 He 4.003																											
		10 Ne 20.18																											
1	2	5	6	7	8	9	10	11	12	13/III	14/IV	15/V	16/VI	17/VII	18/VIII	19	20	21	22										
Li	Be	B	C	N	O	F	Ne	Ar	Ar	Al	Si	P	S	Cl	Kr	Sc	Ti	V	Cr	Mn									
6.941	9.012	10.81	12.01	14.01	16.00	19.00	20.18	20.80	21.80	26.98	28.09	30.97	32.07	35.45	39.95	39.10	40.08	44.96	47.88	50.94									
Na	Mg	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29									
22.99	24.30	22.99	24.30	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40									
K	Ca	Ca	Sc	Sc	Sc	Sc	Sc	Sc	Sc	Sc	Sc	Sc	Sc	Sc	Sc	Sc	Sc	Sc	Sc	Sc									
39.10	40.08	40.08	44.96	44.96	44.96	44.96	44.96	44.96	44.96	44.96	44.96	44.96	44.96	44.96	44.96	44.96	44.96	44.96	44.96	44.96									
Rb	Sr	Rb	Y	Y	Zr	Zr	Nb	Nb	Nb	Ru	Ru	Ru	Ru	Ru	Ru	Ru	Ru	Ru	Ru	Ru									
85.47	87.62	87.62	88.91	88.91	91.22	91.22	92.91	95.94	98.91	101.1	102.9	106.4	107.9	112.4	114.8	118.7	121.8	127.6	131.3	136.9									
Cs	Ba	Ba	La	La	Hf	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn									
132.9	137.3	137.3	178.6	178.6	180.9	180.9	183.8	186.2	186.2	190.2	192.2	195.1	197.0	200.6	204.4	207.2	209.0	210.0	210.0	222.0									
Fr	Ra	Ra	Ac	Unq	104	105	106	107	108	109	Uno	Une									
223.0	226.0	226.0	Lanthanides	Actinides	s block	d block									
57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71
La	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu	La	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu
138.9	140.1	140.9	144.2	144.9	150.4	152.0	157.2	158.9	162.5	164.9	167.3	168.9	173.0	175.0	138.9	140.1	140.9	144.2	144.9	150.4	152.0	157.2	158.9	162.5	164.9	167.3	168.9	173.0	175.0
89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	Ac	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr
227.0	232.0	231.0	238.0	237.0	239.1	243.1	247.1	247.1	252.1	252.1	257.1	256.1	259.1	260.1	227.0	232.0	231.0	238.0	237.0	239.1	243.1	247.1	247.1	252.1	252.1	257.1	256.1	259.1	260.1
f block																													

A-Site

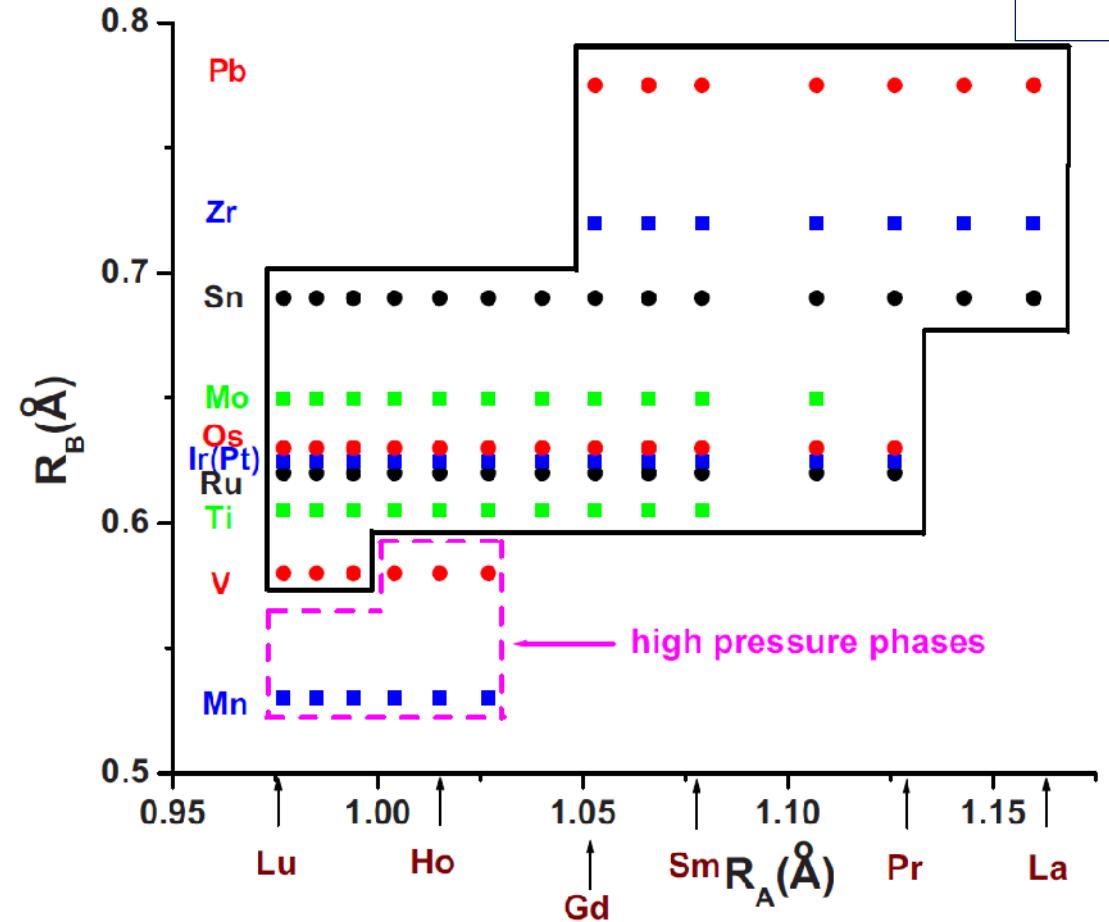


B-Site

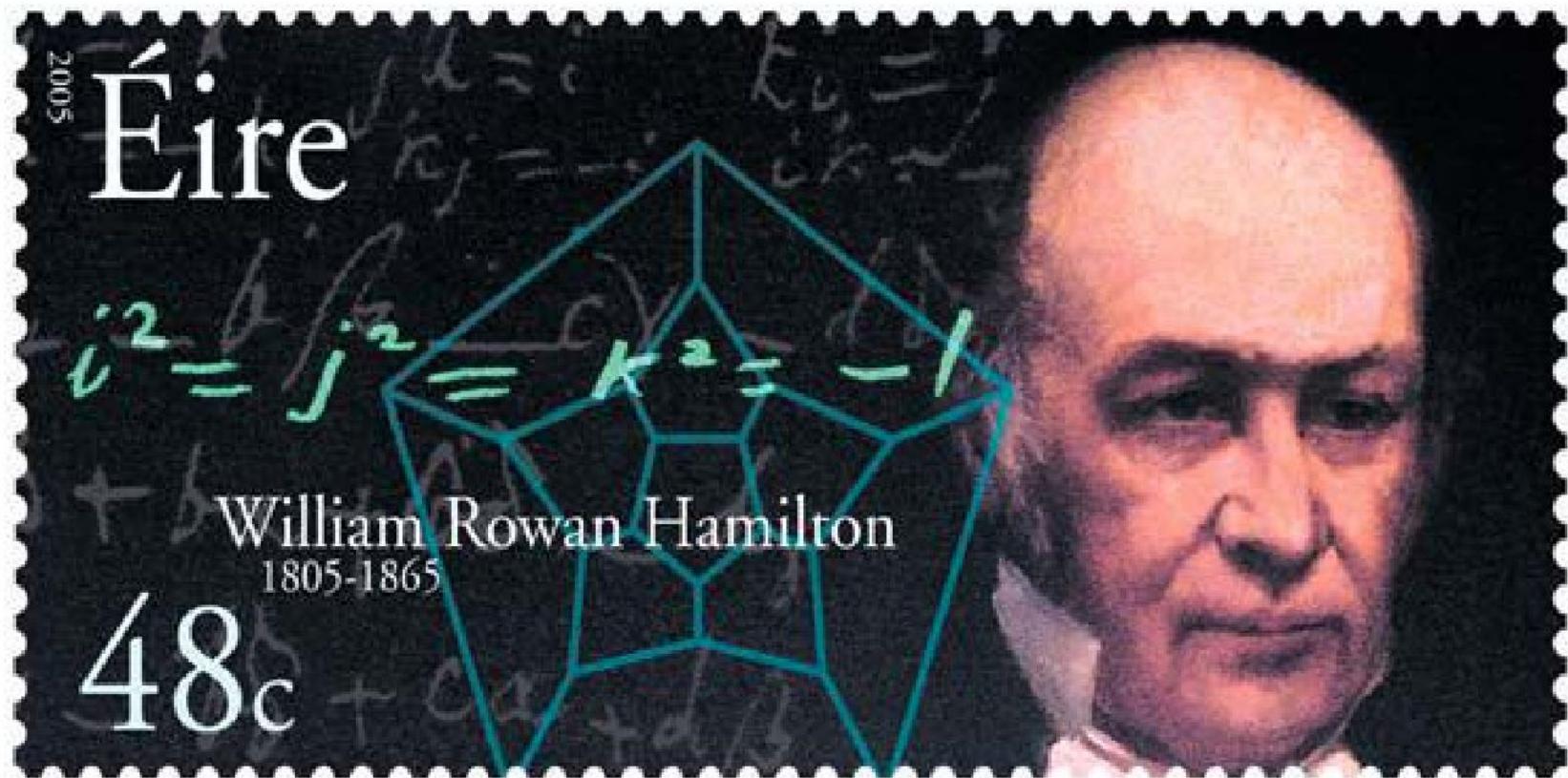


Possible A-site elements
and B site elements

		18/VII																			
		1/H		13/VIII		14/V		15/V		16/VI		17/VII		He 4.003							
Period	Group	1		2		3		4		5		6		7		8		9		10	
		Li	B	Be	C	Sc	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	O	Ge	As	Sb	Br	Kr	
2	3	Li	B	Be	C	Sc	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	O	Ge	As	Sb	Br	Kr	
3	4	Na	Mg	Al	Si	Sc	Ti	Cr	Mn	Fe	Co	Ni	Cu	Zn	O	Ge	As	Sb	Br	Kr	
4	5	K	Ca	Al	Si	Sc	Ti	Cr	Mn	Fe	Co	Ni	Cu	Zn	O	Ge	As	Sb	Br	Kr	
5	6	Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe	He	
6	7	Cs	Ba	La	Hf	Tb	V	W	Pt	Pt	Au	Hg	Tl	Pb	Bi	Pb	Bi	At	Rn		
7	8	Fr	Ra	Ac	Lu	Ho	Er	Pr	Sm	Eu	Dy	Tb	Ho	Lu	Lu	Lu	Lu	Lu	Lu	Lu	



HAMILTONIAN ... HAMILTONIAN ... HAMILTONIAN



What are we talking about?

For rare-earth RE³⁺ ions one typically has:

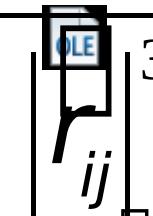
$$H_{\text{Spin-orbit}} \gg H_{\text{crystal-field}} \gg H_{\text{interactions}}$$

A diagram illustrating the addition of angular momentum. It shows two vectors, L and S , represented by arrows originating from the same point. A blue arrow points down to the vector sum $J = L + S$. A red brace groups the vectors L and S , indicating they are being added together.

is a good quantum number

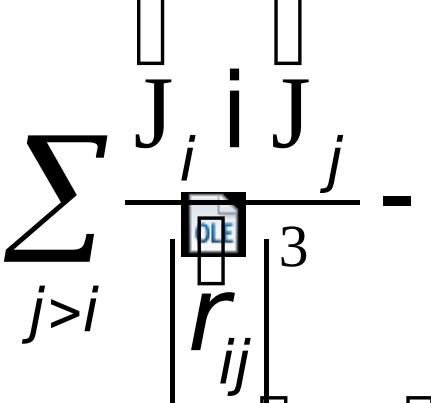
- Free (in vacuum) RE³⁺ ions have $2J+1$ fold degenerate groundstate state
- “Environment” (crystal-field) lift that degeneracy

What are we talking about?

$$H = - \sum_{j>i} I_{ij} \langle J_i | J_j \rangle$$
$$+ \frac{\mu_0}{4\pi} (g\mu_B)^2 \sum_{j>i} \frac{\langle J_i | J_j \rangle}{|r_{ij}|^3} - \frac{3(\langle J_i | r_{ij} \rangle)(\langle J_j | r_{ij} \rangle)}{|r_{ij}|^5}$$
$$+ V_{CF}(J_i^\alpha) - g\mu_B \sum_i \langle J_i | B \rangle$$


Crystal field part of H . This is a single-particle part of the Hamiltonian. It describes how the local electrostatic/chemical environment lifts the otherwise $(2J+1)$ degeneracy of the otherwise free rare-earth ion.

What are we talking about?

$$H = - \sum_{j>i} I_{ij}^{\alpha\beta} J_i^\alpha J_j^\beta$$
$$+ \frac{\mu_0}{4\pi} (g\mu_B)^2 \sum_{j>i} \frac{J_i \cdot J_j}{|r_{ij}|^3} - \frac{3(J_i \cdot r_{ij})(J_j \cdot r_{ij})}{|r_{ij}|^5}$$
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What are we talking about?

$$H = \sum_{i>j} \sum_{K,K'} \sum_{Q,Q'} I_{KK'}^{QQ'} O_K^Q(J_i) O_{K'}^{Q'}(J_j)$$

where $K \leq 2J_i$, $K' \leq 2J_j$;

$$Q = \boxed{K, K} \quad Q' = \boxed{-K', K'}$$

Constraints on the $I_{KK'}^{QQ'}$ coefficients
are imposed by symmetry.

Moment	Symmetry	Operator
Dipoles	Γ_4	J_x
		J_y
		J_z
Quadrupoles	Γ_3	$O_{3z^2-r^2} = 3J_z^2 - J(J+1) \equiv \hat{O}_2^0$
		$O_{x^2-y^2} = J_x^2 - J_y^2 \equiv \hat{O}_2^2$
	Γ_5	$O_{xy} = \overline{J_x J_y}/2 \equiv \hat{O}_2^{-2}$
		$O_{yz} = \overline{J_y J_z}/2 \equiv \hat{O}_2^{-1}$
		$O_{zx} = \overline{J_z J_x}/2 \equiv \hat{O}_2^1$
		$T_{xyz} = (\sqrt{15}/6) \overline{J_x J_y J_z}$
Octupoles	Γ_2	$T_x^\alpha = J_x^3 - (\overline{J_x J_y^2} + \overline{J_z^2 J_x})/2$
		$T_y^\alpha = J_y^3 - (\overline{J_y J_z^2} + \overline{J_x^2 J_y})/2$
		$T_z^\alpha = J_z^3 - (\overline{J_z J_x^2} + \overline{J_y^2 J_z})/2$
	Γ_4	$T_x^\beta = \sqrt{15} (\overline{J_x J_y^2} - \overline{J_z^2 J_x})/6$
		$T_y^\beta = \sqrt{15} (\overline{J_y J_z^2} - \overline{J_x^2 J_y})/6$
		$T_z^\beta = \sqrt{15} (\overline{J_z J_x^2} - \overline{J_y^2 J_z})/6$
		$T_x^\gamma = \sqrt{15} (\overline{J_x J_y J_z})/6$
		$T_y^\gamma = \sqrt{15} (\overline{J_y J_z J_x})/6$
		$T_z^\gamma = \sqrt{15} (\overline{J_z J_x J_y})/6$

TABLE I. Operator equivalents describing active multipoles within a cubic Γ_8 quartet. Bars over symbols indicate the sum with respect to all the possible permutations of the indices, e.g., $\overline{J_x J_y^2} = J_x J_y^2 + J_y J_x J_y + J_y^2 J_x$. The five \hat{O}_2^Q are the usual Stevens' operator equivalents (see below). Adapted from Shiina *et al.* (1998).

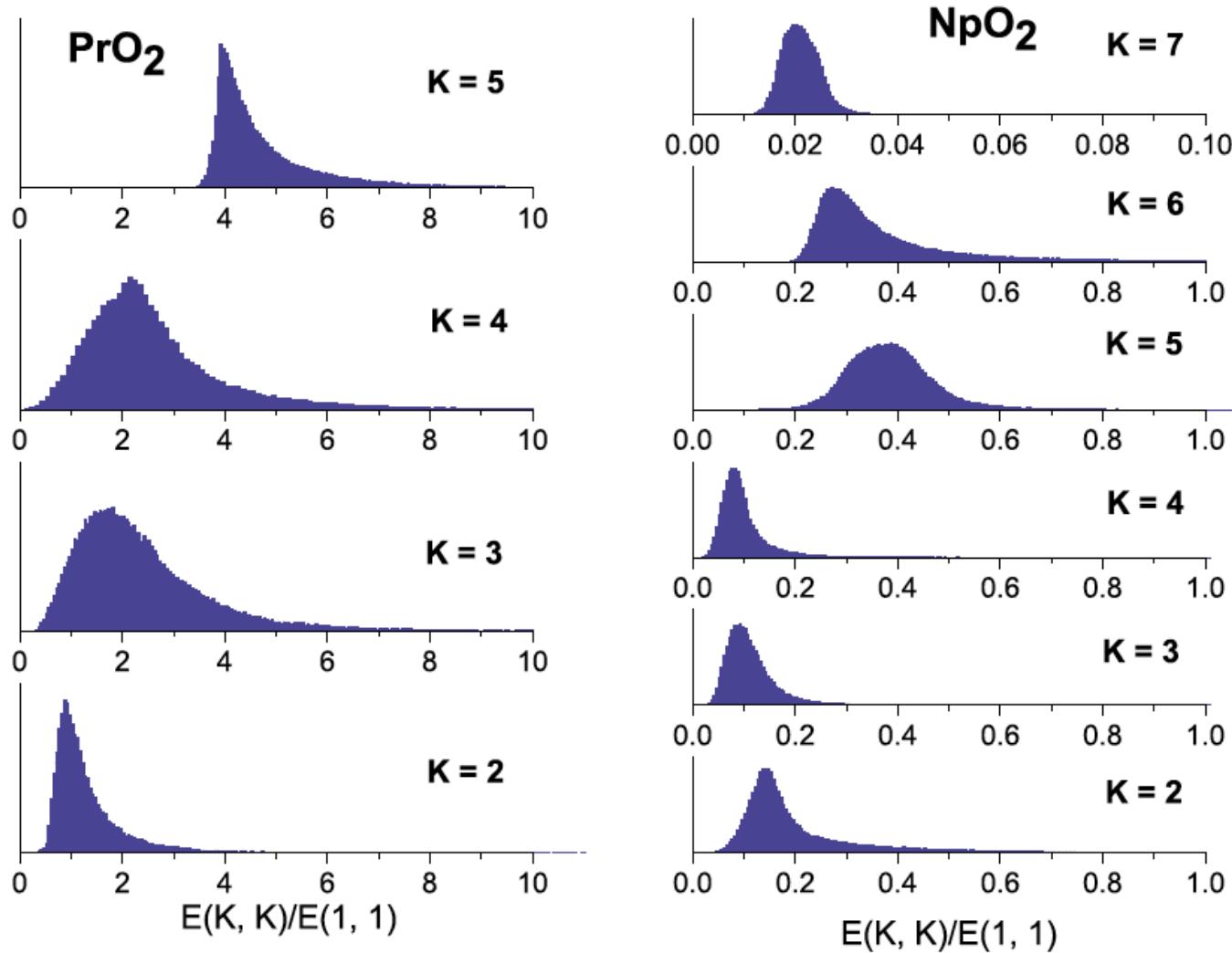
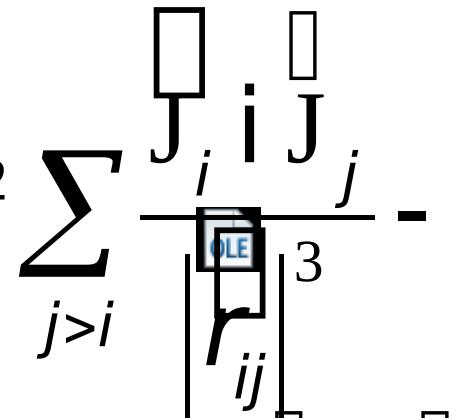


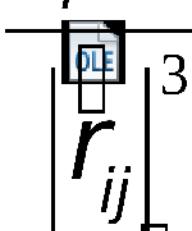
FIG. 2. (Color online) Distribution of the ratio $E(K, K)/E(1, 1)$, with $E(K_i, K_j)$ the ground-state energy of a dimer (C_{2v} bond) of Pr^{4+} and Np^{4+} ions coupled with the part of Eq. (35) associated with a specific pair of ranks K_i, K_j .

What are we talking about?

$$H = - \sum_{j>i} I_{ij}^{\alpha\beta} J_i^\alpha J_j^\beta$$
$$+ \frac{\mu_0}{4\pi} (g\mu_B)^2 \sum_{j>i} \frac{J_i^\alpha J_j^\beta}{|r_{ij}|^3} - \frac{3(J_i^\alpha \mathbf{r}_{ij})(J_j^\beta \mathbf{r}_{ij})}{|r_{ij}|^5}$$
$$+ V_{CF}(J_i^\alpha) - g\mu_B \sum_i J_i^\alpha \mathbf{B}$$


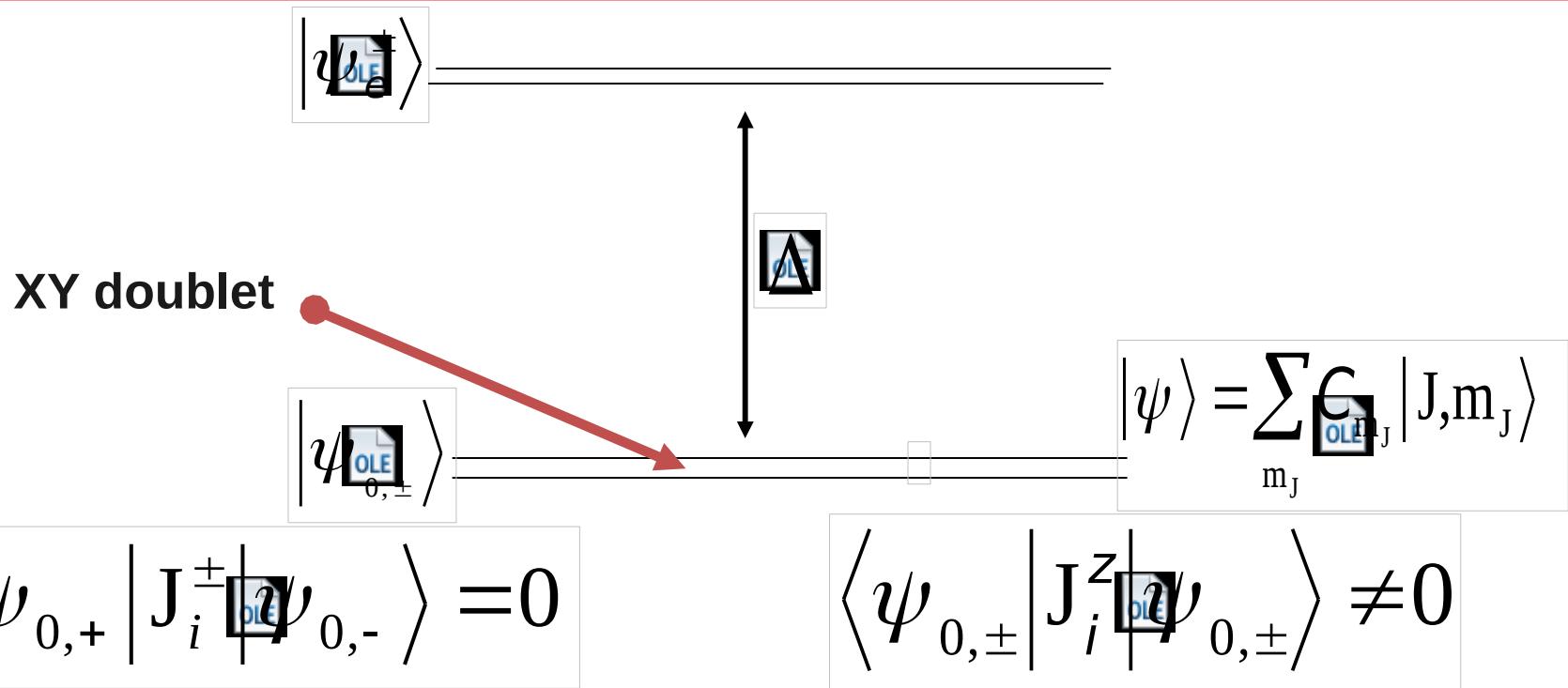
Crystal field part of H . This is a single-particle part of the Hamiltonian. It describes how the local electrostatic/chemical environment lifts the otherwise $(2J+1)$ degeneracy of the otherwise free rare-earth ion.

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$$H = - \sum_{j>i} I_{ij}^{\alpha\beta} J_i^\alpha J_j^\beta$$
$$+ \frac{\mu_0}{4\pi} (g\mu_B)^2 \sum_{j>i} \frac{J_i \cdot J_j}{|r_{ij}|^3} - \frac{3(J_i \cdot r_{ij})(J_j \cdot r_{ij})}{|r_{ij}|^5}$$
$$+ V_{CF}(J_i^\alpha) - g\mu_B \sum_i J_i \cdot B$$


Crystal field part of H . This is a single-particle part of the Hamiltonian. It describes how the local electrostatic/chemical environment lifts the otherwise $(2J+1)$ degeneracy of the otherwise free rare-earth ion.

Crystal field effects in rare-earth pyrochlore oxides



Dy₂Ti₂O₇ & Dy₂Sn₂O₇ : $\Delta \approx 320$ K || spin ice (s.i.)

Ho₂Ti₂O₇ & Ho₂Sn₂O₇ : $\Delta \approx 280$ K || spin ice (s.i.)

Tb₂Ti₂O₇ & Tb₂Sn₂O₇ : $\Delta \approx 20$ K || “soft Ising” ??

Quantum spin ice: Molavian *et al.*: Phys. Rev. Lett. **98**, 157204 (2007); arXiv: 0912.2957;
J. Phys.: Condens. Matter **21**, 172201 (2009).

A comment in anticipation ...

$$H = \sum_{i>j} \sum_{K,K'} \sum_{Q,Q'} I_{KK'}^{QQ'} O_K^Q(\mathbf{J}_i) O_{K'}^{Q'}(\mathbf{J}_j)$$

where $K \leq 2J_i$, $K' \leq 2J_j$;

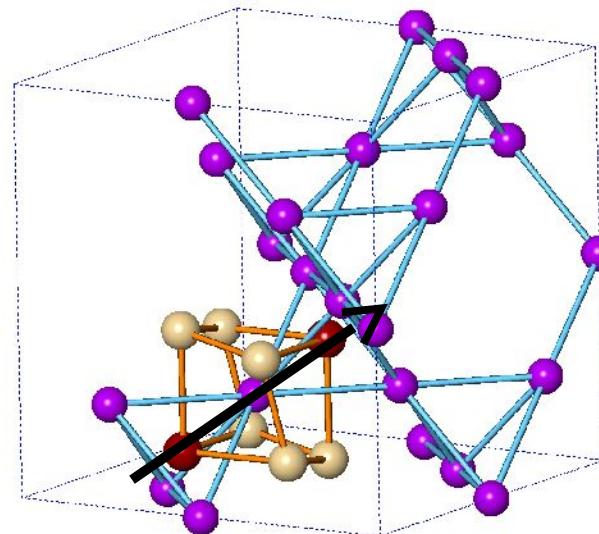
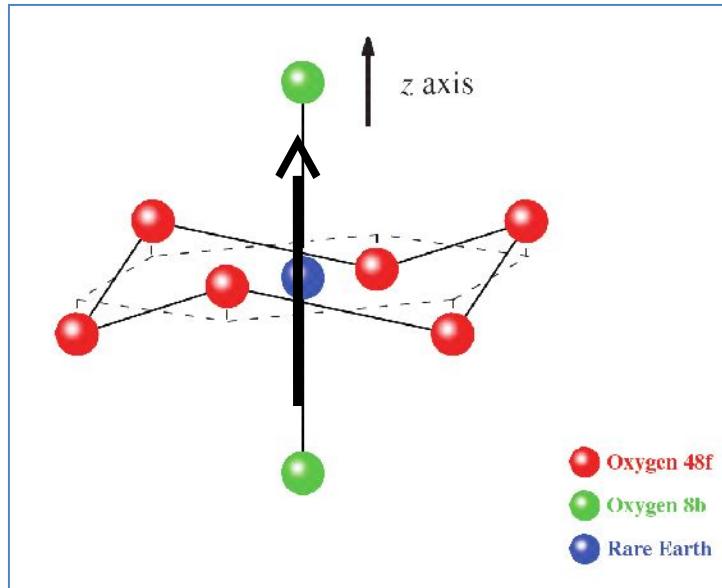
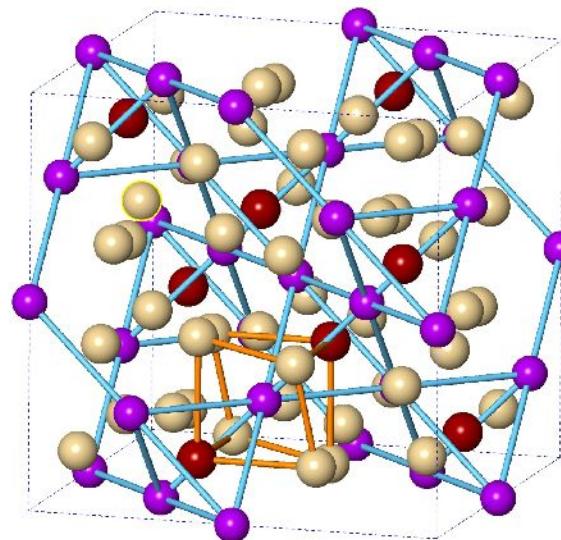
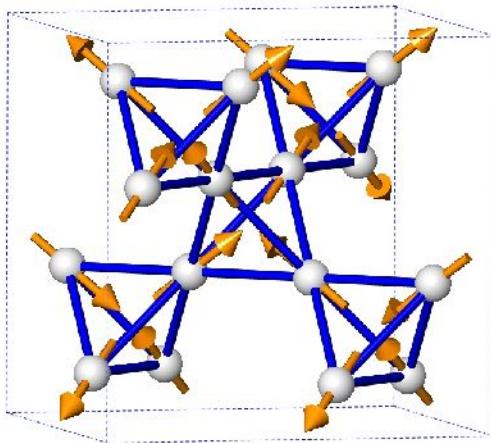
$$Q = \boxed{K, K}; Q' = \boxed{-K', K'}$$

Constraints on the $I_{KK'}^{QQ'}$ coefficients
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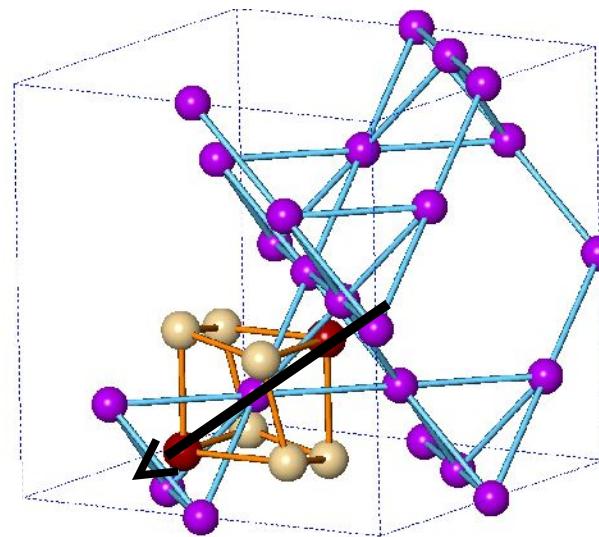
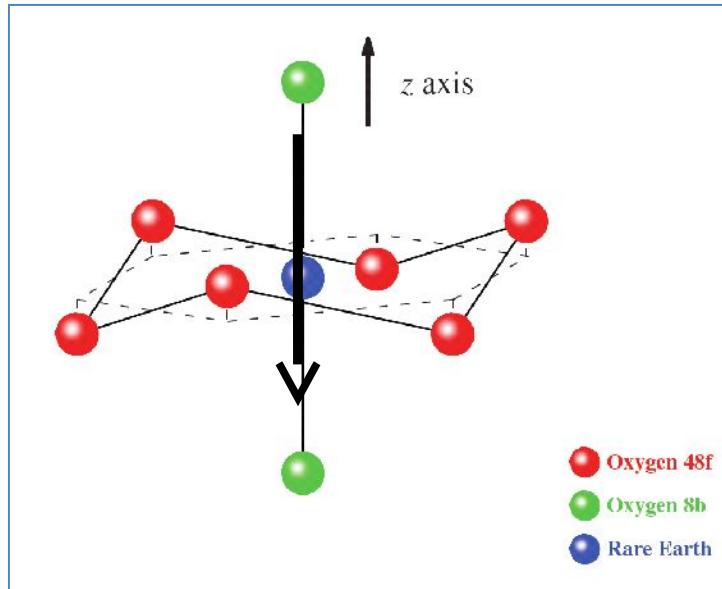
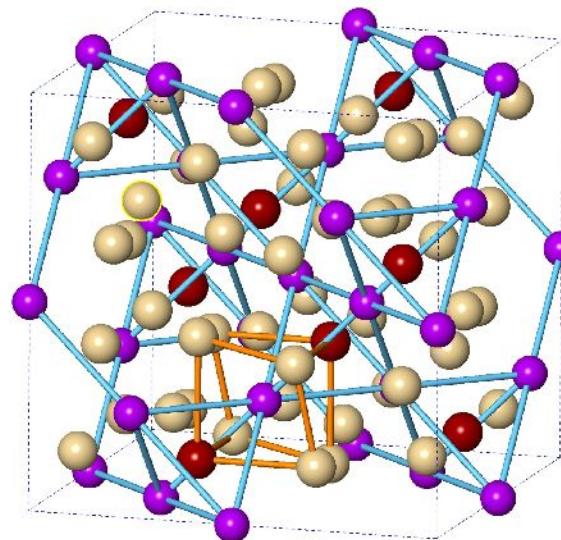
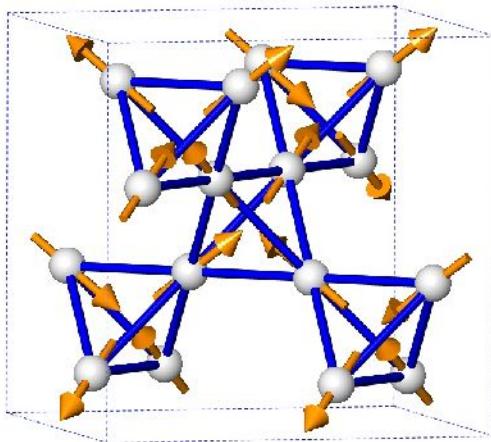
THE POINT IS:

- Quantum fluctuations are “no longer” (not) forbidden as soon as multipolar interactions are considered, even in “classical” Dy₂Ti₂O₇ and Ho₂Ti₂O₇
- This point has been emphasized for Pr₂(Sn,Zr)2O₇ by Onoda and Tanaka [Phys. Rev. B **83**, 094411 (2011)]

Ho₂Ti₂O₇ & Dy₂Ti₂O₇

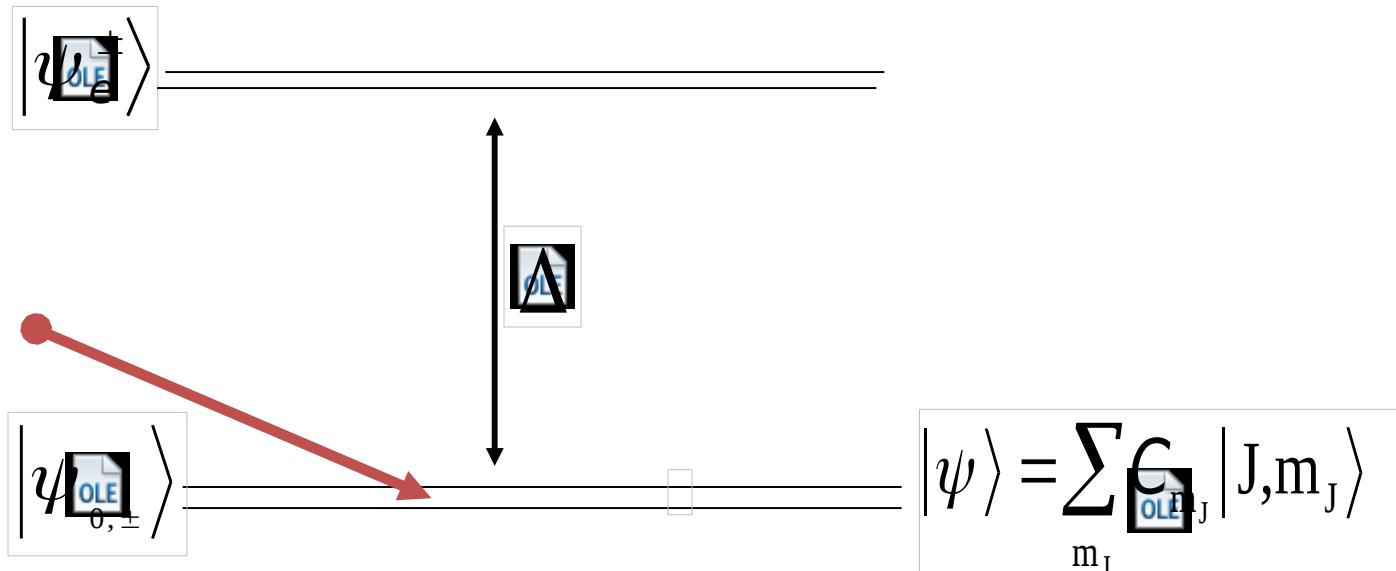


Ho₂Ti₂O₇ & Dy₂Ti₂O₇



Crystal field effects in rare-earth pyrochlore oxides

XY doublet



$$\langle \psi_{0,+} | J_i^{x,y} | \psi_{0,-} \rangle = g_{\perp}$$

$$\langle \psi_{0,\pm} | J_i^z | \psi_{0,\pm} \rangle = g_z$$

Er₂Ti₂O₇ & Er₂Sn₂O₇ : $\Delta \approx 80$ K; AF XY
Yb₂Ti₂O₇ & Yb₂Sn₂O₇ : $\Delta \approx 620$ K; FM XY

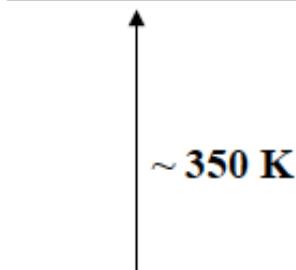
Projection ... Projection ... Projection



$|m_J\rangle$ wavefunction decomposition

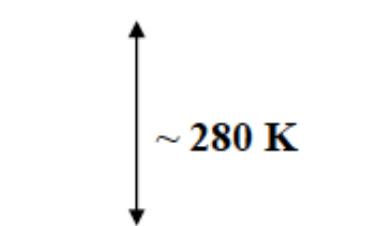
Dy³⁺ ($J=15/2$)

Dy2Ti2O7



$|\pm 15/2\rangle + O(10^{-1})$

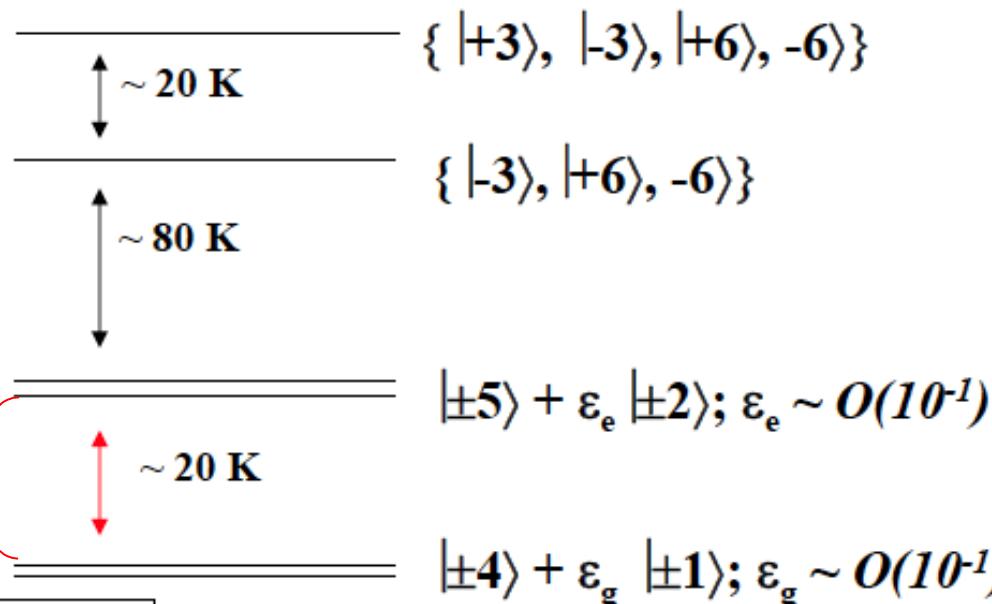
Ho³⁺ ($J=8$)



Ho2Ti2O7

Tb2Ti2O7

Tb³⁺ ($J=6$)
(should be a nonmagnetic singlet, as Tm³⁺ !)



Effective Hamiltonian Method

$$H = H_{\text{cef}} + H_{\text{int}}$$

$$H_{\text{int}} = \sum_{i>j} \sum_{K,K'} \sum_{Q,Q'} I_{KK'}^{QQ'} O_K^Q(J_i) O_{K'}^{Q'}(J_j)$$

$$H_{\text{eff}} = P H_{\text{int}} P + P H_{\text{int}} Q H_{\text{int}} P + \dots$$

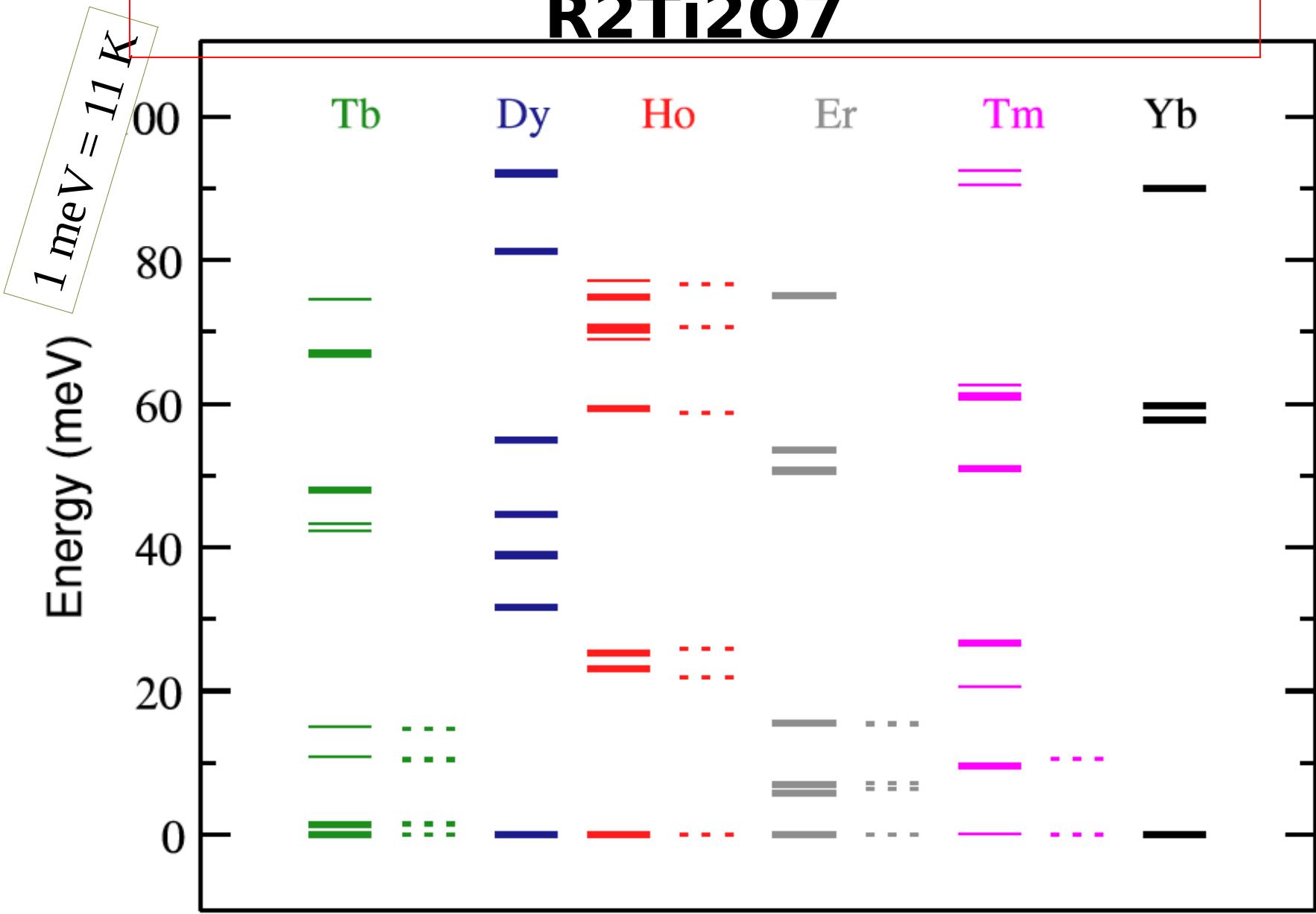
$$P = \sum_{\alpha \in P} |\alpha\rangle\langle\alpha|$$

Large denominator
compared to the energy
scale of H'

$$Q = \sum_{\beta \notin P} \frac{|\beta\rangle\langle\beta|}{E_0^\alpha - E_0^\beta}$$

$$H_{\text{eff}} = P H_{\text{int}} P + P H_{\text{int}} Q H_{\text{int}} P$$

Crystal field energy levels in R₂Ti₂O₇



Symmetry Allowed Hamiltonian on Pyrochlore Lattice

$$H = J_z \sum_{\langle i,j \rangle} S_i^z S_j^z - J_\pm \sum_{\langle i,j \rangle} (S_i^+ S_j^- + S_i^- S_j^+) + J_{z\pm} \sum_{\langle i,j \rangle} \begin{array}{|c|} \hline \text{S}_i^z \\ \hline \end{array} (\xi_{ij} S_j^+ + \xi_{ji}^* S_j^-) + i \leftrightarrow j \begin{array}{|c|} \hline \text{S}_j^z \\ \hline \end{array} + J_{\pm\pm} \sum_{\langle i,j \rangle} (\gamma_{ij} S_i^+ S_j^+ + \gamma_{ij}^* S_i^- S_j^-)$$

$S = \frac{1}{2}$ operator

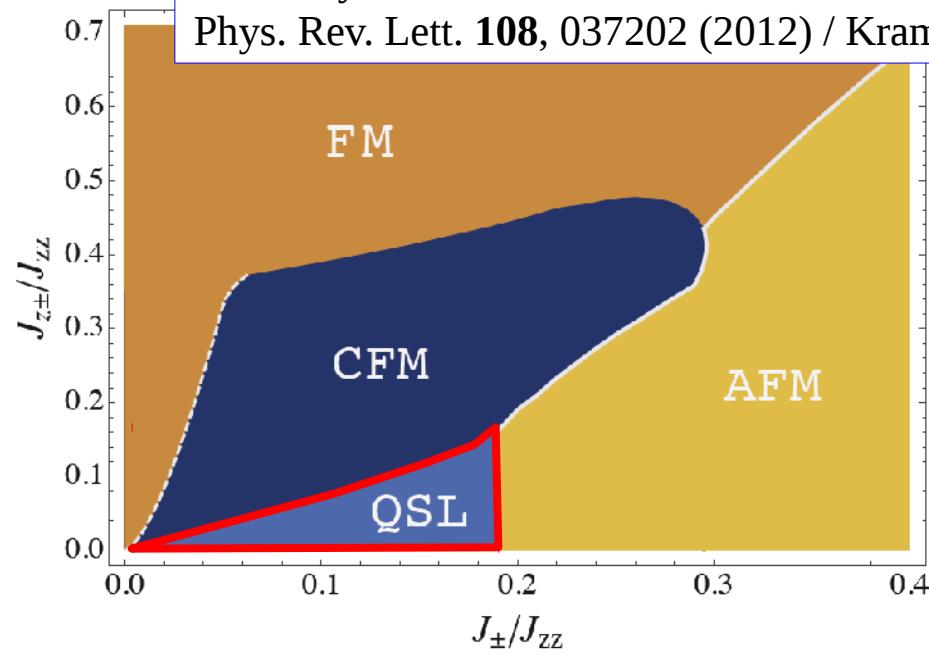
Symmetry Allowed Hamiltonian on Pyrochlore Lattice

$$H = J_{\text{Ising}} \sum_{\langle i,j \rangle} S_i^z S_j^z + J_{\text{iso}} \sum_{\langle i,j \rangle} S_i \cdot S_j + J_{\text{pd}} \sum_{\langle i,j \rangle} (S_i \cdot S_j - 3 S_i \cdot \hat{r}_{ij} \hat{r}_{ij} \cdot S_j) + J_{\text{DM}} \sum_{\langle i,j \rangle} d_{ij} \cdot S_i \times S_j$$

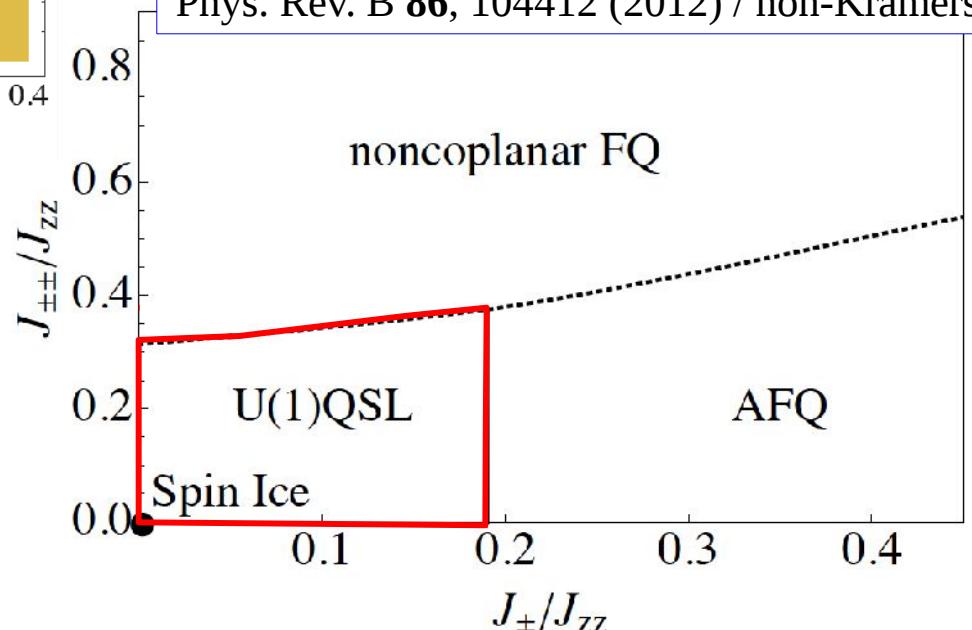
What is possible?

L. Savary and L. Balents,

Phys. Rev. Lett. **108**, 037202 (2012) / Kramers

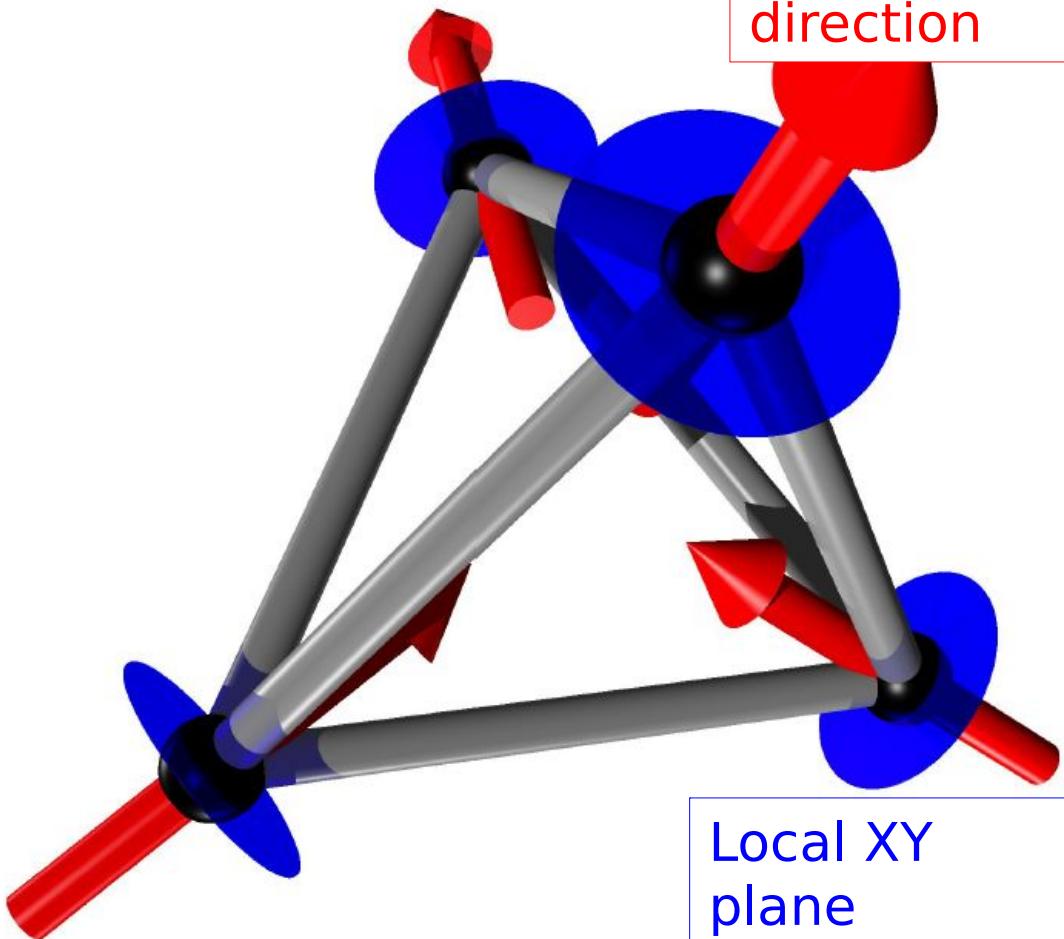


S.-B. Lee, S. Onoda, and L. Balents,
Phys. Rev. B **86**, 104412 (2012) / non-Kramers



Review: “Quantum Spin Ice: A Search for Gapless Quantum Spin Liquids in Pyrochlore Magnets”

P.A. McClarty and M.J.P. Gingras, arXiv:1311.1817



Ising

Ho₂Ti₂O₇
Dy₂Ti₂O₇
Tb₂Ti₂O₇

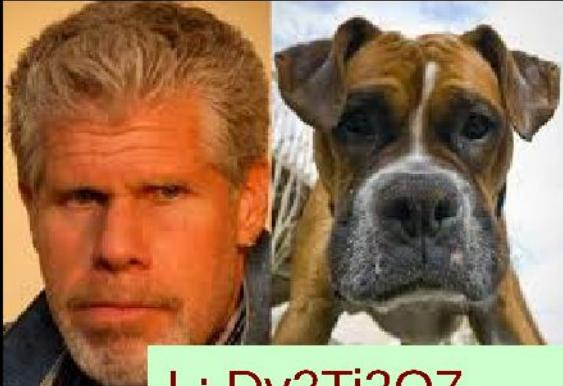
XY

Yb₂Ti₂O₇
Er₂Ti₂O₇
Gd₂Ti₂O₇

R₂Ti₂O₇ vs R₂Sn₂O₇



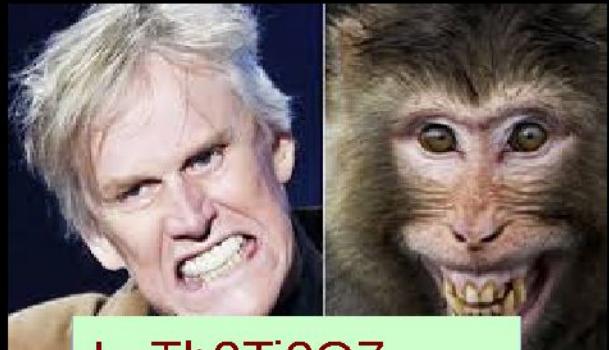
L: Ho₂Ti₂O₇
R: Ho₂Sn₂O₇



L: Dy₂Ti₂O₇
R: Dy₂Sn₂O₇



Iain Dowie
L: Pr₂Sn₂O₇
R: Pr₂Zr₂O₇



L: Tb₂Ti₂O₇
R: Tb₂Sn₂O₇

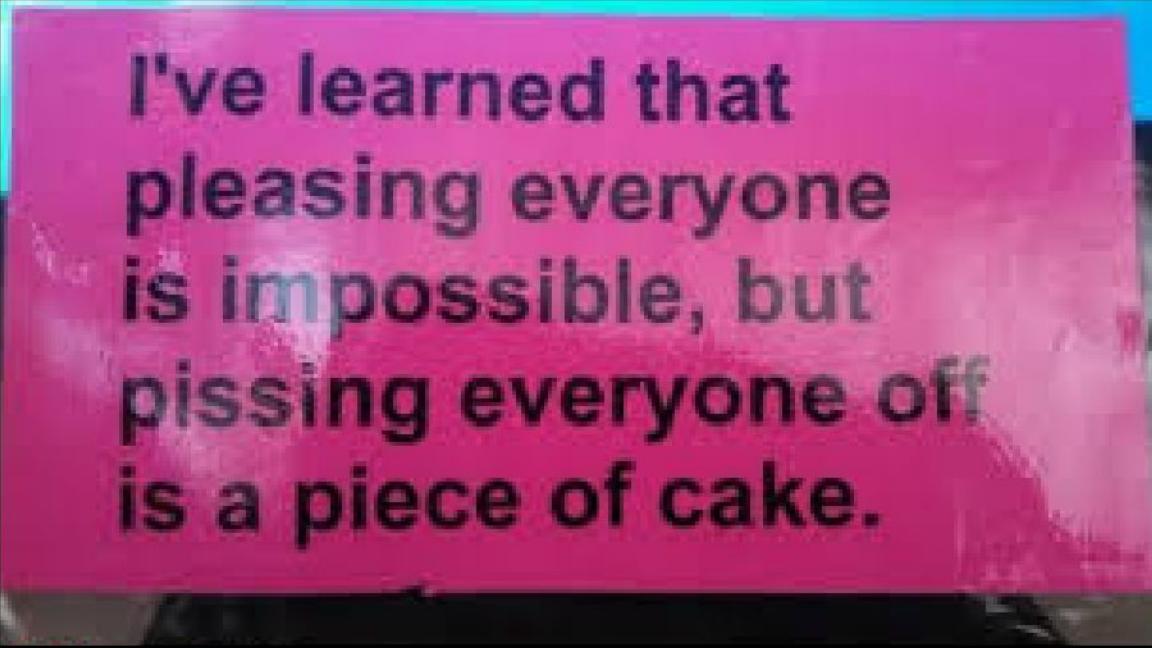


L: Yb₂Sn₂O₇
R: Yb₂Ti₂O₇



L: Gd₂Ti₂O₇
R: Gd₂Sn₂O₇

RE: on references of other people's work
provided henceforth



I've learned that
pleasing everyone
is impossible, but
pissing everyone off
is a piece of cake.

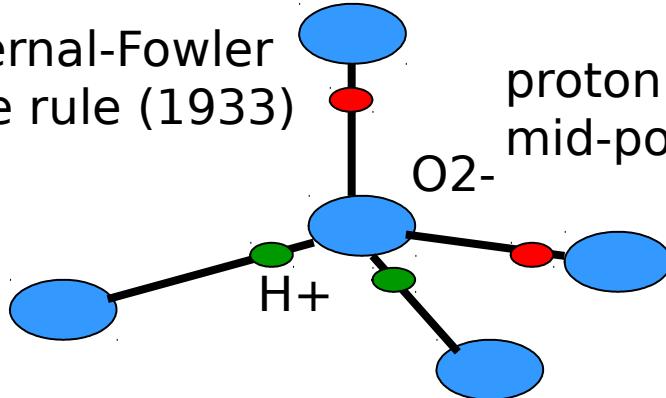
So, in order to be fair, I'll try to p... o... everybody
equally and endeavour to refer as much as possible
only to my own work.

$(\text{Dy},\text{Ho})_2(\text{Ti},\text{Sn},\text{Ge})_2\text{O}_7$

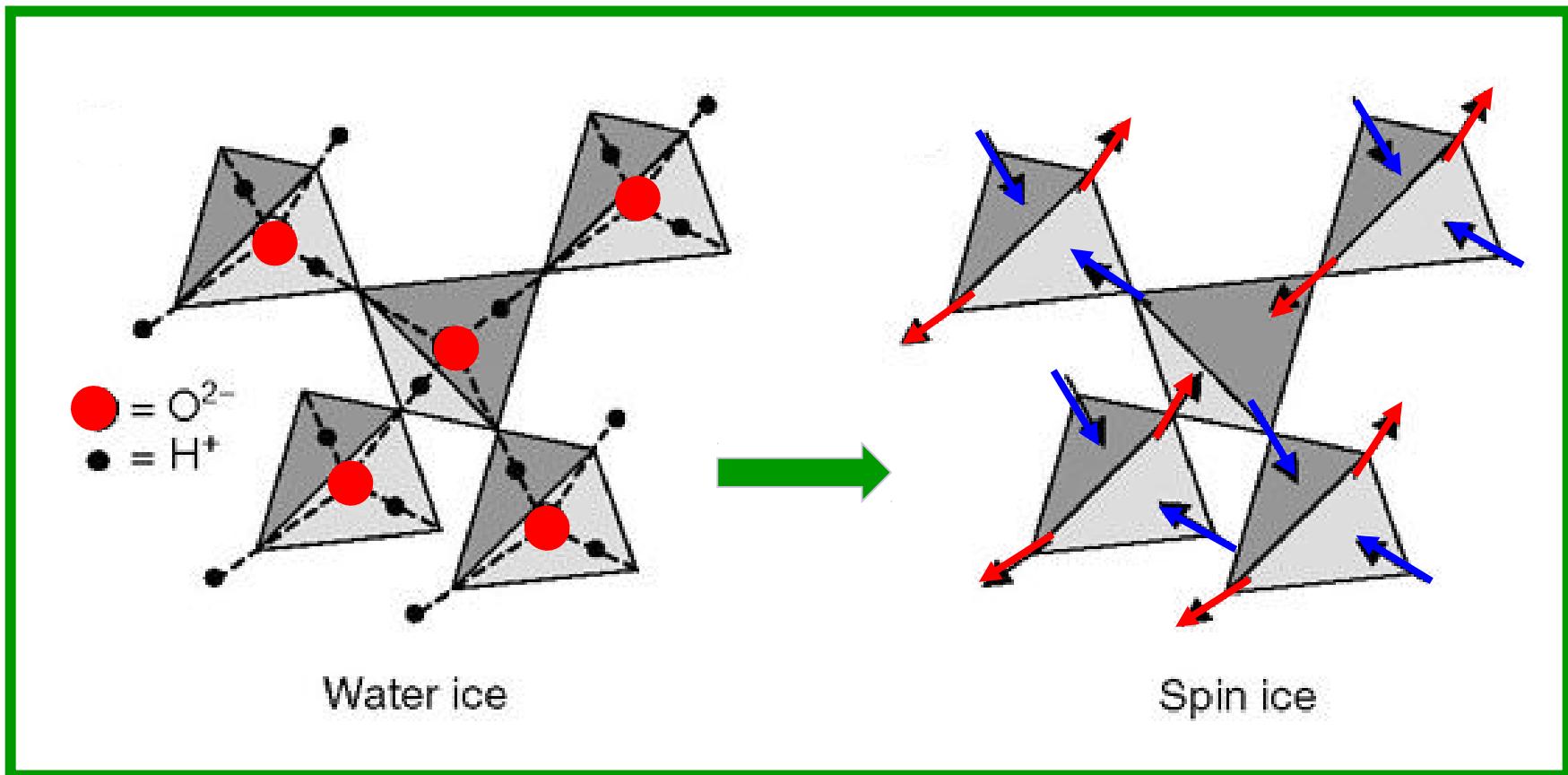
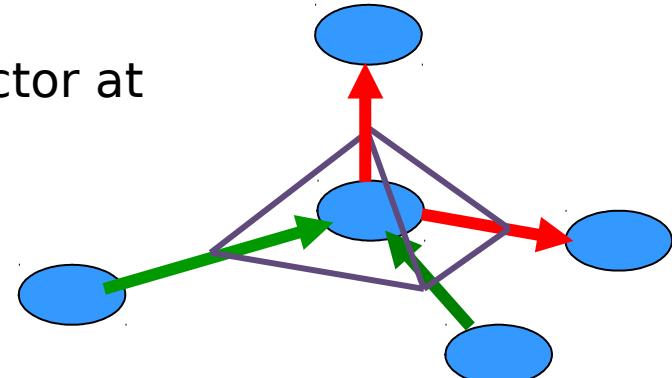
Phenomenology:

Classical dipolar spin ices

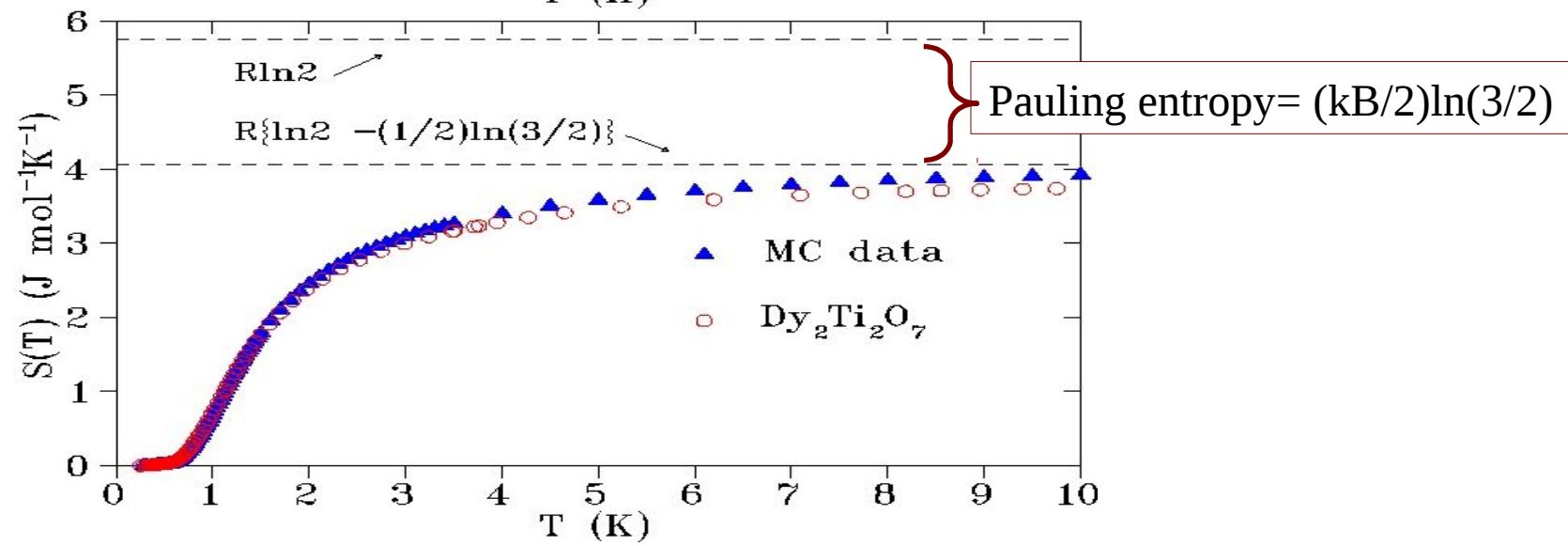
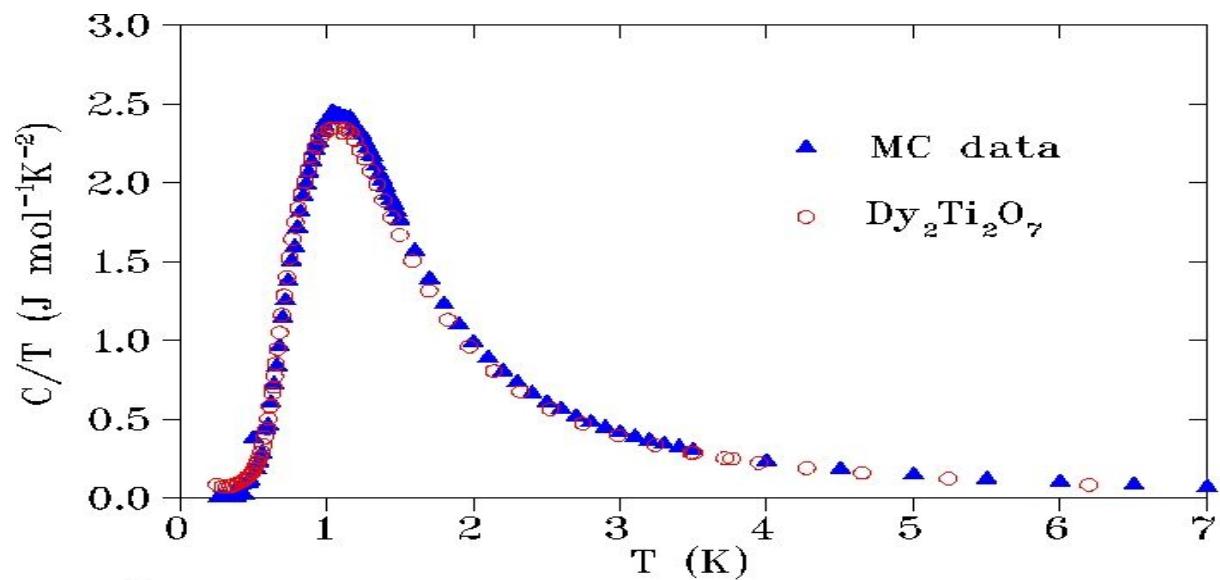
Bernal-Fowler
ice rule (1933)



proton displacement: vector at
mid-point

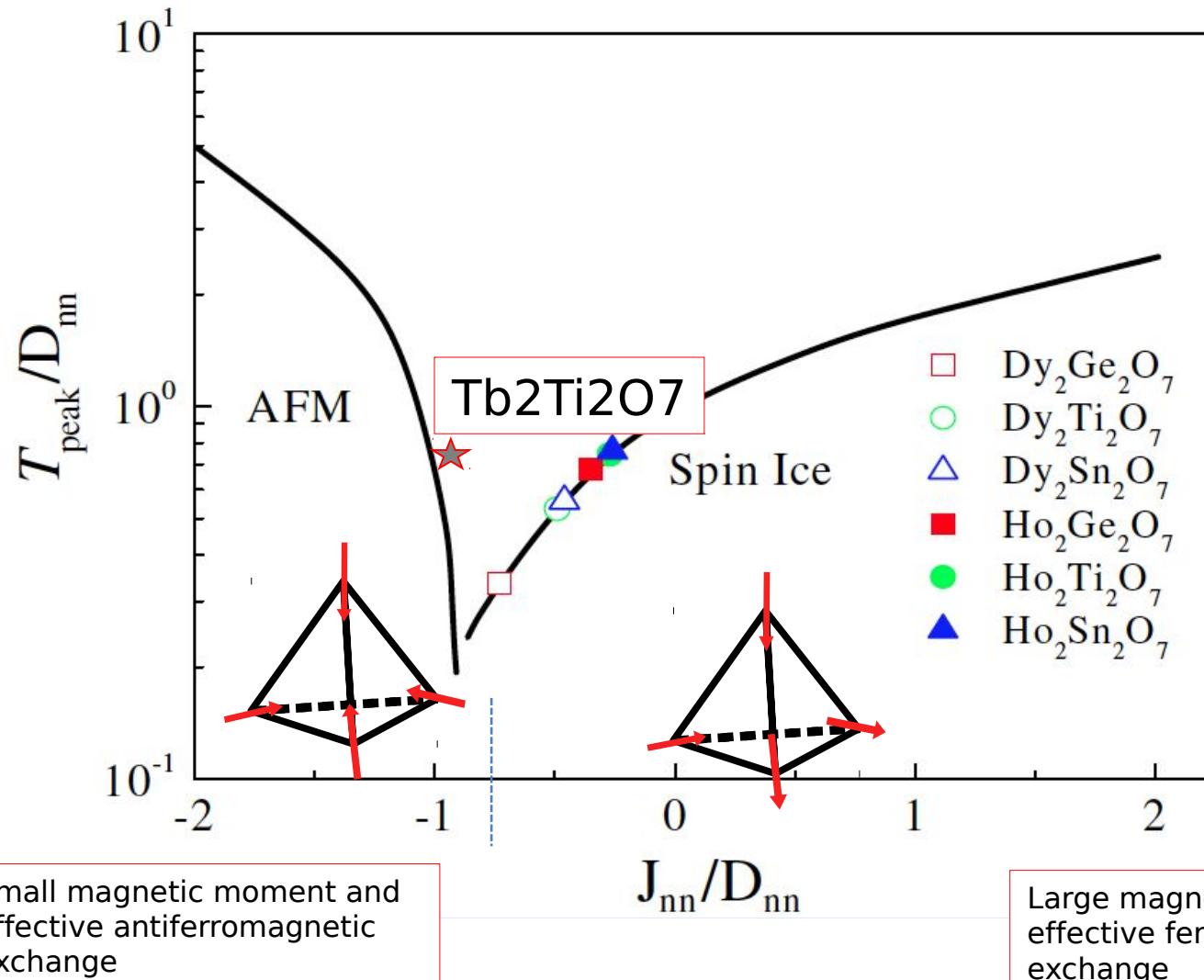


Real materials show manifestations of Pauling's ground state entropy magnetic analogues of water ice



Chemical Pressure Effects on Pyrochlore Spin Ice

H. D. Zhou,^{1,*} J. G. Cheng,² A. M. Hallas,³ C. R. Wiebe,^{1,3,4} G. Li,¹ L. Balicas,¹ J. S. Zhou,² J. B. Goodenough,² J. S. Gardner,^{5,6} and E. S. Choi¹



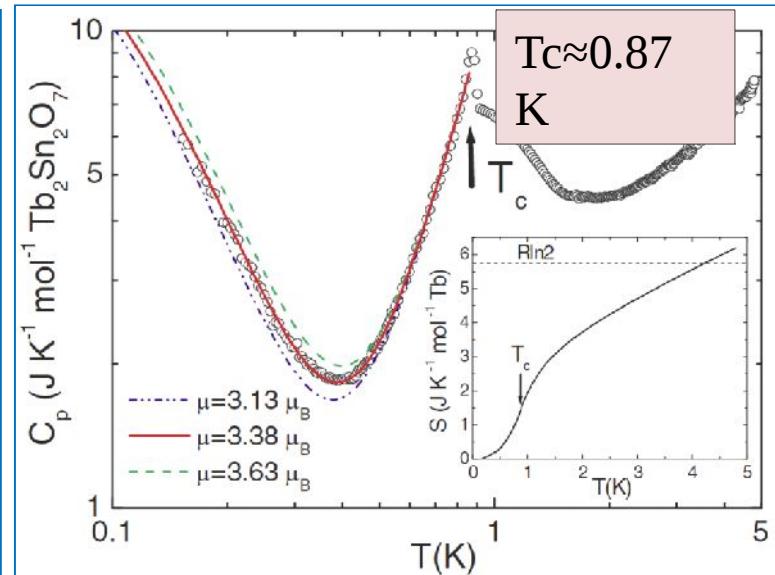
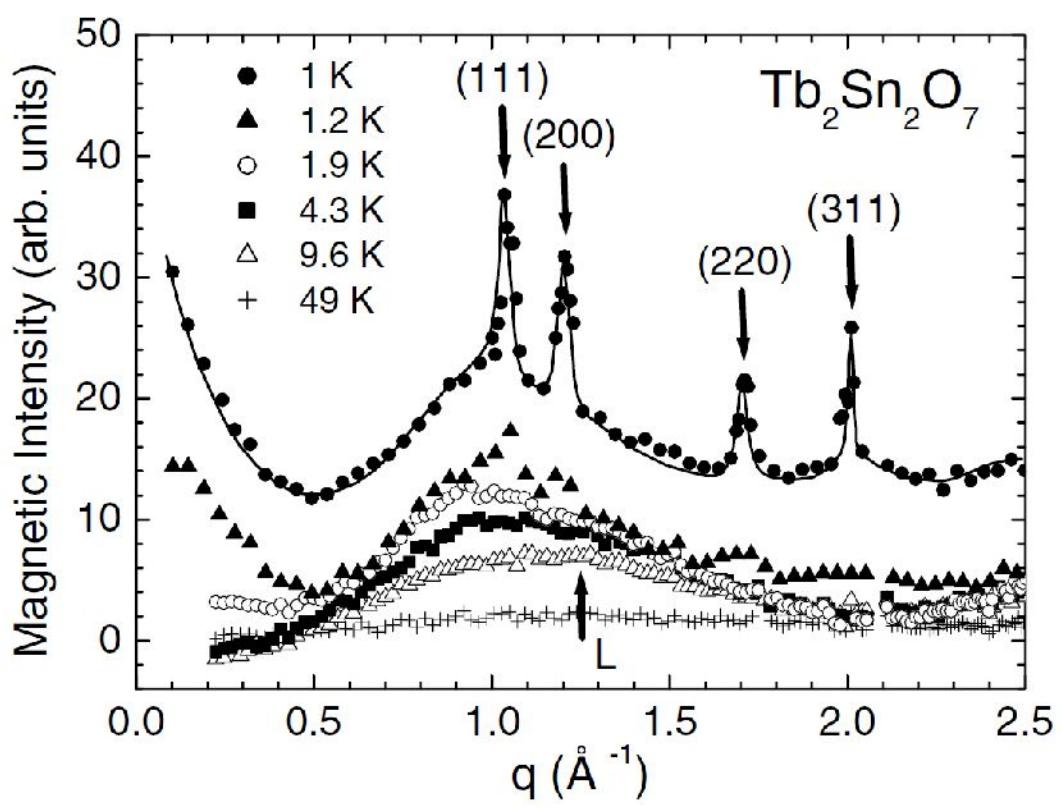
Tb₂Ti₂O₇ and Tb₂Sn₂O₇

Phenomenology:

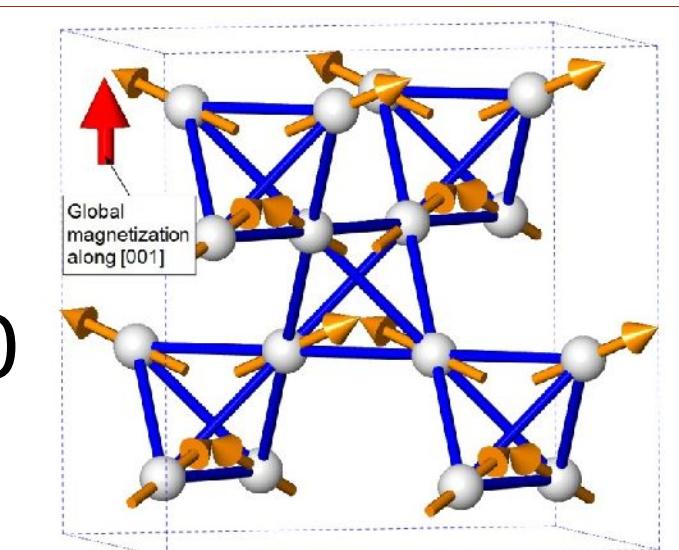


Well, it's a mess alright!

Meanwhile: $Tb_2Sn_2O_7$ has a long range ordered spin ice



$$q=000$$



"Ordered Spin Ice State and Magnetic Fluctuations in $Tb_2Sn_2O_7$ ";
Mirebeau *et al.* Phys. Rev. Lett. **94**, 246402 (2005).

Dynamically Induced Frustration as a Route to a Quantum Spin Ice State in $\text{Tb}_2\text{Ti}_2\text{O}_7$ via Virtual Crystal Field Excitations and Quantum Many-Body Effects

Hamid R. Molavian,¹ Michel J. P. Gingras,^{1,2} and Benjamin Canals^{1,3}

PRL 109, 017201 (2012)

PHYSICAL REVIEW LETTERS

week ending
6 JULY 2012

Power-Law Spin Correlations in the Pyrochlore Antiferromagnet $\text{Tb}_2\text{Ti}_2\text{O}_7$

T. Fennell,^{1,*} M. Kenzelmann,² B. Roessli,¹ M. K. Haas,³ and R. J. Cava³

PHYSICAL REVIEW B 87, 094410 (2013)

Antiferromagnetic spin ice correlations at $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ in the ground state
of the pyrochlore magnet $\text{Tb}_2\text{Ti}_2\text{O}_7$

K. Fritsch,¹ K. A. Ross,^{1,2,3} Y. Qiu,^{3,4} J. R. D. Copley,³ T. Guidi,⁵ R. I. Bewley,⁵ H. A. Dabkowska,⁶ and B. D. Gaulin^{1,6,7}

PRL 111, 087201 (2013)

PHYSICAL REVIEW LETTERS

week ending
23 AUGUST 2013

Anisotropic Propagating Excitations and Quadrupolar Effects in $\text{Tb}_2\text{Ti}_2\text{O}_7$

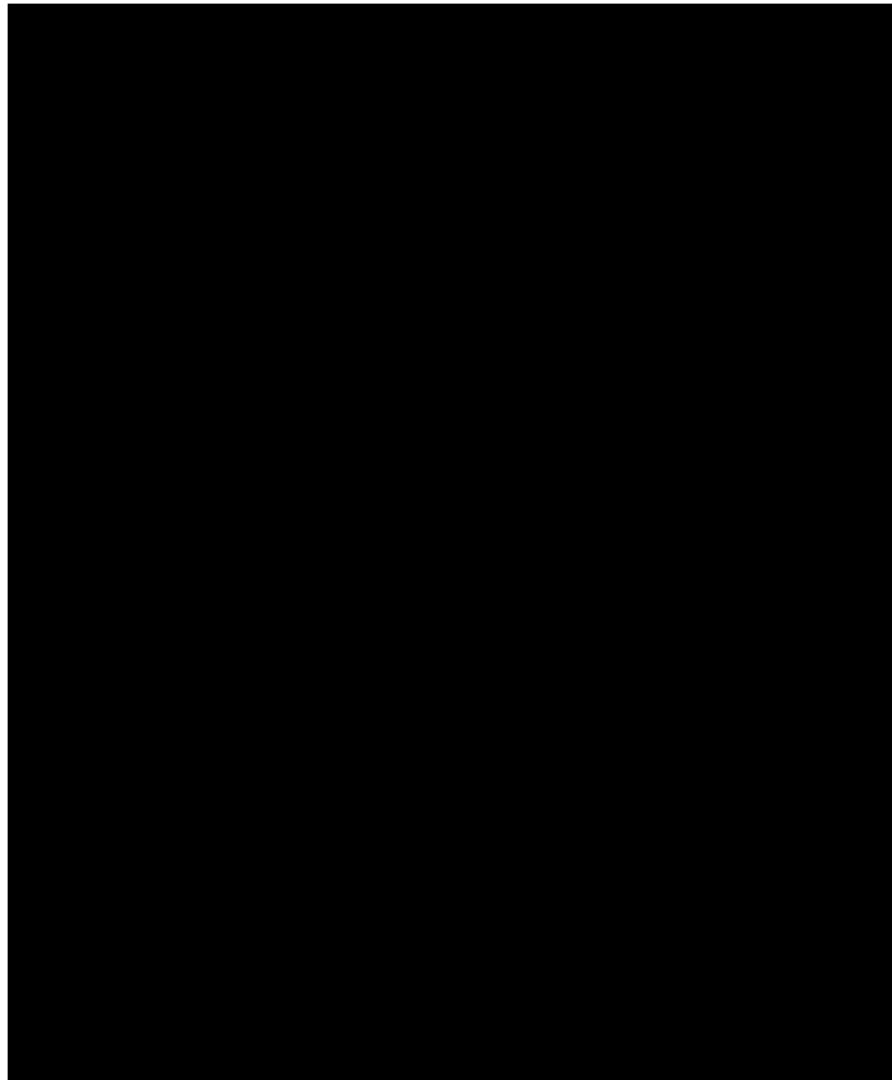
Solène Guitteny,¹ Julien Robert,¹ Pierre Bonville,² Jacques Ollivier,⁴ Claudia Decorse,³ Paul Steffens,⁴
Martin Boehm,⁴ Hannu Mutka,⁴ Isabelle Mirebeau,¹ and Sylvain Petit¹

- Evidence for complex correlations, both in the dynamics and quasi-static
- Evidence for development of extended correlations with peaks at $(1/2, 1/2, 1/2)$
- Evidence for concomitant suggesting quantum spin-ice correlations.

Long-range order and spin-liquid states of polycrystalline $Tb_{2+x}Ti_{2-x}O_{7+y}$

T. Taniguchi,¹ H. Kadokami,¹ H. Takatsu,¹ B. Fák,² J. Ollivier,³ T. Yamazaki,⁴ T. J. Sato,⁵ H. Yoshizawa,⁵ Y. Shimura,⁴ T. Sakakibara,⁴ T. Hong,⁶ K. Goto,¹ L. R. Yaraskavitch,⁷ and J. B. Kycia⁷

But...
evidence for extreme
sample sensitivity





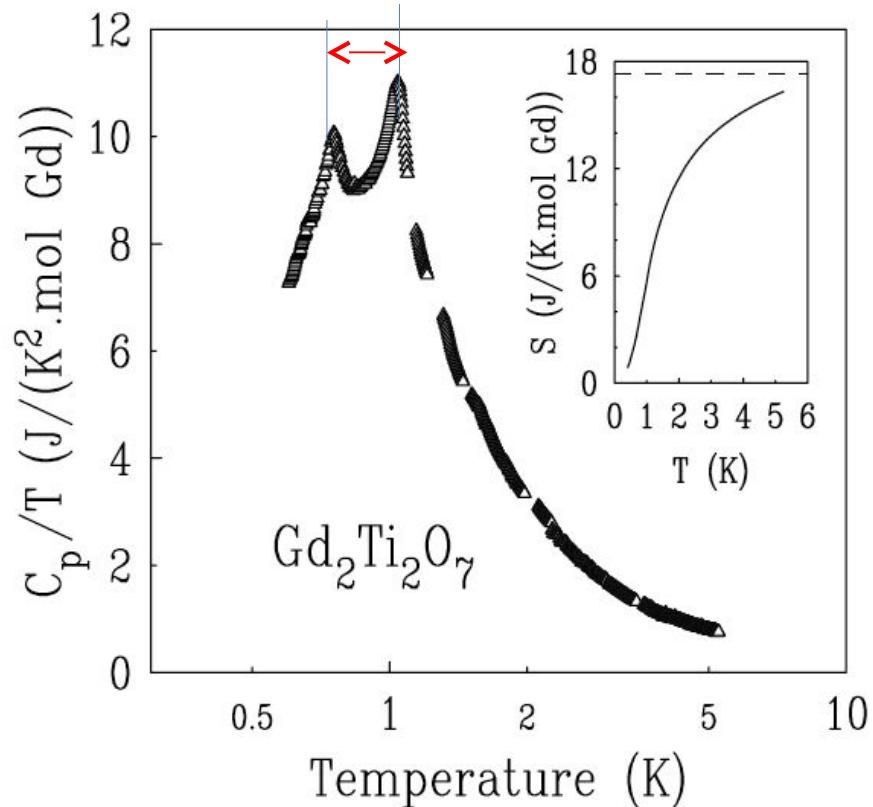
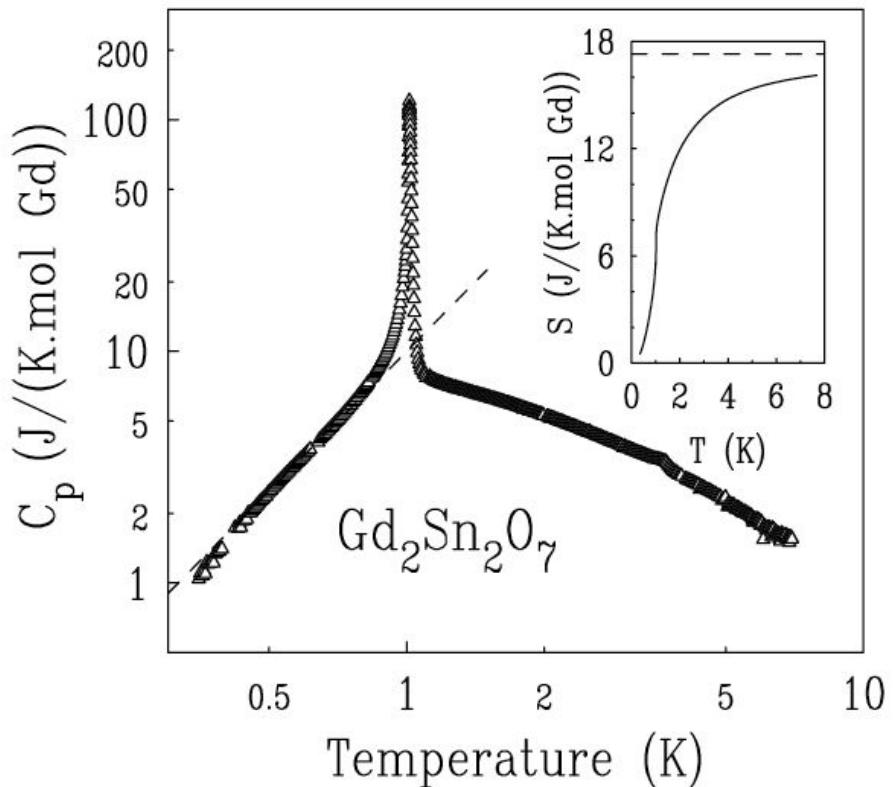
Phenomenology:

- $Gd_2Sn_2O_7$: “simple” long-range order.
[Phys. Rev. Lett. **99**, 097201(2007)]
- $Gd_2Ti_2O_7$:
 - Multiple phase transitions
 - Partially ordered intermediate $4-k$ state
 - Order-by-disorder at T_c
 - Unusual criticality at T_c

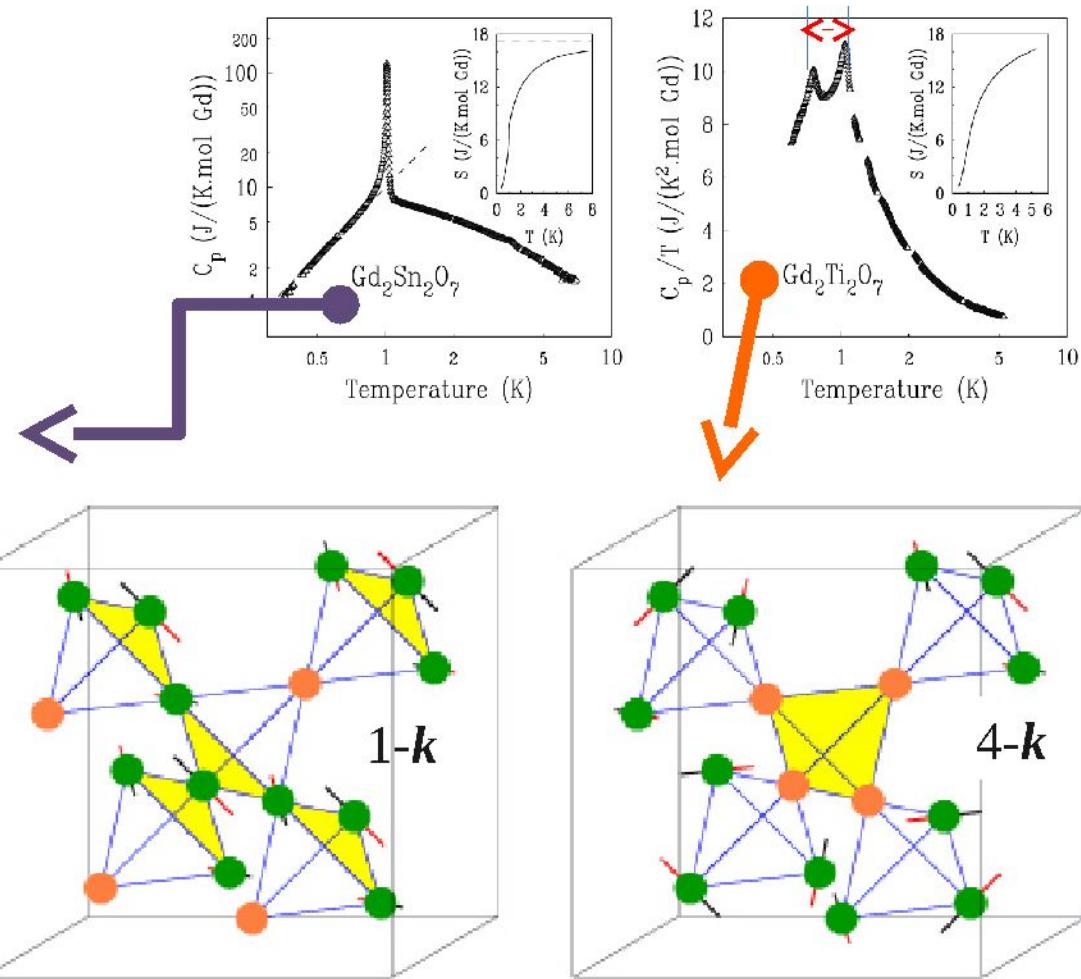
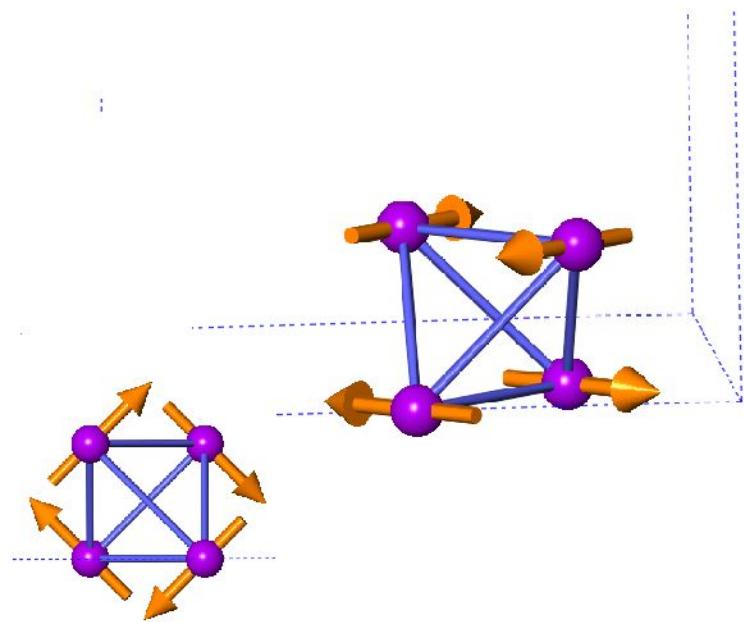
[arXiv:1310.5146](https://arxiv.org/abs/1310.5146); [B. Javanparast](#), [Z. Hao](#), [M. Enjalran](#),
[M. J. P. Gingras](#)

“Fluctuation-Driven Selection at Criticality in a Frustrated
Magnetic System:
the Case of Multiple- k Partial Order on the Pyrochlore Lattice”

Gd₂Ti₂O₇ vs Gd₂Sn₂O₇



Gd₂Ti₂O₇ vs Gd₂Sn₂O₇



Palmer & Chalker.
Phys Rev B **62**, 488
(2000)

[arXiv:1310.5146](https://arxiv.org/abs/1310.5146); [B. Javanparast](#), [Z. Hao](#), [M. Enjalran](#), [M. J. P. Gingras](#)
“Fluctuation-Driven Selection at Criticality in a Frustrated Magnetic System:
the Case of Multiple- k Partial Order on the Pyrochlore Lattice”

Er₂Ti₂O₇ and Er₂Sn₂O₇

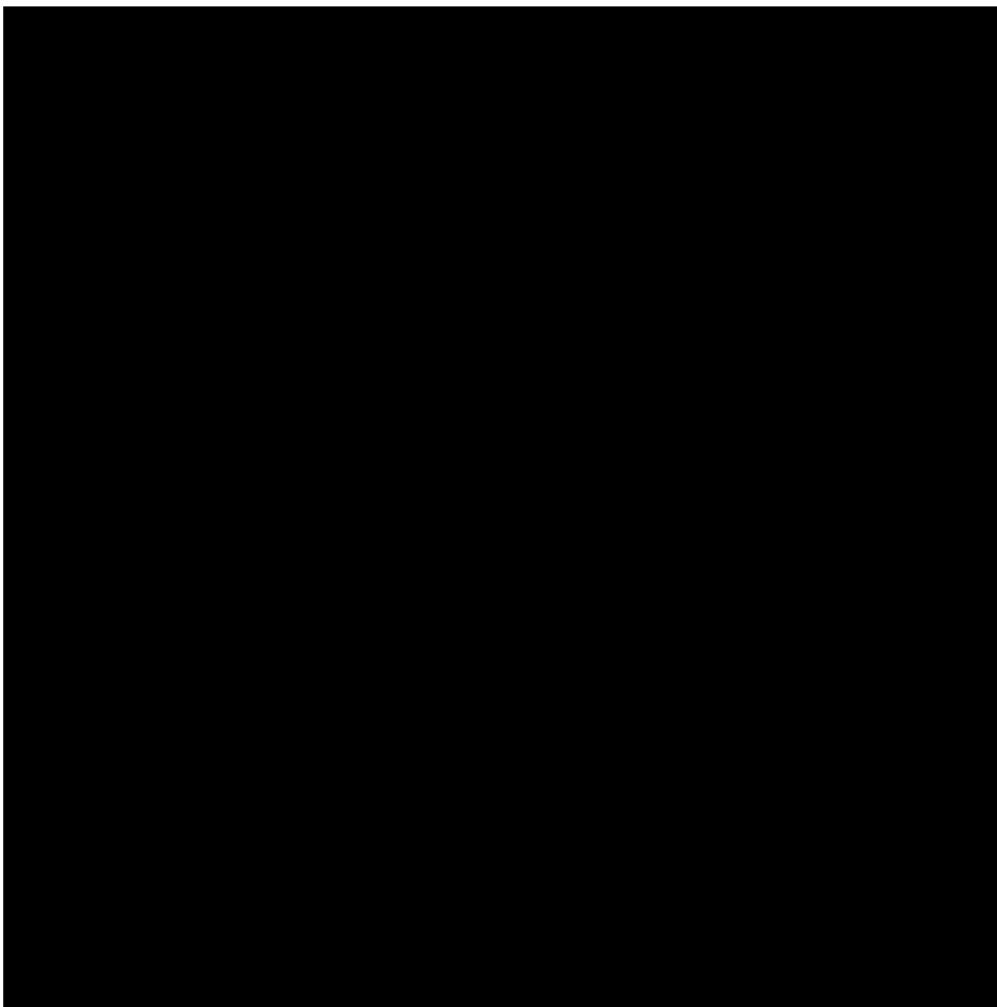
Phenomenology:

- Er₂Ti₂O₇: most likely concurrent quantum & thermal *order-by-disorder*
 - [Quantum Order by Disorder and Accidental Soft Modes](#)
V. Gvozdkova, P. C. W. Holdsworth, and R. Moessner, (2012)
 - [Order by Quantum Disorder in Er₂Ti₂O₇](#) Lucile Savary, Kate A. Ross, Bruce D. Gaulin, Jacob P. C. Ruff, and Leon Balents, Phys. Rev. Lett. **109**, 167201 (2012)
 - [Ground state phase diagram of generic XY pyrochlore magnets with quantum fluctuations](#), Anson W. C. Wong, Zhihao Hao, and Michel J.P. Gingras Phys. Rev. B **88**, 144402 (2013)
 - [Phase transition and thermal order-by-disorder in the pyrochlore quantum antiferromagnet Er₂Ti₂O₇](#), J. Oitmaa, R.R.P. Singh, B. Javanparast, A.G.R. Day, B.V. Bagheri, M.J.P. Gingras; arXiv:1305.2935 (to appear in PRB/RC)
 - [Living on the edge: ground-state selection in quantum spin-ice pyrochlores](#), H. Yan, O. Benton, L..D.C. Jaubert, N. Shannon, arXiv:1311.3501
- Er₂Sn₂O₇: local short-range order akin to Gd₂Sn₂O₇, but not true long-range order down to 100mK
 - [Palmer-Chalker correlations in the XY pyrochlore antiferromagnet Er₂Sn₂O₇](#)
S. Guitteny, S. Petit, E. Lhotel, J. Robert, P. Bonville, A. Forget, and I. Mirebeau; Phys. Rev. B **88**, 134408 (2013)

See Mike hitomirsky's talk this afternoon

$\text{Yb}_2\text{Ti}_2\text{O}_7$ and $\text{Yb}_2\text{Sn}_2\text{O}_7$

Phenomenology:



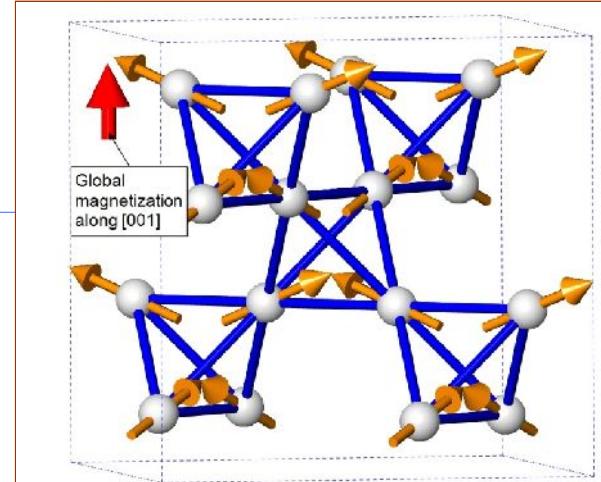
$\text{Yb}_2\text{Ti}_2\text{O}_7$ and $\text{Yb}_2\text{Sn}_2\text{O}_7$

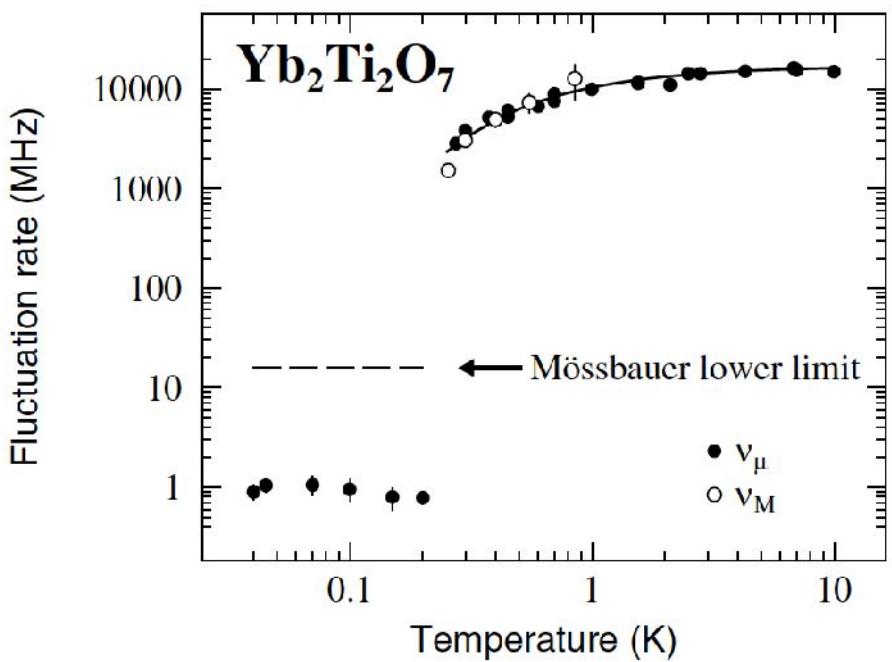
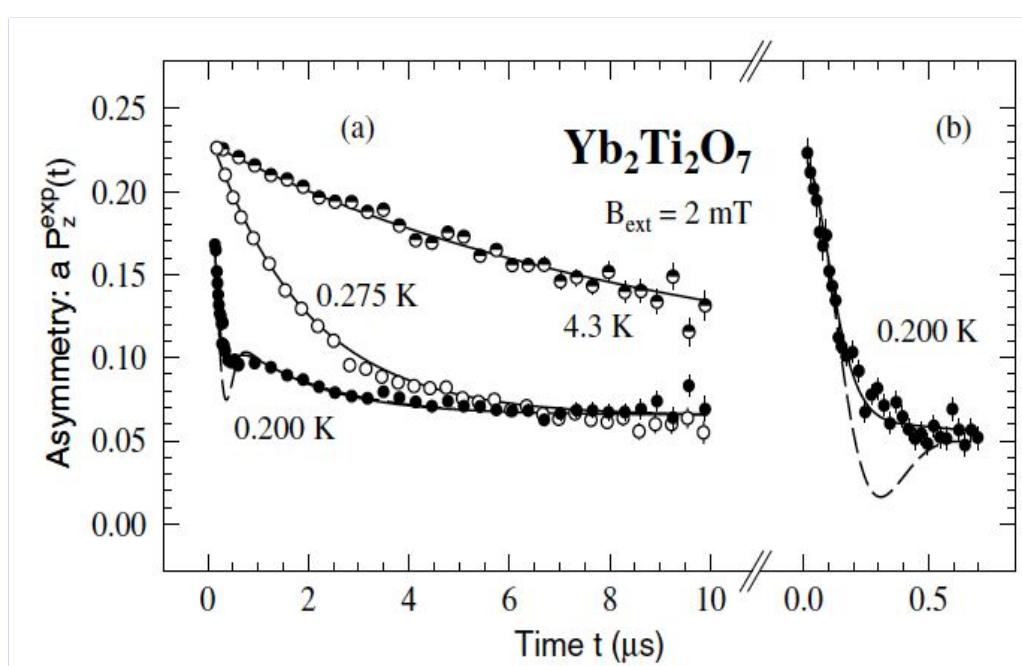
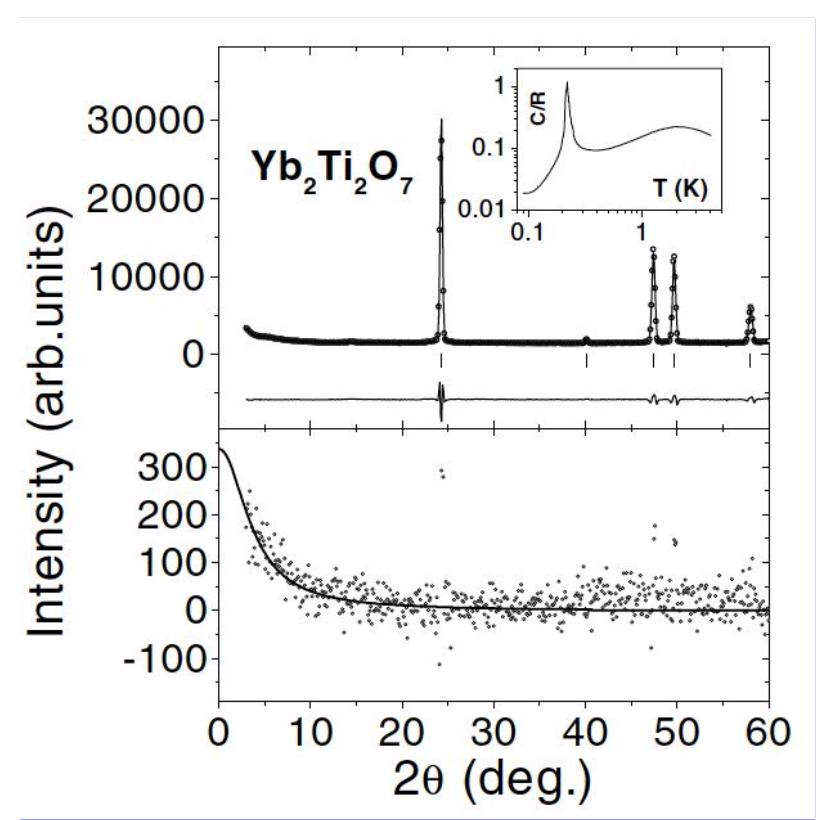
Phenomenology:

- $\text{Yb}_2\text{Sn}_2\text{O}_7$: seems to be a canted (FM) (akin to $\text{Tb}_2\text{Sn}_2\text{O}_7$)
 - [Dynamical Splayed Ferromagnetic Ground State in the Quantum Spin Ice \$\text{Yb}_2\text{Sn}_2\text{O}_7\$](#) , A. Yaouanc *et al.* Phys. Rev. Lett. **110**, 127207 (2013).
- $\text{Yb}_2\text{Ti}_2\text{O}_7$: no agreement yet
 - Long-range (ferrimagnetic) order at $T < \sim 240$ mK (Yasui *et al.* JPSJ, Chang *et al.* Nat. Comm.)
 - No transition down to 20 mK (perhaps related to U(1) QSL of a quantum spin ice?)

Most recent experimental work (?) is:

[Unconventional magnetic ground state in \$\text{Yb}_2\text{Ti}_2\text{O}_7\$](#) R. M. D'Ortenzio *et al.* Phys. Rev. B **88**, 134428 (31 October 2013) / See references therein.





Very confused about Yb₂Ti₂O₇

It would seem that we know the effective $S=1/2$ Hamiltonian (from inelastic neutron scattering in strong field – Ross *et al.* Phys

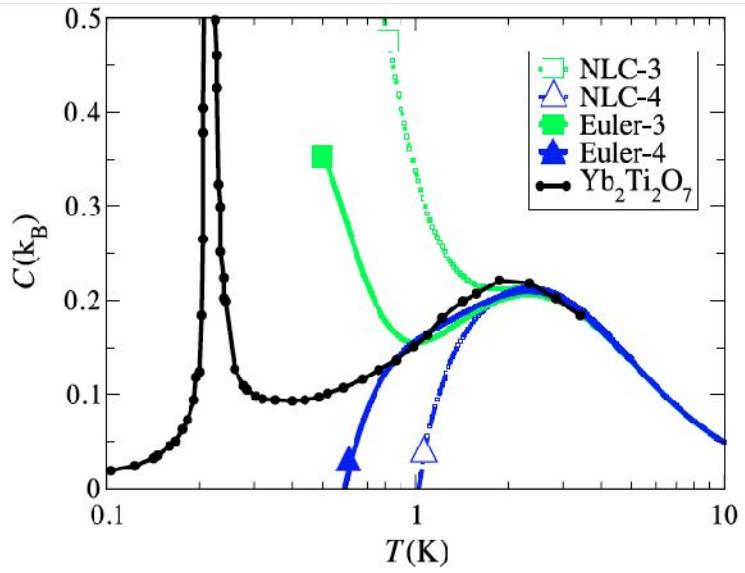
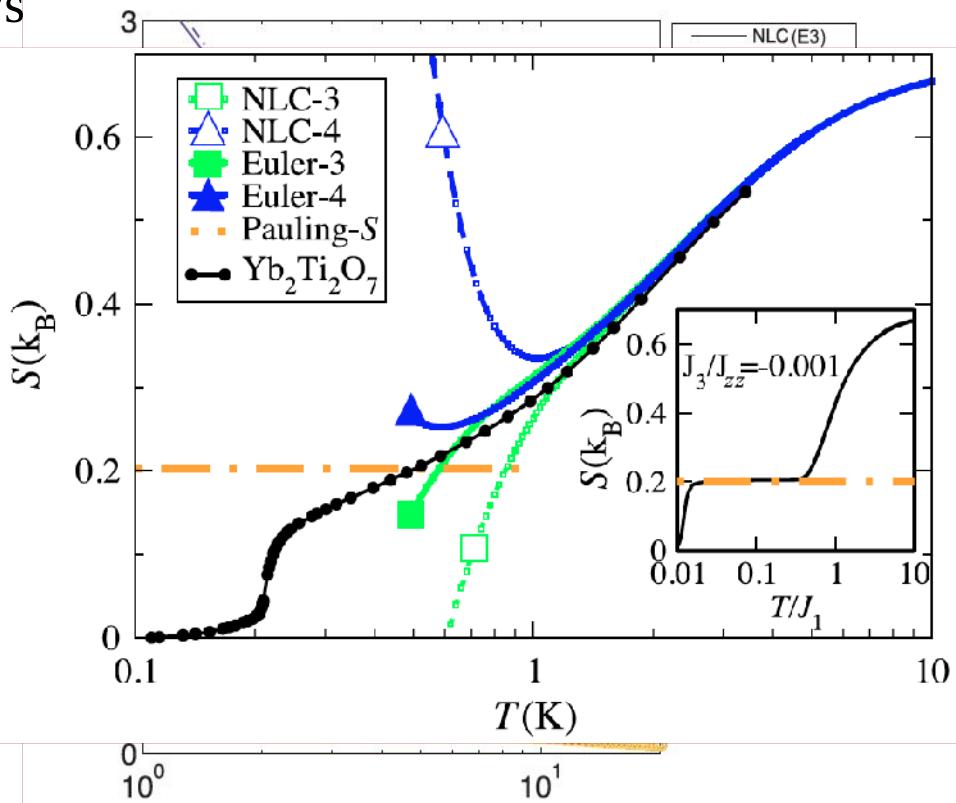
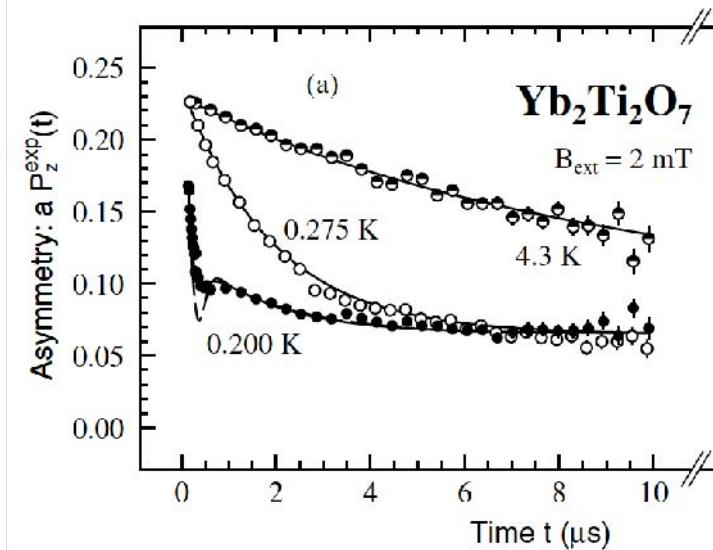
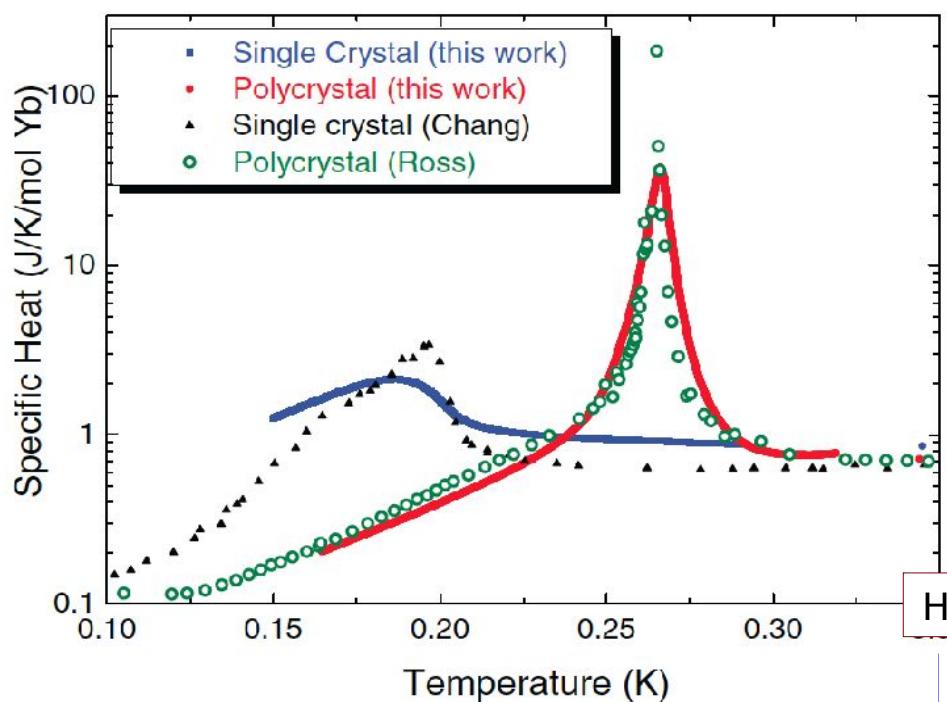


FIG. 2 (color online). Heat capacity $C(T)$ per mole of Yb for the model parameters in Ref. [16], in units of the Boltzmann constant k_B , calculated via NLC (up to the fourth order NLC together with Euler extrapolations) are compared with experimental data for $\text{Yb}_2\text{Ti}_2\text{O}_7$. The filled small black circles are data from Ref. [25].

[Vindication of Yb₂Ti₂O₇ as a Model Exchange Quantum Applegate, Hayre, Singh, Lin, Day and Gingras Phys. Rev. Lett. **109**, 097205 \(2012\).](#)



[Thermodynamic properties of Yb₂Ti₂O₇ pyrochlore as a function of temperature and magnetic field: Validation of a quantum spin ice exchange Hamiltonian; Hayre, Ross, Applegate, Lin, Singh, Gaulin and Gingras, Phys. Rev. B **87**, 184423 \(2013\).](#)



Hodges *et al.* Phys. Rev. Lett. **88**, 077204 (2002)

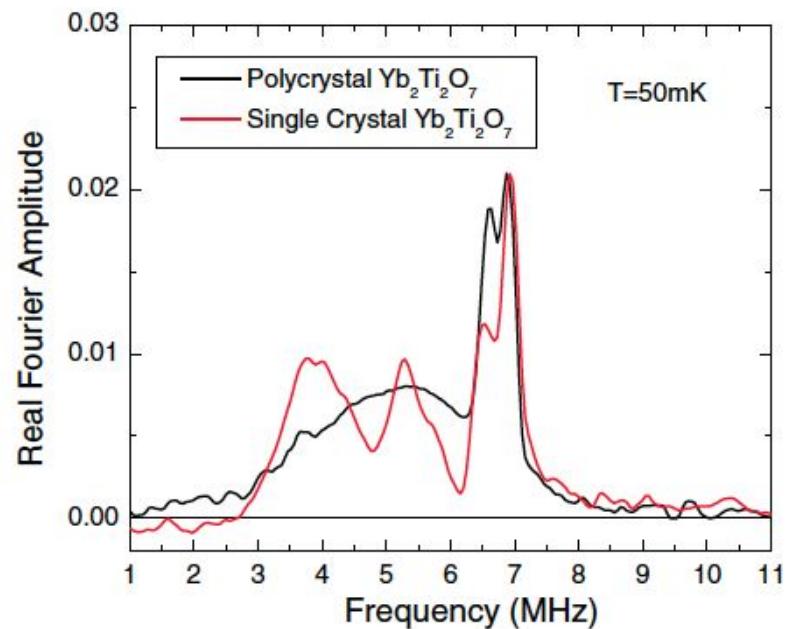
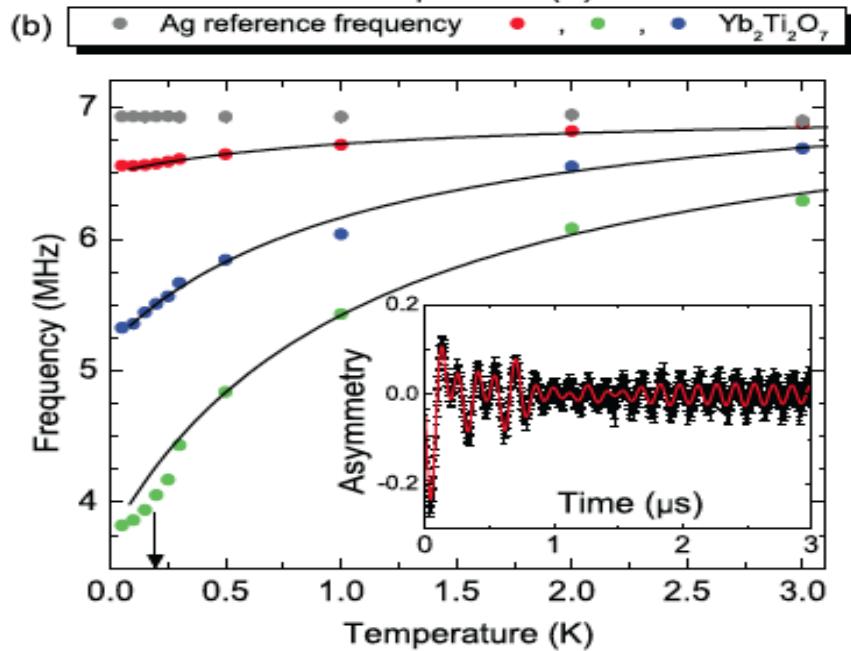
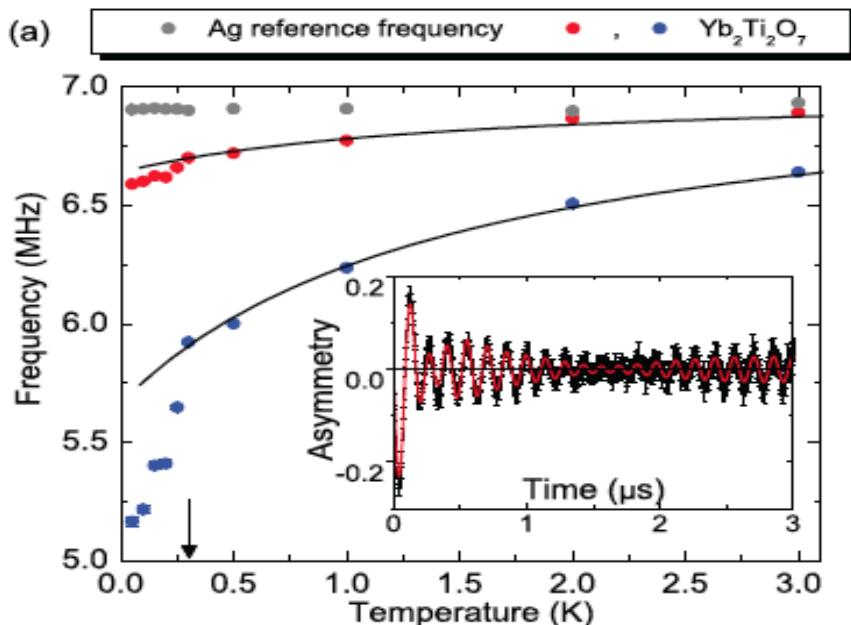
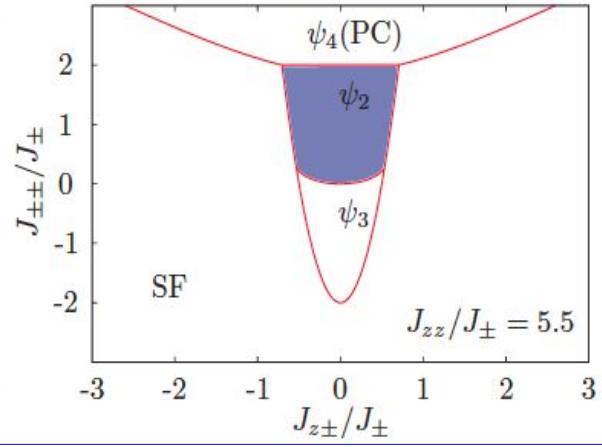
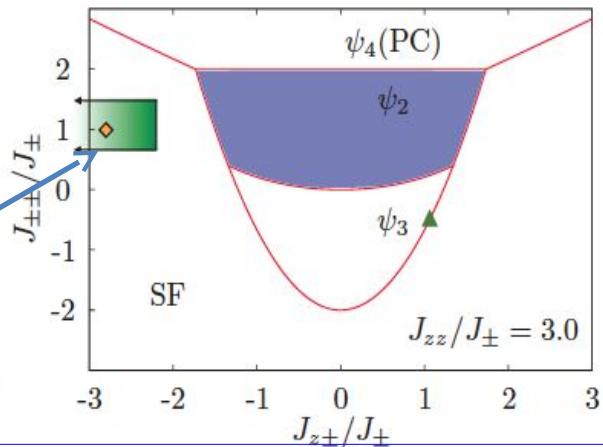
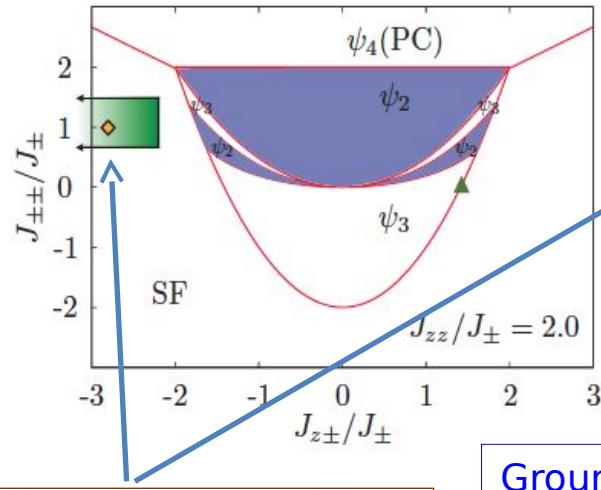
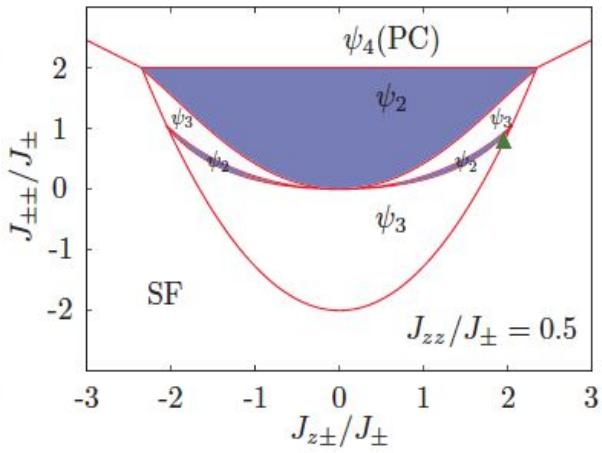
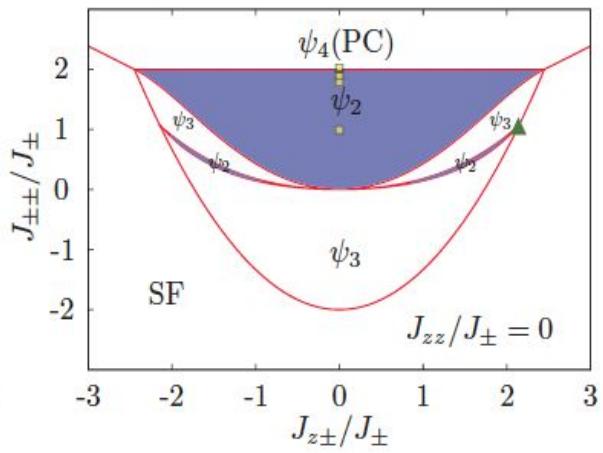
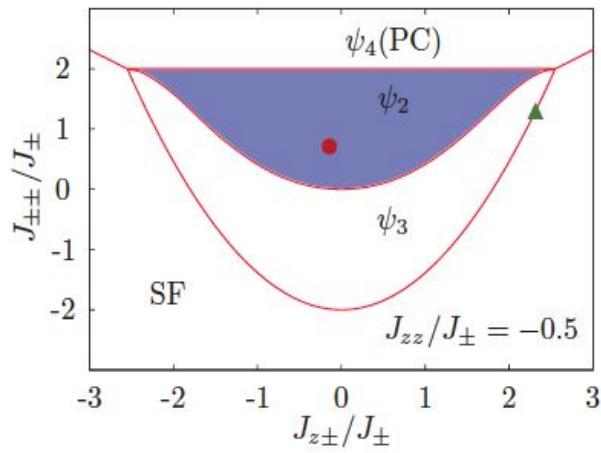


FIG. 3. (Color online) Fourier transform of μ SR asymmetry spectra in a transverse field of 50 mT at $T = 50$ mK. The single-crystal $\text{Yb}_2\text{Ti}_2\text{O}_7$ shows four resolvable frequencies (red) and the polycrystalline $\text{Yb}_2\text{Ti}_2\text{O}_7$ shows three (black). This motivates $n = 4$ for the single-crystal and $n = 3$ in the polycrystalline $\text{Yb}_2\text{Ti}_2\text{O}_7$ for Eq. (1). The sharp signal at approximately 7 MHz for both samples reflects the precession of the muons in the Ag cryostat tails.



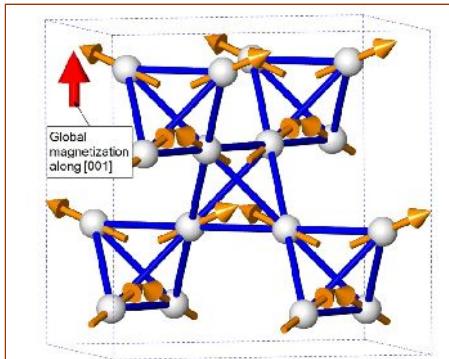
Ground state phase diagram of generic XY

pyrochlore magnets with quantum fluctuations, A.W.C. Wong, Zhihao Hao, and M.J.P. Gingras,
Phys. Rev. B **88**, 144402 (2013)

BUT DIFFERENT POINT OF VIEW IN:

Living

on the edge: ground-state selection in quantum spin-ice pyrochlores,
H. Yan, O. Benton, L.D.C. Jaubert, N. Shannon, arXiv:1311.3501



Conclusion

- Rare earth pyrochlore oxides display a smorgasbord of phenomena
- Much has been understood in the past 2 years (in particular for $\text{Er}_2\text{Ti}_2\text{O}_7$, perhaps for $\text{Gd}_2\text{Ti}_2\text{O}_7$ as well).
- Confusion remains as per $\text{Tb}_2\text{Ti}_2\text{O}_7$ & $\text{Yb}_2\text{Ti}_2\text{O}_7$
 - These two compounds may be related to a *quantum spin ice state* – this is a most exciting prospect. Namely a discovery a $\text{U}(1)$ quantum spin liquid. This remains to be established beyond doubt (in $\text{Pr}_2(\text{Sn},\text{Zr})_2\text{O}_7$ as well)
 - Both materials display extreme sample variation sensitivity (at least in image furnace grown single-crystals)
- So, more experiments & more theory is needed
- A definite resolution of “all” significant problems presented by insulating $\text{R}_2\text{M}_2\text{O}_7$ systems may be reached within a foreseeable future

Morale de l'histoire ...

- “These rare-earth systems are of no-interest in the search for exotic quantum states of matter (e.g. quantum spin liquid) because $J \square 1/2$ ”
- Not so... What matters is the existence and details of the effective spin dynamics/algebra in the low-energy sector.